On Coupling Conditions in Light Nuclei and the Lifetimes of β-Radioactivities

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Two series of β-activities are considered. The first series involves nuclei in which the number n_p of protons exceeds the number n_n of neutrons by 1. The second series involves the nuclei for which $n_p=n_n$ is odd. Similarly two coupling conditions are considered: in approximation 1, the forces do not depend on either ordinary or isotopic spin. In approximation 2 only the independence of isotopic spin is assumed. Apart from these coupling conditions, no use is made of any special model. The lifetimes of the disintegrations of the first series can be calculated on the basis of either coupling condition if one adopts Fermi's original theory. The transitions of the second series are forbidden in this theory. If one adopts the modification of Fermi's theory which has been proposed by Gamow and Teller, one can calculate the lifetimes for both series assuming the validity of the first approximation but this cannot be done if only the validity of the second approximation is assumed. The discussion of the experimental data indicates that the 1st approximation applies quite well for the normal states of nuclei up to a mass of about 30.

1

NORDHEIM and Yost[1] were the first to point out that the absolute magnitude (not only the order of magnitude) of the lifetime of certain β-radioactive elements can be predicted on the basis of Fermi's theory.[2] The transitions they considered were between nuclei in which the number of protons and neutrons differed by ± 1.

The work of Nordheim and Yost was founded on the original formulation of Fermi's theory in which the spin does not change in allowed transitions. Gamow and Teller[3] have pointed out that Fermi's ideas allow another formulation in which the spin change is ± 1 or 0 for these transitions. Breit and Knipp[4] found evidence in the $Be^7 \rightarrow Li^7$ transformation confirming the selection rules of Gamow and Teller. Grönblom[5] considered the β-activities of He^6 and of the elements which were the subjects of Nordheim and Yost's investigation from this point of view. Using Hartree's approximation for the wave functions, he found in all these cases that the β-spectrum should be composite (excepting the case of He^6 where it should be simple). This was in agreement with the hypothesis put forward by Bethe, Hoyle and Peierls[6] who attempted to explain the deviation of the observed energy distribution of the β-rays from the distribution following from Fermi's theory by postulating a composite nature for all transitions in which the energy distribution has been observed so far. For the elements considered in Grönblom's work the intensity formulas derived here are identical with Grönblom's. It is shown, however, that the assumption of a spin independent Hamiltonian, and only this, is necessary for the validity of these formulas.

The experimental information already used by Nordheim and Yost, together with similar data on disintegrations involving elements in which the number of protons is odd and equal to the number of neutrons, will be used in the present paper to determine the amount of deviation from Russell-Saunders coupling and from the inaccuracy of wave functions obtained by using a symmetric Hamiltonian.[7] The former question has become particularly important lately for three reasons. First, the discovery of the quadripole moment of the deuteron[8] made it evident that, at least in this case, the deviation from Russell-Saunders coupling cannot be neglected. The work of Way[9] on the interpretation of magnetic and quadripole moments of heavy elements points strongly in the same direction.

[1] L. W. Nordheim and F. L. Yost, Phys. Rev. **51**, 943 (1937).
[2] E. Fermi, Zeits. f. Physik **60**, 320 (1934).
[3] G. Gamow and E. Teller, Phys. Rev. **49**, 895 (1936).
[4] G. Breit and J. K. Knipp, Phys. Rev. **54**, 652 (1938).
[5] B. O. Grönblom, Phys. Rev. **56**, 508 (1939).
[6] H. A. Bethe, F. Hoyle and R. Peierls, Nature **143**, 200 (1939). Kikuchi, Watase, Itoh, Takeda, Yamaguchi, Proc. Phys. Math. Soc. Japan **21**, 52 (1939). See also A. I. Alichanian and V. Berestezky, Phys. Rev. **55**, 978 (1939).
[7] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936).
[8] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, Jr., and J. R. Zacharias, Phys. Rev. **55**, 318 (1939).
[9] K. Way, Phys. Rev. **55**, 963 (1939).

