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A Simple Picture of the Binding Energies of H^3 and He^5

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It is shown that an approximate application of Wheeler's method of resonating groups to the nuclei, H^3 and He^5, makes it possible to evaluate the magnitude of different contributions to the total energy. The energy expressions are separated into terms representing the internal energy of a favorable sub-group, the kinetic energy of a particle outside the sub-group, and its "ordinary" and "velocity dependent" interaction with the sub-group. For H^3 the sub-group is the deuteron and for He^5 the α-particle. The results indicate considerable polarization of the deuteron and little polarization of the α-particle. The total energies found were $E(H^3)=-6.58$ mMU and $E(He^5)=-19.51$ mMU (for the most reasonable value of one parameter since no true minimum was found for He^5). These results are compared with those of other methods. The current two-body exchange forces were used (see Table I). The approximate method is quick and easy to handle. Application of the complete resonating group method to H^3, however, proved to be very laborious and difficult to make accurate. By this procedure $E(H^3)$ came out -6.68 mMU. The smallness of the improvement over the approximate method is attributed to the fact that the complete method does not take polarization into account. The calculated ground state energy for He^3 was -5.88 mMU by the approximate method and -5.94 mMU by the complete method. No bound excited state was found.

1. INTRODUCTION

In the solution of the problem of atomic energies it is possible to evaluate the separate contributions to the total energy coming from the interaction of the different electrons with the nucleus and with each other. In contrast to the atomic results, calculations of nuclear binding energies do not, in general, yield a similar simple interpretation. This is to be expected for heavy nuclei for which the liquid drop model is a good approximation. In the lightest nuclei, however, there should be some evidence of simple structure, or tendency of particles to collect into subgroups, which it should be possible to bring to light in the calculation of their binding energies. Such evidence should give some insight into the general situation in these nuclei. This would be especially desirable since the comparison of the light nuclei calculations with experiment will probably be a critical test for nuclear forces proposed in the future.

It is shown in this paper how the method of resonating group structure developed by Wheeler[1] makes it possible to give a simple interpretation to nuclear interactions in H^3 and He^5. The current two-body exchange force is used (see Table I for V_{ij}). It now seems clear that this force cannot account for many of the facts about light nuclei; nevertheless it seems reasonable to suppose that the evidence of structure will depend more on the wave function and the state of motion than on the exact nature of the interaction between nuclear particles.

The only important sub-group in H^3 would seem to be the deuteron, and in He^5 the α-particle. From stability considerations the grouping $2n+p$ in H^3 and either H^2+He^3 or He^2+H^3 in He^5 should be relatively unimportant. In the simple case of only one grouping the method of resonating group structure starts out with a wave function, Ψ, for the system written as the product of the internal wave function, φ, of the particles in the sub-group, and a function, F, which depends on the position of the outside particle with respect to the centroid of the sub-group. When Ψ is normalized and made properly antisymmetrical with respect to neutrons and protons[2] and inserted in the expression

$$E=\int\Psi^*H\Psi d\tau\Big/\int\Psi^*\Psi d\tau, \tag{1}$$

the result is

$$E=E_0+\frac{\hbar^2}{2\mu}\int\nabla F(\mathbf{X})\cdot\nabla F(\mathbf{X})d\mathbf{X}+\int V(\mathbf{X})F^2(\mathbf{X})d\mathbf{X}$$
$$+\int\int F(\mathbf{X})[J(\mathbf{X},\xi)-EI(\mathbf{X},\xi)]F(\xi)d\mathbf{X}d\xi. \tag{2}$$

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[1] J. A. Wheeler, Phys. Rev. 52, 1083 and 1107 (1937).

[2] See appendix for complete resonating group wave functions for H^3 and He^5.

