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Spin-Dependence in the Electron-Positron Theory of Nuclear Forces

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An interaction between heavy particles and electrons is considered which includes a dependence upon relative spin orientation. The anomalous magnetic moments of neutron and proton can be accounted for by an adjustment of constants. Correct spin-dependence in the interaction of two heavy particles can be obtained only through an angular dependence of forces. An estimate of the angular dependence in this theory indicates that it is of sufficient magnitude for this purpose and of the right sign to give a "cigar-shaped" deuteron.

INTRODUCTION

PROGRESSIVE stages of an electron-positron field theory of nuclear forces have been presented from time to time.[1–3] It is the purpose of this paper to extend the most recent of these considerations (this issue of *Physical Review*) to account for spin-dependent phenomena. Definite experimental results correlating properties of nuclear particles with their spins are: (1) the magnetic moments of proton and neutron; (2) the greater binding experienced by a neutron and a proton with parallel spins (triplet deuteron) than between heavy particles with opposite spins; and (3) the electrical quadrupole moment of the deuteron. Simple explanations of these properties of one- and two-body systems are sought under the assumption that the neutron and proton interact strongly (compared with Coulomb interactions at nuclear distances) with the electron-positron field.

In the preceding formulation[3] of the spin-independent theory it was found convenient to adopt an interaction between heavy particles and electron which is proportional to the scalar spin-operator, ρ_3. Let V' represent the dependence of the interaction on coordinates and momentum of the electron. Then, neglecting the small components of the heavy particle wave function, the Dirac equation for an electron in the presence of a heavy particle may be written:

$$[E+\rho_1(\sigma,\, cp)+\rho_3(mc^2-V')]\psi=0. \tag{1}$$

Since in this approximation (nonrelativistic for heavy particles) the heavy particle state is not changed the calculations are simplified; also, no spin-dependent phenomena are to be expected to result from them. Relativistic invariance was abandoned in another respect, namely: functions of finite radius, $u(x)$, were used to replace delta-functions, $\delta(x)$, in the invariant form of interaction (see Eqs. (2) and (3) of reference 3). The average value of $\rho_3 V'$ used in the spin-independent theory may be written

$$\int \bar{\psi}\rho_3 V'\psi dx = -\eta\int dx\int dx'\rho_{ik}{}^3\psi_i(x)^\dagger\psi_k(x')u(x)u(x')^*. \tag{2}$$

η is the proportionality constant for the energy when normalized ψ and u-functions are used. The function $u(x)$ is centered about the heavy particle. It is clear that the radius of $u(x)$ is substantially the range of force between heavy particle and electron. This range of force may be expected to be comparable to that found between nuclear particles, $\sim e^2/mc^2$, so that the average kinetic energy in the state $u(x)$ would be of the order of $c\hbar/(e^2/mc^2)=137mc^2$. Electron states for which the integral (2) is relatively important have, therefore, kinetic energies large compared with mc^2. It was assumed that interactions between electrons could be neglected and that the kinetic energy of every free electron state could be written as $E_{\rm kin}=cp$. The influence of $\rho_3 V'$ on each electron state was calculated under these assumptions. Summing up the

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[1] G. Gamow and E. Teller, Phys. Rev. **51**, 289 (1937).
[2] C. Critchfield and E. Teller, Phys. Rev. **53**, 812 (1938).
[3] E. Wigner, C. Critchfield and E. Teller, Phys. Rev. this issue.

changes in energy caused by $\rho_3 V'$ (associated with one heavy particle) over all negative energy levels led to Eq. (21) of reference 3. We shall write this result as

$$\Delta E = 2\int_{-\infty}^{0} F(\eta, E)dE \qquad (3)$$

with

$$F(\eta, E) = \frac{1}{\pi}\operatorname{arctg}\frac{(4\pi^2/c^3)E^2 v(E/c)^2 f(E)}{1/\eta^2 - f(E)^2 + (4\pi^4/c^6)E^4 v(E/c)^4},$$

$$f(E) = 4\pi\int_0^{\infty}\frac{p^2 v(p)^2 - (E^2/c^2)v(E/c)^2}{E^2 - c^2p^2}E dp,$$

$$v(p) = (2\pi\hbar)^{-\frac{3}{2}}\int_0^{\infty} u(x)\exp i(p, x)/\hbar dx.$$

The argument of the arctangent is constrained by the boundary conditions to lie in the first or second quadrants.