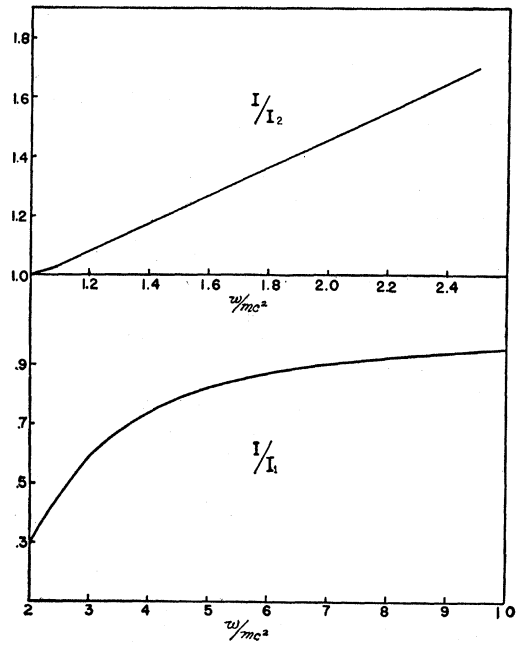

FIG. 1. Correction factors for the approximate formulas (2) for Fermi's function $I(w)$ of (1a).

Finally, the meson theory of nuclear forces[10] yields a Hamiltonian on the basis of which the Russell-Saunders coupling cannot be expected to be valid in nuclei. On the other hand, Barkas' work[11] shows that one can successfully systematize the mass defects of the lighter elements by using a picture on the basis of which one would expect the above-mentioned simple coupling conditions to be prevalent in these nuclei.

2

We shall use for our calculations the formalism[12] which has been employed before by Breit and Knipp.[4] The transition probability λ_{if} is given[13] in Fermi's theory by

$$\lambda_{if}=GI(w), \tag{1}$$

$$I(w)=(w^2-1)^{\frac{1}{2}}(w^4/30-3w^2/20-2/15)+\tfrac{1}{4}w\ln(w+(w^2-1)^{\frac{1}{2}}), \tag{1a}$$

where $w=1+\epsilon_l/mc^2$ with m the electronic mass and ϵ_l the upper limit of the energy of the emitted β-ray. One can represent (1a) for large and small w, respectively, by the formulas

$$I_1(w)=w^5/30, \tag{2}$$
$$I_2(w)=(16\sqrt{2}/105)(w-1)^{7/2}=0.215(w-1)^{3.5},$$

and the correction factors I/I_1 and I/I_2 are plotted in Fig. 1. The G is given, in Fermi's original theory by $G_0=g_0^2f_0$ where

$$f_0=|(\psi_f,\tfrac{1}{2}(\tau_{\eta 1}+\tau_{\eta 2}+\cdots+\tau_{\eta n})\psi_i)|^2. \tag{3a}$$

Here, g_0 is the interaction constant in Fermi's theory, ψ_i and ψ_f the wave functions of the disintegrating and product nuclei in the formalism using isotopic spin.[14] The $\tau_{\eta i}$ are the same operators as the s_{yi}, acting, however, on the isotopic spin coordinate τ_i instead of the ordinary spin coordinate σ_i. In the formulation of Gamow and Teller,[3] there are, in addition to G_0 three other terms of the character $g_1^2f_x$ where

$$f_x=|(\psi_f,\tfrac{1}{2}(\tau_{\eta 1}s_{x1}+\tau_{\eta 2}s_{x2}+\cdots+\tau_{\eta n}s_{xn})\psi_i)|^2. \tag{3b}$$

The G of (1) is then given by

$$G=g_0^2f_0+g_1^2f_1=g_0^2f_0+g_1^2(f_x+f_y+f_z). \tag{4}$$

The decay constant λ_i of the state ψ_i is, of course, $\Sigma_f\lambda_{if}$.

The form (1a) is characteristic for Fermi's theory.[15] In most of the following applications $I_1(w)$ could be substituted for $I(w)$ without greatly affecting the conclusions.