TABLE I. *Form of interaction, force constants, and approximate forms of F and ϕ used in calculation of binding energies of* H^3 *and* He^5 *by the resonating group method.*

$$V_{ij}=-B\exp(-br_{ij}^2)(1-g-g_1-g_2)P_{ij}^M+gP_{ij}^H+g_1\mathbf{1}+g_2P_{ij}^B$$

where P^M, P^H, *and* P^B *are the Majorana, Heisenberg, and Bartlett operators, respectively.*

$$B=39.45\text{ mMU}\quad b=1.566(mc^2/e^2)^2\quad g+g_2=0.22$$
$$=36.72\text{ Mev}\quad b^{-\frac{1}{2}}=2.25\times10^{-13}\text{ cm}^2$$

FUNCTION	H^3 (1 AND 2 NEUTRONS)	He^5 (1, 2, AND 3 NEUTRONS)
$\mathbf{X}$	$\mathbf{r}_1-(\mathbf{r}_2+\mathbf{r}_3)/2$	$\mathbf{r}_1-(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4+\mathbf{r}_5)/4$
$F(\mathbf{X})(L=0)$	$\exp(-\beta X^2)$	$\exp(-\beta X^2)$
$F(X)(L=1)$		$\lvert X\rvert\cos\theta\exp(-\beta X^2)$
φ	$\exp(-\alpha r_{23}^2)$	$\exp[-\alpha(r_{23}^2+r_{24}^2+r_{25}^2+r_{34}^2+r_{35}^2+r_{45}^2)]$

μ is the reduced mass of the sub-group and the outside particle. $\mathbf{X}$ is the vector between the outside particle and the centroid of the sub-group and ξ is a similar vector when an exchange of particles has been made. $V(\mathbf{X})$, $J(\mathbf{X},\xi)$ and $I(\mathbf{X},\xi)$ can be found when φ is known and the integrations in (1) are carried out over the coordinates other than $\mathbf{X}$ and ξ and the coordinate of the centroid of the system.

The first term in (2), E_0, represents the internal energy of the sub-group; the second term the kinetic energy of the outside particle; and the third and fourth terms the interaction of the outside particle with the sub-group.

$$\int V(\mathbf{X})F^2(\mathbf{X})d\mathbf{X}$$

may be thought of as an "ordinary" potential energy and

$$\int\int F(\mathbf{X})[J(\mathbf{X},\xi)-EI(\mathbf{X},\xi)]F(\xi)d\mathbf{X}d\xi$$

as an effective velocity-dependent potential arising from the exchange of the outside particle with similar particles in the sub-group.

The resonating group method then makes use of the variation principle and proceeds to find the function, F, which makes E a minimum. Without going through this complicated procedure, however, interesting results can be obtained by using an approximate form for F in (2). The forms chosen for F and φ for both H^3 and He^5 are listed in Table I. The force constants used are also given there.

2. Results for H^3 and He^5 with Approximate Resonating Group Wave Functions

The energies of H^3 and He^5 obtained with these resonating group wave functions depend on the parameters α and β in φ and F and, in the case of He^5, on the combination[3] $G_3=1+g-5g_1-3g_2$. G_3 is a factor of the V term which turns out to be positive, indicating a repulsive "ordinary" force. The requirement that heavy nuclear shall not have large spins leads however to the condition[4] $G_3\geqslant 0$. The value $G_3=0$ will therefore lead to the greatest binding.

For H^3, the minimum with respect to α and β is easily found. For He^5 there is a minimum with respect to the α-particle parameter, α, at $\alpha=0.505$. ($(1/\alpha)^{\frac{1}{2}}$ is proportional to the radius of the α-particle.) But for this value of α and for $G_3=0$, the only minimum with respect to β ($(1/\beta)^{\frac{1}{2}}$ is proportional to the mean separation of the neutron) is at $\beta=0$, a value which represents the outside neutron as being at an infinite distance from the α-particle group. Curves $RG(L=0)$ and $RG(L=1)$ of Fig. 1 show the resonating group results for states of He^5 of angular momenta $L=0$ and $L=1$. The state for $L=1$ is clearly lower than that for $L=0$. The point of inflection on the $L=1$ curve at about $\beta=1$ may be an indication of the existence of a virtual level.