A study of limiting cases for $\Delta E(\eta)$ has shown that for small η, $\Delta E \sim -\eta^2/E_{\mathrm{kin}}$ and for large η, $\Delta E \sim -|\eta|$. Since, with a simple form for $u(x)$, $\Delta E(\eta)$ should be a simple function of η, $\eta\partial\Delta E/\partial\eta$ is negative for all values of η. The quantity $\partial^2\Delta E/\partial\eta^2$ is negative for small η and zero for large so that in most simple examples it would be negative everywhere. These estimates of sign will be useful when spin-dependence is taken into account, for we shall direct particular attention to those cases in which the spin interaction may be considered as a perturbation and the perturbed expression for ΔE expanded in a series.

Spin-Dependent Interaction

Consider one heavy particle in the presence of all electrons in negative states. The electron states may be divided into two classes: those forming singlets with the heavy particle and those forming triplets. This classification is invariant to rotations in space and also time independent if there are no spin-orbit interactions (as here assumed). We shall, therefore, introduce spin-dependence by assuming that the interaction constant, η, is different when heavy and light particles form a singlet than when they form a triplet. In terms of the five invariants discussed in the previous work such a modification of the interaction is equivalent to adding the tensor-tensor product to the already assumed scalar-scalar product. The approximate form of the new interaction, corresponding to Eq. (2), then becomes

$$\bar{V} = -\eta\int dx\int dx' \rho_{is}{}^3[\delta_{mn}\delta_{sk} + \lambda(\sigma_{mn}, \sigma_{sk})]$$
$$\times \Psi_m{}^\dagger \Psi_n \psi_i(x)^\dagger \psi_k(x') u(x) u(x')^*, \qquad (4)$$

where λ is a new constant and Ψ_m, $\Psi_m{}^\dagger$ are the (quantized) amplitudes of the heavy particle wave. Heavy particle functions must now be considered since the spin of the heavy particle may change as a consequence of the interaction.

Of the five invariant forms mentioned above it is also true that the product of pseudovectors would lead to spin-dependence without involving the negative states of heavy particles; but this operator will not be considered since it commutes with the operator on electron states, ρ_2, and may therefore modify the electric charge density in the lowest state of a heavy particle. The interaction (4) which we have chosen for further discussion anticommutes with ρ_2 and thus, according to the argument given in the former work, will present no difficulty with charge density. By the same argument it can be proved that, assuming (4), no contribution to the angular momentum of the lowest state of a heavy particle system will arise in the electron-positron field. It is thus assured that the multiplicity of the complete system of heavy particle and electrons in the lowest state is the same as for the heavy particle alone. The proof is as follows. The Hamiltonian,

$$H = -\rho_1(\sigma, cp) - \rho_3[mc^2 - V(p, q, \sigma)],$$

anticommutes with ρ_2; hence, $\rho_2 H \rho_2 = -H$, and the transformation ρ_2 changes all negative levels into positive levels and vice versa. On the other hand, the spin operator of the light particles, σ, commutes with ρ_2; the total spin when all positive energy levels are filled and all negative levels empty is then the same as when all negative levels are filled and all positive ones empty. But if all positive and negative energy levels were filled at the same time the total spin would be zero; hence, the electron spin in the lowest state of H vanishes also.