Clearly, (4) is only an approximate expression for G, valid, according to Fermi, if the dimension of the nucleus is very much smaller than the wave-length of the emitted β-particle and neutrino. If this is not the case, additional terms will enter, in analogy to the terms corresponding to the quadripole etc. radiation in ordinary light emission.[16] The three terms (3b) can be thought of as corresponding to polarizations of

[10] H. Fröhlich, W. Heitler and N. Kemmer, Proc. Roy. Soc. **A166**, 154 (1939). H. J. Bhabha, Proc. Roy. Soc. **A166**, 501 (1938). N. Kemmer, Proc. Camb. Phil. Soc. **34**, 354 (1938). H. A. Bethe, Phys. Rev. **55**, 1261 (1939).

[11] W. H. Barkas, Phys. Rev. **55**, 691 (1939).

[12] E. Wigner, Phys. Rev. **51**, 106, 947 (1937).

[13] Cf. reference 2 and E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 7 (1935).

[14] W. Heisenberg, Zeits. f. Physik **77**, 1 (1932). J. H. Bartlett, Phys. Rev. **49**, 102 (1936). W. Elsasser, J. de phys. et rad. **7**, 312 (1936). B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936) and reference 12.

[15] G. E. Uhlenbeck and S. Goudsmit have shown that (1a) is the sum of statistical factors and has as simple a form as can be expected. For modifications of (1a), due to the Coulomb field, cf. reference 13.

[16] Cf. F. Hoyle, Proc. Roy. Soc. **A166**, 249 (1939).

the electron neutrino field in the X, Y and Z directions. The term (3a) would correspond to the emission of a scalar (longitudinal) wave.

It should be mentioned, perhaps, in this connection that the operators of (3a) and (3b) are both even and thus correspond to the β-particle and neutrino being emitted in the $^1S_0$ and $^3S_1$ states, respectively. Making use of a four-valued spin for the heavy particles, operators can be found, which—though not involving the space coordinates,—have an odd character and correspond to the electron-neutrino pair to be emitted in P states. In the case of such an interaction one would obtain in the approximation in which the nuclear dimension can be neglected only transitions from even to odd terms, and conversely. In the present theory one has, in the same approximation, only transitions without change of parity. Operators of the above-mentioned character would give no β-decay for a free neutron and the β-decay would, in general, depend on relativistic effects and on the part of the wave function which corresponds to negative kinetic energies.

3

We shall consider, first of all, the matrix elements (3a) and (3b) from the point of view of the "first approximation" of reference 12. In this approximation, the ordinary spin and isotopic spin variables play the same role which the spin variables play in the theory of atomic spectra, assuming Russell-Saunders coupling. The matrix elements (3a) and (3b) can be calculated under these conditions without making further special assumptions about the wave functions, just as, for instance, the Zeeman splitting can be calculated in the corresponding theory of atomic spectra. The operators

$$T_\eta = \tfrac{1}{2}(\tau_{\eta 1} + \tau_{\eta 2} + \cdots + \tau_{\eta n}) = \tfrac{1}{2}\sum_k \tau_{\eta k}, \quad (5a)$$

$$Y_{\eta x} = \tfrac{1}{2}\sum \tau_{\eta k} S_{xk}; \quad Y_{\eta y} = \tfrac{1}{2}\sum \tau_{nk} S_{yk}; \quad Y_{\eta z} = \tfrac{1}{2}\sum \tau_{\eta k} S_{zk}, \quad (5b)$$

are infinitesimal operators of rotation acting in the four-dimensional unitary space of the σ, τ just as the S_x, S_y, S_z were infinitesimal operators of rotation acting in the two-dimensional space of the σ alone. It follows from this, first of all, that only transitions between the "fine structure components" of a multiplet are "allowed," transitions between different multiplets always "forbidden," both for the Fermi and the Gamow-Teller matrix elements.