For $L=1$, $\beta=1$, the contributions to the energy of He^5 from the different terms in (2) are listed in Table II and compared with similar terms for the values of the parameters which make $E(H^3)_{L=0}$ a minimum.

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[3] In the notation of G. Breit and E. Wigner, Phys. Rev. **53**, 998 (1938), $G_3=-c_1=-(4V-M-2H+2B)$.

[4] N. Kemmer, Nature **140**, 192 (1937).

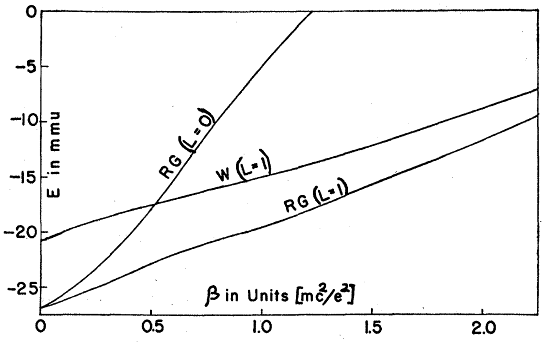

FIG. 1. Energy of He^5 as a function of the "outside" neutron parameter, β, for approximate resonating group wave functions, $RG(L=0)$ and $RG(L=1)$, and for the determinantal wave function of Watanabe, $W(L=1)$.

The values of E_0 in the tabulation show that neither the deuteron nor α-particle sub-group wave function has been very well chosen. For He^5 the value -26.94 mMU is the lowest which can be obtained for any value of α. For H^3, $\alpha=0.54$ gives a minimum for E_0 of -1.17 mMU, but this value of α gives a higher total energy for H^3. This can be interpreted as meaning that there is considerable polarization of the deuteron by the neutron in H^3 and very little polarization of the α-particle by the neutron in He^5.

The values of the kinetic energy of the neutron outside the sub-group are nearly the same in both cases and approximately equal to the average kinetic energy of a particle in the He^4 nucleus, 11 mMU.[5] This is approximately the same value found by Watanabe[5] for the p neutron after second-order perturbation corrections were applied to the regular Hartree function.

The "ordinary" potential is attractive in the H^3 case and zero for He^5 since G_3 was taken equal to zero. The "velocity dependent" potential is greater than the kinetic energy for H^3 but smaller for He^5. Coulomb terms which would still further increase the potential energy have not been included for He^5.

The breakdown of the total energy by this approximate method shows then that the interaction of Table I binds a neutron tightly to a deuteron group and not at all to an α-particle group.

For He^3 the grouping is H^2+p. The He^3 binding energy can be obtained from the H^3 value by merely adding the proper Coulomb terms. The result is -5.88 mMU. (Exp. value $=-8.1$ mMU.)

3. COMPARISON OF APPROXIMATE RESULTS WITH RESULTS USING OTHER FUNCTIONS

The results for the total energy by the resonating group method using the above approximate wave functions are compared with results for other simple wave functions in Table III.

For He^5, Watanabe[5] has tried a determinantal wave function involving two parameters similar to the α and β of the resonating group method. His calculations assume $g+g_2=0.25$. When the other values of the constants used here are substituted in his equation (15) and after this expression has been minimized with respect to α the curve in Fig. 1 marked $W(L=1)$ is found. It gives less binding than the resonating group $L=1$ curve for all values of β and thus, according to the variation principle, indicates that the determinantal function is less accurate than the resonating group function.