The indicated operator on spin-indices in (4) may be written in the notation of Dirac as

$$O_s=1+\lambda(\sigma,\sigma^h), \qquad (5)$$

where σ^h is the heavy particle spin and σ the light particle spin. Proper states of O_s are, of course, proper states of (σ,σ^h), i.e., the triplet belonging to $+1$ and the singlet belonging to -3. Proper values of O_s are therefore $1+\lambda$ in the triplet and $1-3\lambda$ in the singlet. Interaction (4) may then be described as two interactions analogous to (2) with η replaced by $\eta(1+\lambda)$ when the heavy particle and the electron form a triplet and by $\eta(1-3\lambda)$ for singlet systems. A sum of energy changes in the electron levels taken over all possible spin configurations then leads to*

$$3\int F(\eta+\lambda\eta,E)dE+\int F(\eta-3\lambda\eta,E)dE. \qquad (6)$$

In obtaining (6) heavy particle states of both spins are taken into account. Since the light particle field does not contribute to the angular momentum, (6) is the sum over two members of a doublet system. No difference in energy is to be expected between these states so that the change in energy caused by a single heavy particle is half of (6):

$$\Delta E_1=\tfrac{3}{2}\int F(\eta+\lambda\eta,E)dE +\tfrac{1}{2}\int F(\eta-3\lambda\eta,E)dE. \qquad (7)$$

Expression (7) therefore replaces (3) when a spin-dependent interaction (4) is considered for one heavy particle.

Magnetic Moments of Proton and Neutron

It is well known that the observed magnetic moments of neutron and proton are not what would result from a relativistic theory of heavy particles analogous to the Dirac theory of the electron. One should suppose the magnetic moment of the neutron to be zero and that of the proton to be 1/1840 as much as the positron magnetic moment. Experiment shows, however, that the proton has about 1.8 nuclear magnetons more moment than would be expected on this basis and the neutron approximately as much less, i.e., the neutron apparently has a magnetic moment of the same sign as the electron and between one and two magnetons in magnitude. In order to explain these discrepancies it was proposed by Wick[4] that an interaction between heavy particles and a light particle field which would explain nuclear forces might also give rise to an anomalous magnetic moment. We shall, therefore, choose the spin-dependent interaction of neutron and of proton with the electron field in such a way as to account for their magnetic moments and then investigate the consequences of the choice on the forces between two heavy particles.

Let the electron which is governed by Eq. (1) be subject also to a small magnetic field represented by the vector potential, A. The change in proper energy may then be determined by a perturbation calculation. From the average value of the addition, $e\rho_1(\sigma,A)$, to the Hamiltonian we can obtain an expression for the operator of the magnetic moment of the electron

$$E'=-\int\bar{\psi}e\rho_1(\sigma,A)\psi dx$$
$$=\frac{-e}{2E}\int\bar{\psi}[H\rho_1(\sigma,A)+\rho_1(\sigma,A)H]\psi dx. \qquad (8)$$

$\rho_1(\sigma,A)$ does not anticommute with ρ_3V' but the result is proportional to ρ_2 and its average value in the state ψ which is a proper state of a combination of ρ_1 and ρ_3 must vanish.

$$(\sigma,cp)(\sigma,A)+(\sigma,A)(\sigma,cp) =\hbar c(\sigma,\mathrm{curl}\,A)-i\hbar\,\mathrm{div}\,A+2(A,p). \qquad (9)$$

The part of the perturbing energy which is due to the action of the magnetic field, curl A, on the spin is then

$$E_\mu=-e\hbar c(\sigma,\mathrm{curl}\,A)/2E \qquad (10)$$

from which we conclude that the magnetic moment of the electron is $-e\hbar c\sigma/2E$, where E is the unperturbed energy.

We shall consider a heavy particle with upward spin. Let ax_1 and ax_1' be the average dis-

* Integrals over E will be understood to extend from $-\infty$ to zero.