Since, in general, the initial state of a β-transition is the lowest state of that element, it is, according to Fig. 4, reference 12, the term of a multiplet which has the largest possible $T_\zeta = \frac{1}{2}(n_n - n_p)$ for that multiplet.[17] The reason for this is that the energy of a multiplet, in general, increases with increasing value of the largest T_ζ of that multiplet so that the lowest term of any element belongs to a multiplet, the largest T_ζ of which is as small as possible, i.e., equal to the T_ζ of the element in question. If the element makes a β-transition, T_ζ will change; if it increases (positron emission) the transition will naturally go to another multiplet. If T_ζ is decreased by the transition, this could lead to another component of the same multiplet —were it not that all other components have higher energies, the energy decreasing with increasing T_ζ within a multiplet on account of the Coulomb energy. These conditions are represented in Fig. 2, which is a partial reproduction of Fig. 4, reference 12. It follows from this that

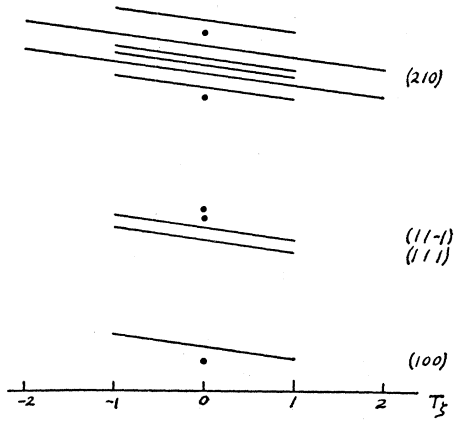

FIG. 2. The energy of a $(PP'P'')$ (numbers at right) multiplet increases with increasing P. The P is the highest T_ζ occurring in the multiplet. Within the multiplet the energy decreases with increasing T_ζ. Every multiplet is the lowest for $T_\zeta = P$ and $T_\zeta = -P$, only (100) is the lowest for all three T_ζ, *viz.* -1, 0, 1.

[17] The largest T_ζ of a multiplet is the P of the $(PP'P'')$ symbol of the multiplet. The $(PP'P'')$ symbol is defined in the papers of reference 12 where it is denoted by (STY). The term multiplet (without qualification) is used in the present paper for a $(PP'P'')$ "supermultiplet."

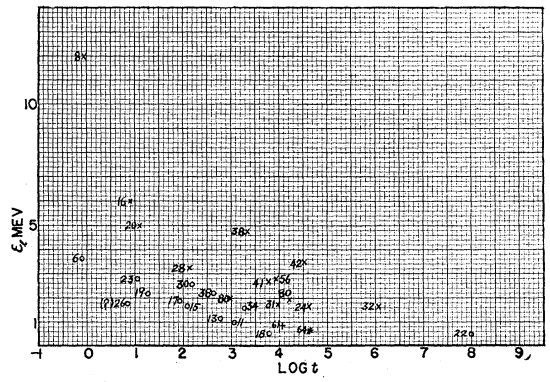

FIG. 3. Energy *vs.* $\log_{10}$ of lifetime for light nuclei. Small circles represent disintegrations considered in the present paper, + positron emitters, × electron emitters.

practically all β-transitions are "forbidden" transitions.

There are two exceptions to this: the lowest states of elements with negative T_ζ belong to multiplets the highest T_ζ of which is $|T_\zeta|$. The elements $T_\zeta=-\frac{1}{2}$ give the well-known series of positron emitters discussed already in reference 1. Second, the multiplet (100), occurring for nuclear masses of the form $4k+2$, is the lowest one both for the elements with $T_\zeta=0$ and $T_\zeta=\pm 1$. One will expect, therefore, "allowed" transitions for these elements. It is in agreement with this that for light elements only transitions involving elements of this character are on the first Sargent curve (Fig. 3). Several of the transitions occurring in heavier elements are, however, at any rate very near to this curve and we shall return to this point later.

In order to calculate the absolute value of expressions (3a) and (3b) it is simplest to calculate first

$$(\psi_i,\ T_\eta^2\psi_i)=\sum_f |(\psi_f,\ T_\eta\psi_i)|^2 \qquad (6a)$$

and

$$(\psi_i,\ (Y_{\eta x}^2+Y_{\eta y}^2+Y_{\eta z}^2)\psi_i = \sum_f |(\psi_f,\ Y_{\eta x}\psi_i)|^2+|(\psi_f,\ Y_{\eta y}\psi_i)|^2 + |(\psi_f,\ Y_{\eta z}\psi_i)|^2, \qquad (6b)$$

where f runs over all possible states. The expressions (6a), (6b) represent the sum of the G of one initial state with respect to all possible final states.