More complicated methods yield, of course,

TABLE II. *Contributions of different terms in (2) to the total binding energies of* H^3 *and* He^5 *for approximate resonating group wave functions.*

TERM*	H^3 $L=0$ $S=\frac{1}{2}$ $\alpha=0.8$ $\beta=0.8$	He^5 $L=1$ $S=\frac{1}{2}$ $\alpha=0.54$ $\beta=1.00$ $G_3=0$
E_0	-0.67 mMU (Exp. val. $=-2.33$)	-26.94 mMU (Exp. val. $=-30.23$)
$\hbar^2/2\mu\int\nabla F(\mathbf{X})\cdot\nabla F(\mathbf{X})d\mathbf{X}$	$+10.12$	$+11.25$
$\int V(\mathbf{X})F^2(\mathbf{X})d\mathbf{X}$	-2.02	0.00
$\int\int F(\mathbf{X})[J(\mathbf{X},\xi)-EI(\mathbf{X},\xi)]F(\xi)d\mathbf{X}d\xi$	-14.01	-3.82
E (Total)	-6.58 mMU (Exp. val. $=-8.94$)	-19.51 mMU (Exp. val. $=-29.3$**)

* For algebraic expressions for all terms see appendix. Coulomb terms not included for He^5.
** Value of Williams *et al.* See discussion below.

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[5] S. Watanabe, Zeits. f. Physik **112**, 159 (1939).

TABLE III. *Comparison of approximate resonating group results with those for other simple functions.*

WAVE FUNCTION*	E (H^3) $L=0$ $S=\frac{1}{2}$	E (He^5) $L=1$ $S=\frac{1}{2}$
Resonating group	-6.58 mMU	-19.51 mMU ($\beta=1$)
Variation	-6.34**	
Hartree	-6.31**	-10.6***

* See appendix for forms of these functions.
** Above constants used. The difference of the resonating group and variation values from the ones given by Wheeler is due to use of slightly different constants.
*** Above constants used except $g+g_2=0.25$. Coulomb terms not included for either He^5 result.

better results. Margenau and Warren[6] using a variation function with twelve terms find $E(H^3) = -7.74$ mMU. By perturbation method Tyrrell, Carroll, and Margenau[7]* find $E(He^5) = -22$ mMU. Also using perturbation method Watanabe[5] finds $E(He^5) = -16.6$ mMU. The constants used by these authors differ slightly from each other and from the ones used here.

The present experimental data on He^5 may be summed up as follows. Investigation of the α-particles[8] and neutrons[9] from the reaction

$$D^2+Li^7 \rightarrow He^4+He^5,$$
$$He^5 \rightarrow He^4+n$$

seems to point clearly to a virtual level of He^5 0.84 Mev above the energy of a free neutron and α-particle, and a binding energy therefore of 29.3 mMU. Experiments[10] on the scattering of neutrons by α-particles agree with this result and indicate the level is a p state. From the bombardment of deuterons by low energy α-particles Joliot[11] finds evidence for a stable He^5 with binding energy 32.4 mMU. His experiments have not yet been repeated by other workers.

4. Rigorous Application of Resonating Group Method to H^3 and He^3

In the three-body problem, instead of the approximate functions for F and φ used above in the resonating group wave function, one can use a numerical function for φ, which is an exact solution of the deuteron wave equation, and then proceed to find the form of F which makes the energy a minimum. The complete method puts at one's disposal the adjustable function, F, rather than a number of adjustable parameters. Wheeler[2] shows that the radial equation for F can be written

$$f_L(r)+\int S_L(r,\rho)f_L(\rho)d\rho=0,$$

where S_L depends on V, J_L, I_L, and the energy E. The requirement that the Fredholm determinant of this equation vanish gives the eigenvalue of E. When this is known, $f_L(r)$ is easily found. The corresponding F is then the "best" function in the sense of the variation principle.

The results of carrying out this complicated procedure are $E(H^3) = -6.68$ mMU and $E(He^3) = -5.94$ mMU, very little improvement over the minima (-6.58 mMU and -5.88 mMU) obtained with the approximate functions. The complete method gives E_0, the deuteron binding energy, its full value of 2.33 mMU and does not therefore allow for any polarization. Its results are thus more strictly comparable with the approximate results when the E_0 there is required to have its minimum value of -1.17 mMU instead of the value -0.67 mMU which makes the total energy a minimum, that is when again no polarization is allowed. If this requirement is

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[6] H. Margenau and B. E. Warren, Phys. Rev. **52**, 790 (1937).