[4] G. C. Wick, Accad. Lincei Atti **21**, 170 (1935).

placements of energy levels, E, of electrons with upward spin and let ax_2 and ax_2' be the average displacement when the spin is down. If the light and heavy particle spins are both up they certainly form a triplet so that expressions for x_1 and x_1' as functions of E are the same as formerly found for x (Eqs. (18) and (19), reference 3) except that η is replaced by $\eta+\lambda\eta$. If the electron spin is down the state is equally mixed from singlet and triplet states (i.e., a superposition of two states, one even, one odd under a reversal of spins) and thus an average over x and x' expressions in which η has been replaced by $\eta+\lambda\eta$ and by $\eta-3\lambda\eta$ must be taken. The magnetic moment arising from a given level, of energy E, is then

$$\begin{aligned} M_E = -\tfrac{1}{2}e\hbar c\{&(E+ax_1)^{-1}+(E+ax_1')^{-1} \\ &-(E+ax_2)^{-1}-(E+ax_2')^{-1}\} \\ &\cong (e\hbar c/2E^2)a(x_1+x_1'-x_2-x_2'). \end{aligned} \tag{11}$$

Summing over all negative energy levels the total magnetic moment due to the electron-positron field is

$$M=\tfrac{1}{4}\sigma_z{}^h e\hbar c\left\{\int F(\eta+\lambda\eta, E)\frac{dE}{E^2} - \int F(\eta-3\lambda\eta, E)\frac{dE}{E^2}\right\}. \tag{12}$$

In the special case that $|\lambda|\eta\ll\eta$, (12) may be expanded in a series of powers of λ:

$$M\cong\sigma_z{}^h e\hbar c\lambda\eta\int\frac{\partial F(\eta, E)}{\partial\eta}\frac{dE}{E^2} \tag{13}$$

from which it is apparent that the sign of the magnetic moment depends upon the sign of λ. From the former considerations[3] concerning the integration of ΔE one would conclude that

$$\eta\int\frac{\partial F(\eta, E)}{\partial\eta}\frac{dE}{E^2}\sim\begin{cases} -\eta^2/E_0{}^3 & \eta\ll E_0 \\ -1/|\eta| & \eta\gg E_0 \end{cases}, \tag{14}$$

where E_0 is the absolute value of the kinetic energy characteristic of the spatial extent of the wave functions $u(x)$. Now with $E_0\sim 137mc^2$, $e\hbar c/E_0\sim 30$ nuclear magnetons. Hence to account for the anomalous magnetic moment of the proton $-\lambda\eta^2/E_0{}^2\cong 1.8/30\cong\frac{1}{16}$ if η is small and $-\lambda E_0/|\eta|=\frac{1}{16}$ for large η. It is quite probable that with $\frac{1}{16}<|\lambda|<1$ the anomalous magnetic moment of the proton can be accounted for by assuming an interaction of the form (4) and with a negative "λ," $O_s{}^p=1-\lambda(\sigma, \sigma^h)$. The magnetic moment of the neutron would then be obtained by introducing a λ of the same absolute value but of positive sign, $O_s{}^n=1+\lambda(\sigma, \sigma^h)$.

Two Heavy Particles

When two heavy particles are considered two special configurations are readily treated. In one case the heavy particles are so widely separated that the $u(x)$ associated with one particle may be considered to vanish in the region in which the $u(x)$ centered on the other particle is appreciable. Then the V' operators commute and the influence of the heavy particles on the energy of the light particle field is simply additive. ΔE for two neutrons, two protons or proton and neutron at infinite separation is thus obtained by taking the corresponding sum of ΔE^n and ΔE^p. The superscripts refer to neutron and proton, respectively. Assuming $\frac{1}{16}<\lambda<1$ the results obtained for magnetic moments lead to the following expressions:

$$\begin{aligned} \Delta E^p &= \tfrac{3}{2}\int F(\eta-\lambda\eta, E)dE + \tfrac{1}{2}\int F(\eta+3\lambda\eta, E)dE, \\ \Delta E^n &= \tfrac{3}{2}\int F(\eta+\lambda\eta, E)dE + \tfrac{1}{2}\int F(\eta-3\lambda\eta, E)dE. \end{aligned} \tag{15}$$