Since the T_η^2 etc. are infinitesimal operators of the unitary group of the σ, τ space, the expressions (6a) and (6b) will assume the same values for all ψ which belong to the same row of the same representation of this group. The representation of the unitary group depends only on the multiplet so that we can calculate (6a) and (6b) with a particularly simple example of that multiplet.[18] For the $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ multiplet we can choose a problem of one single particle. In this case $T_\eta^2=\frac{1}{4}\tau_\eta^2=\frac{1}{4}$ and similarly $Y_{\eta x}^2=Y_{\eta y}^2=Y_{\eta z}^2=\frac{1}{4}$. This holds for all wave functions of such a multiplet. Hence, in this case

$$(\psi_i,\ T_\eta^2\psi_i)=\tfrac{1}{4}, \qquad (7a)$$

$$(\psi_i,\ (Y_{\eta x}^2+Y_{\eta y}^2+Y_{\eta z}^2)\psi_i)=\tfrac{3}{4}. \qquad (7b)$$

For the multiplet $(\frac{1}{2}\frac{1}{2}-\frac{1}{2})$ which occurs for masses of the form $4k+3$ we can choose a three-particle problem in which all three particles are in equivalent s states. It follows then from the theory of holes[19] that Eq. (7a), (7b) remain valid for this multiplet also as the matrix elements for the states of the closed shell all vanish.

There are six states in the multiplet (100) and the corresponding representation of the four-dimensional unitary group is six-dimensional. The simplest example in this case contains two particles. Dropping the factor of the wave function which depends on the space coordinates, we can write for the six wave functions

$$\begin{aligned} &b_{-1}(\tau_1\tau_2)a(\sigma_1\sigma_2),\quad b_0(\tau_1\tau_2)a(\sigma_1\sigma_2),\quad b_1(\tau_1\tau_2)a(\sigma_1\sigma_2),\\ &\qquad a(\tau_1\tau_2)b_1(\sigma_1\sigma_2),\\ &\qquad a(\tau_1\tau_2)b_0(\sigma_1\sigma_2),\\ &\qquad a(\tau_1\tau_2)b_{-1}(\sigma_1\sigma_2). \end{aligned} \qquad (8)$$

Here $a(uv)$ is the wave function of the singlet state $a(u,v)=2^{-\frac{1}{2}}[(u-1)(v1)-(u1)(v-1)]$ and b_1, b_0, b_{-1} the three triplet wave functions

$$\begin{aligned} b_1(uv)&=(u1)(v1),\\ b_0(uv)&=2^{-\frac{1}{2}}[(u-1)(v1)+(u1)(v-1)],\\ b_{-1}(uv)&=(u-1)(v-1), \end{aligned}$$

where $(u-1)$ and $(u1)$ stand for δ_{u-1} and δ_{u1}, respectively. The arrangement of the wave func-

[18] Cf. E. Wigner, *Gruppentheorie* etc. (Braunschweig, 1931), p. 302.

[19] Cf. E. U. Condon and G. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, 1935), p. 284.

tions in (8) corresponds to Fig. 2. Calculating $(\psi, T_\eta^2\psi)$ for the six functions of (8), we obtain, respectively,

$$\begin{matrix} \frac{1}{2} & 1 & \frac{1}{2} \\ & 0 & \\ & 0 & \\ & 0 & \end{matrix} \qquad (9a)$$

The values of $(\psi, (Y_{\eta x}^2+Y_{\eta y}^2+Y_{\eta z}^2)\psi)$ are, in the same arrangement,

$$\begin{matrix} \frac{3}{2} & 0 & \frac{3}{2} \\ & 1 & \\ & 1 & \\ & 1 & \end{matrix} \qquad (9b)$$