[7] Tyrrell, Carroll, and Margenau, Phys. Rev. **55**, 790 (1939). Value given estimated from graph.

* *Note added in proof.*—W. A. Tyrrell, in a more recent detailed paper on He^5 (Phys. Rev. **56**, 250 (1939)), finds -20 Mev as the probable lower bound of the ^2P state. In addition the ^2S state is found to have practically the same energy. The two-parameter determinantal wave function for the state $L=1$ gives a curve very similar to that calculated from the equation of Watanabe since the values of the constants used differ only slightly. This curve is therefore quite similar to RG ($L=1$) curve. A two-parameter determinantal function for the state $L=0$, however, gives a curve not at all like the RG ($L=0$) curve but one which shows a distinct minimum at about -17 Mev for a finite value of the α-particle-neutron separation. This indicates superiority of the determinantal over the resonating group wave function in this case.

[8] Williams, Shepherd, and Haxby, Phys. Rev. **51**, 889 (1937).

[9] H. Staub and W. E. Stephens, Phys. Rev. **55**, 845 (1939).

[10] H. Staub and W. E. Stephens, Phys. Rev. **55**, 131 (1939).

[11] F. Joliot, J. de phys. et rad. **9**, 403 (1939).

imposed, the approximate functions give $E(\mathrm{H}^3) = -5.84$ and $E(\mathrm{He}^3) = -5.20$ mMU. The complete procedure thus gives an improvement over the approximate one of about 0.8 mMU. If polarization effects, or the dependence of φ on F, were taken into account, the results of the complete method might be considerably improved. The fact that in the case of He^5 the approximate resonating group method gives results which are comparable to those of much more complicated methods seems attributable to the small polarization of the α-particle. Watanabe[5] finds some evidences of polarization from a corrected Hartree wave function.

The complete procedure is very laborious on account of the large number of numerical integrations involved, many of which would be necessary even if an analytic expression for φ were used. It is also difficult to know when the size of the steps used in the numerical integrations has been made sufficiently small. Indeed, in the work leading to the above results energy values were obtained with large steps which were better than those presented. Increasing the number of steps must ultimately lead to an increase in accuracy but doubling the number of steps, for instance, increases the work fourfold. Also in an investigation of the $L=0$, $S=\frac{3}{2}$ state of H^3 a very queer result was obtained which was finally shown to be spurious and due to having taken the size of the steps too large.[12]

The resonating group method also presents in some cases another type of difficulty which can, however, be removed. The solution of the integral equation (3) proves in certain problems to be unstable with respect to small errors in the calculations. This is due to the singular nature of the numerator and denominator of the fraction (1). On choosing $F=e^{-\beta x^2}$ both numerator and denominator may vanish identically for a certain value of β, say β_0. This arises from the fact that the entire resonating group wave function itself, in the cases in question, vanishes identically for this choice of the intergroup wave function. The difficulty may be avoided in either of two ways. One can multiply the wave function by a suitably chosen power of $1/(\beta-\beta_0)$ and go to the limit $\beta=\beta_0$, where the wave function is finite (equivalent to calculating the limit of the fraction (1)). This procedure was actually followed in the He^5 calculations above for the case $L=0$, $\beta=5\alpha$. Or one may remove the difficulty by introducing a new function $\bar{F}$ connected with F by an operator equation $F=S\bar{F}$, chosen in such a way that neither numerator nor denominator of the fraction (1) vanishes for any nonzero choice of $\bar{F}$. This method is developed and applied in an unpublished paper by J. A. Wheeler on the interaction between two α-particles.

5. Excited State of He^3

Inspection of the p state functions J_1, I_1, and V as calculated by the complete method indicated that they would not lead to a bound state for He^3. An exact calculation for the state $L=0$, $S=\frac{3}{2}$ of this nucleus also showed that it could not be bound for any value of the combination, $1-3g_1-3g_2$, allowed by stability requirements of heavy nuclei.