The other special configuration occurs when two heavy particles occupy the same point in space. Two neutrons or two protons in a singlet state at a certain point form a spherically symmetrical system so that the spin coupling with the electron field vanishes. We shall use the subscript S on ΔE to denote a singlet state and double superscripts pp, nn, np in case of two particles at coincidence. Then

$$\Delta E_S{}^{nn}=\Delta E_S{}^{pp}=2\int F(2\eta, E)dE. \tag{16}$$

A proton and a neutron at the same point in space present a problem similar to that of a single particle. Consider one electron in a spherical state centered on the same point. The eight possible spin-states may be represented as one quartet and two doublets, i.e., proper states of $Q\equiv\lambda(\sigma, \sigma^n-\sigma^p)$. Taking the square and fourth power of Q

$$Q^4=12\lambda^2Q^2, \tag{17}$$

from which we gather that the quartet belongs to zero and the doublets to $2\lambda\sqrt{3}$ and $-2\lambda\sqrt{3}$. The changes in energy of all electrons in negative states summed over all possible spin configurations of the heavy particles are then

$$\sum\Delta E^{np}=4\int F(2\eta, E)dE +2\int F(2\eta+2\lambda\sqrt{3}\eta, E)dE +2\int F(2\eta-2\lambda\sqrt{3}\eta, E)dE. \tag{18}$$

There are four spin configurations for two heavy particles: three members of a triplet and one singlet. The triplets are symmetric to an interchange of heavy particles so that they alone are affected by the quartet states in (18). Since the operator Q is antisymmetric to an interchange of neutron and proton the doublets are equally mixed of antisymmetric and symmetric functions. The terms in (18) which depend explicitly on λ are therefore half-singlets and half-triplets. From these considerations we obtain

$$\Delta E_T{}^{np}=\tfrac{4}{3}\int F(2\eta, E)dE +\tfrac{1}{3}\int F(2\eta+2\lambda\sqrt{3}\eta, E)dE +\tfrac{1}{3}\int F(2\eta-2\lambda\sqrt{3}\eta, E)dE, \tag{19}$$

$$\Delta E_S{}^{np}=\int F(2\eta+2\lambda\sqrt{3}\eta, E)dE +\int F(2\eta-2\lambda\sqrt{3}\eta, E)dE.$$

The expressions (19) may be expanded in series and the following approximate energies for two coincident particles obtained:

$$\Delta E_S{}^{nn}=\Delta E_S{}^{pp}=2\int F(2\eta, E)dE,$$

$$\Delta E_S{}^{np}=2\int F(2\eta, E)dE +6\lambda^2\eta^2\int \frac{\partial^2F(2\eta, E)}{4\partial\eta^2}dE, \tag{20}$$

$$\Delta E_T{}^{np}=2\int F(2\eta, E)dE +4\lambda^2\eta^2\int \frac{\partial^2F(2\eta, E)}{4\partial\eta^2}dE.$$

In consequence of the general conclusion that the second derivative of ΔE is negative we see from (20) that the lowest state in this approximation is the singlet deuteron, the next lowest the triplet deuteron and highest the singlet like-particle states. Experimental results[5] of scattering protons by protons and neutrons by protons indicate that the depths of the singlet wells should be the same for like and unlike particles. Relations (20) therefore force the conclusion that

$$\lambda^2\eta^2\int \frac{\partial^2F(2\eta, E)}{\partial\eta^2}dE$$

be negligible compared with the maximum binding potential, $\equiv D$. Under this condition all the ΔE's in (20) become practically the same and the value of D is independent of λ:

$$D=2\int F(2\eta, E)dE-4\int F(\eta, E)dE \qquad \left|\lambda^2\eta^2\int \frac{\partial^2F(2\eta, E)}{\partial\eta^2}dE\right|\ll|D.| \tag{21}$$

Relations (21) lead to an apparently unsatisfactory result in that the depth of potential wells for singlet and triplet deuteron should be

[5] M. Tuve, N. Heydenburg, L. Hafstad, Phys. Rev. **50**, 806 (1936); **53**, 239 (1938); G. Breit, E. Condon and R. Present, *ibid.* **50**, 825 (1936).