It is easy now to calculate the expressions (3a), (3b) themselves by means of the ordinary theory of spectra, i.e., considering the τ as parameters. The operator T_η is a scalar-scalar operator (i.e., scalar in the space coordinates and also scalar in the ordinary spin coordinates). Hence, the selection rules are $\Delta J=0$, $\Delta L=0$, $\Delta S=0$, $\Delta m=0$. Thus only one matrix element is different from zero for any initial state in which the isotopic spin is as large as it can be for this multiplet. The square of this matrix element is $\frac{1}{4}$ in the case of the $(\frac{1}{2}\frac{1}{2}\pm\frac{1}{2})$ multiplet and $\frac{1}{2}$ in the case of the (100) multiplet. In both cases the matrix element is independent of the azimuthal quantum number. The β-spectra of the elements in which the number of protons differs by one from the number of neutrons becomes simple. Since the lowest state of the nuclei with $2k+1$ protons and $2k+1$ neutrons is assumed to be a triplet state, the β-transition between this state and the lowest state of the nucleus with $2k$ protons and $2k+2$ neutrons is forbidden because the latter is a singlet state. The allowed transitions with intensities are represented in Fig. 4(a) and 5(a) for the $(\frac{1}{2}\frac{1}{2}\pm\frac{1}{2})$ and the (100) multiplets, respectively.

On the other hand, the operators $T_{\eta x}$, $T_{\eta y}$, $T_{\eta z}$ form the components of a scalar-vector operator (scalar in space, vector in ordinary spin coordinates) and the intensities of the transitions from the different components of a spin multiplet to the different components of another spin multiplet can be computed by the well-known Hönl-Kronig intensity formulas.[20] Of course, the role of S and L has to be interchanged in these as the operator responsible for light emission is a vector in its dependence on space coordinates, while our operators are vectors in their dependence on (ordinary) spin coordinates. In case of the multiplet (100) the transitions from the $b_1(\tau_1\tau_2)a(\sigma_1\sigma_2)$ state all lead to the $a(\tau_1\tau_2)b(\sigma_1\sigma_2)$ states, none to the $b_0(\tau_1\tau_2)a(\sigma_1\sigma_2)$ states, as can be seen from the fact that according to (9b) the sum of the intensities of all transitions starting from or leading to $b_0(\tau_1\tau_2)a(\sigma_1\sigma_2)$ is zero. This allows the calculation of all intensities and these are represented in Figs. 4(b) and 5(b) and (c).

The results contained in Fig. 4(a) were given before by Breit and Knipp.

Before going over to the discussion of the "second" etc. approximation, it should be remarked that all terms of the multiplet (100), e.g., have the same energy in the "first approximation." Their splitting is due to forces which involve the ordinary and isotopic spin coordinates. The wave functions of the states with $J=L+1$ and $J=L-1$ are, for very small splitting, independent of the form of the forces involving the isotopic and ordinary spin coordinates, and this is the situation for all wave functions in case of S terms. The situation is, however, in general different for the two states with $J=L$. Fig. 5(b) and (c) is drawn in the

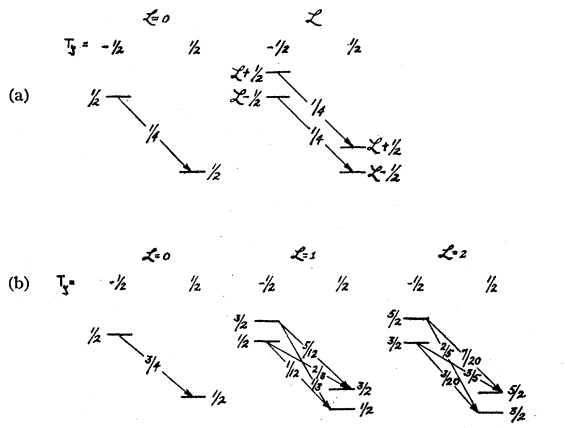

FIG. 4. (a) Value of f_0 of (3a) for multiplets $(\frac{1}{2}\frac{1}{2}\pm\frac{1}{2})$ i.e., for the stable states of elements in which the difference of the number of protons and neutrons is 1. The number at each level denotes the J value of that level. (b) Value of f_1 of (4), (3b) for the same elements. The arrangement of the levels is such that the lower J has the lower energy. This is purely arbitrary and the reverse arrangement occurs just as frequently as that used in the figure. Cf. Table II.