6. Conclusions

If it turns out to be true that the quantum mechanics is strictly applicable to nuclear problems, a knowledge of the reliability and ease of handling different quantum-mechanical approximation methods will be important for trying out new ideas about forces. With regard to the resonating group method the above results lead to the following conclusions.

1. The use of the resonating group wave function with approximate analytic expressions for F and φ leads quickly and easily to energy values lower than those obtained with other simple functions. This is especially true when there is small polarization of the sub-groups.
2. The method permits a separation of the contributions to the total energy in such a way that one can see approximately what is going on in the nucleus.
3. The complete method, which uses the "best" form for F rather than an analytical approximation, is laborious and somewhat difficult to make accurate on account of the necessity for numerical integrations.

The writer is very much indebted to Professor John A. Wheeler of Princeton University

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[12] Results for this state obtained with a Hartree function and very kindly sent by Dr. Tyrrell of Yale University were helpful in finding the source of the difficulty.

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APPENDIX 1

Resonating group wave functions:

H^3. Particles 1 and 2 neutrons.

$$\Psi=S_1F[1-(2+3)/2]\varphi(2\;3)-S_2F[2-(1+3)/2]\varphi(1\;3),$$
$$S_1=12^{-\frac{1}{2}}[\alpha(1)\alpha(2)\beta(3)+\alpha(1)\beta(2)\alpha(3)-2\beta(1)\alpha(2)\alpha(3)],$$
$$S=12^{-\frac{1}{2}}[\alpha(2)\alpha(1)\beta(3)+\alpha(2)\beta(1)\alpha(3)-2\beta(2)\alpha(1)\alpha(3)].$$

He^5. Particles 1, 2, and 3 neutrons.

$$\Psi=S_1F[1-(2+3+4+5)/4]\varphi(2345)+S_2F[2-(1+3+4+5)/4]\varphi(3145)+S_3F[3-(1+2+4+5)/4]\varphi(1245),$$
$$S_1=2^{-1}\alpha(1)[\alpha(2)\beta(3)-\beta(2)\alpha(3)]\times[\alpha(4)\beta(5)-\alpha(5)\beta(4)],$$
$$S_2=2^{-1}\alpha(2)[\alpha(3)\beta(1)-\beta(3)\alpha(1)]\times[\alpha(4)\beta(5)-\alpha(5)\beta(4)],$$
$$S_3=2^{-1}\alpha(3)[\alpha(1)\beta(2)-\beta(1)\alpha(2)]\times[\alpha(4)\beta(5)-\alpha(5)\beta(4)].$$

Variation wave function. H^3.

$$\Psi=2^{-\frac{1}{2}}\alpha(3)[\alpha(2)\beta(1)-\beta(2)\alpha(1)]\times\exp\left[-\tfrac{1}{2}\nu[r_{23}^2+r_{13}^2]-\tfrac{1}{2}\mu r_{12}^2\right].$$

Hartree wave function.

H^3. $\Psi=(3\,!)^{-\frac{1}{2}}|s\eta_1\cdot s\eta_2\cdot s\eta_3|$.
He^5. $\Psi=(5\,!)^{-\frac{1}{2}}|s\eta_1\cdot s\eta_2\cdot s\eta_3\cdot s\eta_4\cdot p\eta_1|$.

$$\eta_1=\delta(s_z,\tfrac{1}{2})\delta(\tau_\zeta,-\tfrac{1}{2}),\qquad \eta_3=\delta(s_z,\tfrac{1}{2})\delta(\tau_\zeta,\tfrac{1}{2}),$$
$$\eta_2=\delta(s_z,-\tfrac{1}{2})\delta(\tau_\zeta,-\tfrac{1}{2}),\qquad \eta_4=\delta(s_z,-\tfrac{1}{2})\delta(\tau_\zeta,\tfrac{1}{2}),$$

where τ_ζ is the isotopic spin quantum number

$$s=(\alpha/\pi)^{3/4}\exp(-\alpha r^2/2),$$
$$p=(\alpha/\pi)^{3/4}\sqrt{2}\alpha z\exp(-\alpha r^2/2).$$

For the He^5 two parameter determinantal wave function of Watanabe, the α in p is set equal to β.