the same. Calculations with arbitrary, spherical, short range potentials have led to the result that the depth of the triplet well should be from 30 to 100 percent deeper than the singlet well. This apparent disagreement of our theory with experience might be rectified by dropping the assumption that λ is small and finding two values of the order unity which, when substituted in (12), would give the correct magnetic moments for neutron and proton. Whether such an assumption would lead also to satisfactory results with regard to forces in the two-particle systems cannot be determined, in general, without adopting a particular form for $u(x)$. There is one positive indication that $\lambda\eta \gg \eta$ would not be satisfactory because repulsions of the order of $\lambda\eta$ should arise as two heavy particles are brought together. These repulsions are due to the fact that, in general, the lowest proper states of individual heavy particles cannot be filled simultaneously when the heavy particles are close together. It is not within the scope of the present paper to investigate particular forms for $u(x)$, so that the possibility of using larger values of λ will be considered only for cases satisfying (21) with $\frac{1}{16} < \lambda < 1$.

It is not necessary that the potential well be deeper for the triplet than for the singlet deuteron if there is an angular dependence in the forces between neutron and proton. Forces of this type have been discussed by several authors in connection with nuclear forces and the electrical quadrupole moment of the deuteron.[6] It is proposed in these discussions that the forces between neutron and proton contain a part analogous to the interaction of electric or magnetic dipoles. Under such forces the lowest state of the two-particle system would not be spherical but would be elongated or flattened according to the sign of the interaction energy. The lowest state is then a superposition of spherical harmonics of higher order on the spherically symmetrical state than has been customarily assumed. A neutron and a proton in a spherical state with parallel spins can make a transition into a D state without violating the conservation of angular momentum but an analogous transition for particles with opposite spin (singlet) is impossible. The lowest triplet state of the deuteron may then be expected to lie lower than the singlet although the maximum depth of the potential wells is the same. In fact, Bethe has shown[6] for the neutral meson theory that the entire difference in binding for triplet and singlet deuteron can be ascribed to such an angular dependence. We propose, therefore, that the greater binding in the triplet is due alone to the angular dependence of the forces and thus is not in contradiction with the assumption $\frac{1}{16} < \lambda < 1$. It will not be possible to demonstrate quantitatively that the correct relation of triplet to singlet binding can be obtained without assuming a particular form for $u(x)$ and extending the calculation considerably. It will be shown, however, that an angular dependence in the potential which is of importance compared with the potential itself may be expected in the electron-positron field theory presented here.

Angular Dependence and Quadrupole Moment

An angular dependence in forces between neutron and proton would also account for the observed electrical quadrupole moment of the deuteron.[7] Present indications are that the lowest state of the triplet deuteron is prolate.[6] A potential which favors the prolate configuration may be written as a function of the unit vector, X, joining the two particles and of the distance, r, between particles:

$$V'' = -(\sigma^n, X)(\sigma^p, X)V''(r), \qquad (22)$$

where $V''(r)$ is assumed to be positive. We now assume, as before, a spin interaction constant, λ, for the neutron and $-\lambda$ for the proton. It is not proposed to find an exact solution of the two-body problem similar to (3); we shall rather consider a neutron and a proton not quite at the same point but sufficiently close that a description of electron states in spherical coordinates referred to the center of mass is adequate. If, at such small distances, a potential of the form (22) obtains it might be expected that the consequences discussed above will hold qualitatively for the type of interaction assumed and

[6] See H. A. Bethe, Phys. Rev. **55**, 1261 (1939).

[7] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. **55**, 318 (1939).

quadrupole moment as well as binding energy could be correctly accounted for.