APPENDIX 2

Resonating group energy expressions for $H^3(L=0,\,S=\frac{1}{2})$ and $He^5(L=1,\,S=\frac{1}{2})$ with the approximate wave functions of Table I.

H^3.

$$E_0=3\hbar^2\alpha/M-B/(1+b/2\alpha)^{\frac{3}{2}},$$

$$\hbar^2/2\mu\int\nabla F(\mathbf{X})\cdot\nabla F(\mathbf{X})d\mathbf{X}=9\hbar^2\beta/4M,$$

$$\int V(\mathbf{X})F^2(\mathbf{X})d\mathbf{X}=64B(\alpha\beta)^{\frac{3}{2}}\times(-\tfrac{1}{2}+\tfrac{3}{2}g-\tfrac{3}{2}g_1+\tfrac{3}{2}g_2)/(16\alpha\beta+2b\beta+8\alpha b)^{\frac{3}{2}},$$

$$\int\int F(\mathbf{X})I(\mathbf{X},\,\xi)F(\xi)d\mathbf{X}d\xi=256(\alpha\,\beta)^{\frac{3}{2}}/[(4\alpha+\beta)(4\alpha+9\beta)]^{\frac{3}{2}},$$

$$\int\int F(\mathbf{X})J(\mathbf{X},\,\xi)F(\xi)d\mathbf{X}d\xi$$
$$=\hbar^2 3\times64^2(4\alpha+3\beta)(\alpha\beta)^{5/2}/M[(4\alpha+\beta)(4\alpha+9\beta)]^{5/2}-512B(\alpha\beta)^{\frac{3}{2}}\{(2-3g-\tfrac{3}{2}g_1-3g_2)/[(4\alpha+\beta)(4\alpha+9\beta+8b)]^{\frac{3}{2}}+1/[(4\alpha+\beta)(4\alpha+9\beta)+16\alpha b+20b\beta]^{\frac{3}{2}}\}.$$

He^5. $G=1-g-g_2$,

$$E_0=18\hbar^2\alpha/M-6GB/(1+b/4\alpha)^{\frac{3}{2}},$$

$$\hbar^2/2\mu\int\nabla F(\mathbf{X})\cdot\nabla F(\mathbf{X})d\mathbf{X}=2\hbar^2\beta/M,$$

$$\int V(\mathbf{X})F^2(\mathbf{X})d\mathbf{X}=G_3B2^{10}(\alpha\beta)^{5/2}\times(32+3b/\alpha)/[64\alpha\beta+2b(25\alpha+3\beta)]^{5/2},$$

$$\int\int F(\mathbf{X})I(\mathbf{X},\,\xi)F(\xi)d\mathbf{X}d\xi=2^{13}(\alpha\beta)^{5/2}/[(3\alpha+\beta)(25\alpha+3\beta)]^{5/2},$$

$$\int\int F(\mathbf{X})J(\mathbf{X},\,\xi)F(\xi)d\mathbf{X}d\xi=2^{13}(\alpha\beta)^{5/2}$$
$$\times\{[188\hbar^2\alpha/5M-3GB/(1+b/4\alpha)^{\frac{3}{2}}]/[(3\alpha+\beta)(25\alpha+3\beta)]^{5/2}-4\hbar^2\alpha[2775\alpha^2+458\alpha\beta-49\beta^2]/5M[(3\alpha+\beta)(25\alpha+3\beta)]^{7/2}-B(6b-16\alpha)(3G+G_3)/16\alpha[(3\alpha+\beta)(25\alpha+3\beta)+50\alpha\beta+9b\beta]^{5/2}-B6G(4\alpha)^{\frac{3}{2}}(4\alpha+b)/[25\alpha^2(12\alpha+5b)+34\alpha\beta(4\alpha+b)+\beta^2(12\alpha+b)]^{5/2}\}.$$