When two heavy particles are considered the state $v(p)$ with its interaction constant 2η applies only at coincidence. At finite separation it is convenient to set up orthogonal superpositions of $v_1(p)$ centered at one heavy particle and $v_2(p)$ at the other. Such an orthogonal set is, of course, the sum and difference. If two particles are close together and joined by a vector having the direction of the unit vector, X, the difference $v_1(p)-v_2(p)$ is approximated by the directional derivative of $v(p)$ and the sum $v_1(p)+v_2(p)$ by $v(p)$ itself. In the normalized form, therefore, we shall consider the function $v(p)$ centered on the center of mass and having the interaction constant $(\eta-\epsilon)$ and the function $(X,w)\sqrt{3}v(p)$ having the interaction constant ϵ. Here, $w=p/|p|$. The wave equation of an electron in this field then becomes

$$\begin{aligned}(E-\rho_3E_p)\varphi'(p)+(\eta-\epsilon)v(p)\\ \times\textstyle\int T(p)\rho_3[1+\lambda(\sigma,\sigma^n-\sigma^p)]T^{-1}(p')\\ \times v(p')^*\varphi(p')dp'+3\epsilon(w,X)v(p)\\ \times\textstyle\int T(p)\rho_3[1+\lambda(\sigma,\sigma^n-\sigma^p)]\\ \times T^{-1}(p')(w',X)v(p')^*\varphi'(p')dp'=0\end{aligned}\tag{23}$$

with $T=1/\sqrt{2}[1+\rho_3\rho_1(\sigma,w)]$. We shall consider only very small values of ϵ, $\epsilon\ll\eta$. Then the functions $\varphi'(p)$ for which the influence of the interaction with the heavy particle is different from zero must contain a spherically symmetrical part. Following the steps of the previous treatment we accordingly choose $\varphi'(p)$ of the form $S(p)+(\sigma,w)P(p)$ where $S(p)$ and $P(p)$ are functions of the absolute value of p and also of heavy and light particle spins. Thus each $S(p)$ and $P(p)$ has sixteen components. Define

$$\xi_S=4\pi\int_0^\infty S(p)v(p)^*p^2dp$$

and

$$\xi_P=4\pi\int_0^\infty P(p)v(p)^*p^2dp.$$

Then if $\epsilon=0$, (23) becomes

$$\begin{aligned}(E-\rho_3E_p)[S(p)+(\sigma,w)P(p)]\\ +\tfrac{1}{2}\eta v(p)\rho_3[1+\rho_1\rho_3(\sigma,w)]\\ \times[1+\lambda(\sigma,\sigma^n-\sigma^p)][\xi_S+\rho_1\rho_3\xi_P]=0.\end{aligned}\tag{24}$$

Let $\xi=\xi_S+\rho_1\rho_3\xi_P$.

$$\begin{aligned}\varphi'(p)&=S(p)+(\sigma,w)P(p)\\ &=-\tfrac{1}{2}\eta\frac{E\rho_3+E_p}{E^2-E_p{}^2}v(p)[1+\rho_1\rho_3(\sigma,w)]\\ &\qquad\times[1+\lambda(\sigma,\sigma^n-\sigma^p)]\xi.\end{aligned}\tag{25}$$

The proper values, E, in (23) can be approximated by adding the average value in the state $\varphi'(p)$ of the term proportional to ϵ to the energy in that state. The perturbing energy is thus:

$$\begin{aligned}P=\epsilon\int\varphi'(p)^\dagger v(p)T(p)\rho_3\\ \times\int[1+\lambda(\sigma,\sigma^n-\sigma^p)]\\ \times T^{-1}(p')v(p')^*\varphi'(p')dpdp'\\ -3\epsilon\int\varphi(p)^\dagger(w,X)v(p)T(p)dp\rho_3\\ \times\int[1+\lambda(\sigma,\sigma^n-\sigma^p)]\\ \times T^{-1}(p')(w',X)v(p')^*\varphi'(p')dp'.\end{aligned}\tag{26}$$

Integrating over $|p|$ and the angles:

$$\begin{aligned}P=\tfrac{1}{2}\epsilon\bar{\xi}\rho_3[1+\lambda(\sigma,\sigma^n-\sigma^p)]^3\xi\\ +\tfrac{2}{3}\epsilon\beta^2\bar{\xi}\rho_3\{1+3\lambda(\sigma,X)(\sigma^n,X)\\ -3\lambda(\sigma,X)(\sigma^p,X)+6\lambda^2\\ -6\lambda^2(\sigma^n,X)(\sigma^p,X)+3\lambda^3(\sigma,X)(\sigma^n,X)\\ -3\lambda^3(\sigma,X)(\sigma^p,X)-\lambda^3(\sigma,\sigma^p)+\lambda^3(\sigma,\sigma^n)\},\end{aligned}\tag{27}$$

where

$$\beta=2\pi\eta\int_0^\infty\frac{E_pv(p)^2p^2dp}{E^2-E_p{}^2}.$$

The angularly dependent part of P is therefore

$$P_a(E)=-4\epsilon\beta(E)^2\bar{\xi}\rho_3\lambda^2(\sigma^n,X)(\sigma^p,X)\xi.\tag{28}$$

The effect of P_a on the electron levels may be summed over all filled levels in an approximate way, sufficient for an order of magnitude estimate. If the perturbation had been simply a change in the interaction constant $\eta\rightarrow\eta-\epsilon$ only the first term in (27) would have been obtained. A sum over the first term is, therefore, practically equal to

$$-2\epsilon\int\frac{\partial F(\eta, E)}{\partial\eta}dE.$$

The same sum modified by multiplication with $2\beta^2$ occurs as a factor in (28). β^2 can be estimated from its relation to the solutions of the secular equations derived from (24) (see Eqs. (14) and (15) of reference 3).

$$2\pi\eta\int_0^\infty\frac{Ev(p)^2p^2dp}{E^2-E_p{}^2}=\pm\tfrac{1}{2}$$

whence $2\beta^2\sim\frac{1}{2}E_0{}^2/E^2$. Thus

$$\sum P_a \cong 4\epsilon\lambda^2(\sigma^n, X)(\sigma^p, X)E_0{}^2 \times\int\frac{\partial F(\eta, E)}{\partial\eta}\frac{dE}{E^2}. \quad (29)$$

According to the considerations of magnetic moments, Eq. (13) and the subsequent discussion

$$\lambda\eta\int\frac{\partial F(\eta, E)}{\partial\eta}\frac{dE}{E^2}=\frac{-1}{16E_0}, \quad \lambda>0,$$

whence

$$\sum P_a \cong -\tfrac{1}{4}\epsilon\lambda\frac{E_0}{\eta}(\sigma^n, X)(\sigma^p, X). \quad (30)$$

$\epsilon E_0/\eta$ in most cases should be of the order of magnitude of the change in total potential experienced by two particles as they are slightly separated. The angular part of the change in interaction between neutron and proton is, therefore, related to the total change in interaction by the fraction $\frac{1}{4}\lambda$. This estimate is very rough and applies only for small separations but it appears within the range of possibility that the angular dependence over the whole range of force is quite sufficient to give the desired nonspherical character to the potential well in the deuteron.

It is further satisfactory that when the heavy particles are separated in a direction parallel to their spins the energy perturbation (30) is negative. This means that the lowest state of the deuteron should be prolate, in agreement with indications from the quadrupole moment.

Conclusion

One would conclude that a satisfactory theory of nuclear forces may be built upon an interaction of neutrons and protons with the electron-positron field. This is apparent from the above work for such functions $\Delta E(\eta, \lambda)$ for which an expansion in a series of powers of λ is valid. The value of λ has been found from magnetic moment requirements to be larger than $\frac{1}{16}$. To obtain a significant angular dependence, however, λ may not be very much less than unity. The expansions in series are valid, therefore, only for those $\Delta E(\eta, \lambda)$ which are very nearly linear functions of η. The functions $\Delta E(\eta)$ become nearly straight lines for values of η larger than E_0. According to the above considerations a satisfactory theory would require: (1) $\eta \gg E_0$, (2) $\lambda E_0/\eta \cong \frac{1}{16}$, (3) λ comparable to, but less than, unity. These conditions are self-consistent.

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