[20] Cf. reference 19, p. 241 or reference 18, p. 297.

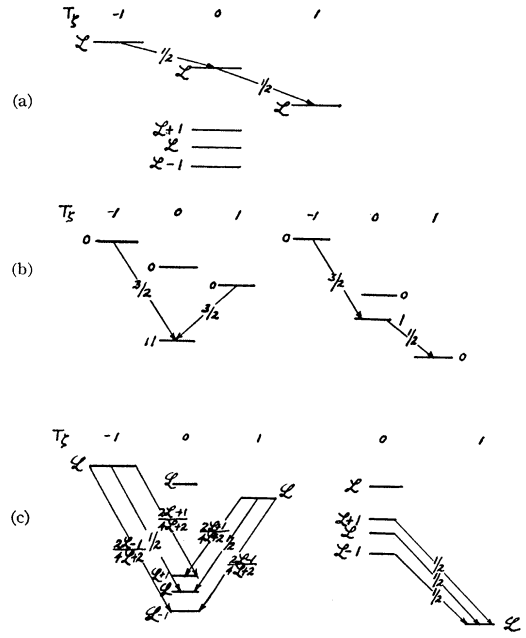

FIG. 5. (a) Allowed transitions and values of f_0 of (3*a*) for the multiplet (100). The transitions to and from the lowest state of elements with odd number of protons and neutrons is forbidden in the original theory of Fermi. (b) For S terms. Allowed transitions and values of f_1 of (3*b*) for the same elements. In the diagram at the left the odd-odd elements are the stable ones (Li^6, B^{10}, N^{14}). The diagram at the right represents the conditions at higher A where the odd-odd element ($T_\zeta=0$) is unstable. (c) For higher L. Values of f_1 of (3*b*) for the same elements as in (b), drawn, however, on the assumption that the lowest term is not an S term. The number at each level denotes the J value of that level. The arrangement of the levels is again arbitrary (cf. Fig. 4(b)). The $T_\zeta=-1$ part is omitted in the last diagram.

customary way which involves in this case the assumption that forces involving both ordinary and isotopic spin coordinates can be neglected when determining the splitting. This is, however, questionable and it is possible, therefore, that the two states with $J=L$ are actually linear combinations of triplet and singlet states.[21] The transition probabilities to these two states will be proportional to the amount of singlet wave function contained in them if Fermi's original formulation is adopted. It is proportional to the amount of triplet wave functions contained in them if the matrix elements of 5(b) are responsible for the transition.

[21] The L–S coupling is not a consequence of the first approximation, except in special cases. All terms have a definite L in this approximation but not necessarily a definite S, unless there is only one S for the corresponding element in the whole ($PP'P''$) multiplet. This is, according to the theory, always true for the lowest states (giving $S=0$ for even, $S=\frac{1}{2}$ for odd elements), with the exception mentioned in the text, *viz.* elements with odd number of protons and odd number of neutrons.

4

From the point of view of the "second approximation" (forces identical for protons and neutrons, depending, however, on ordinary spin) the matrix elements of T_η still can be calculated on the basis of general considerations[4] because T_η plays in this theory the same role which S_y plays in ordinary atomic theory. The result is, of course, the same as given in the previous section. On the other hand, the matrix elements of $Y_{\eta x}$ etc. cannot be calculated by assuming only approximation 2, and no regular relation between energy and lifetime could be expected if this operator determined the transition probabilities. In fact, the matrix elements connecting states of different multiplets would become as great as the matrix elements between states of a single multiplet. Since the sum of squares of the matrix elements between one state and all others is limited and there are very many of them, each single matrix element would become very much smaller than one would expect from the previous section. The same would hold for both operators T_η and $Y_{\eta x}$, etc., in the third and fourth approximation, i.e., if the forces affecting protons and those affecting neutrons were so different that no particular relations existed between the wave functions of isobars with different isotopic numbers. These relations are summarized in Table I. The column App. 1 is based upon the assumption that one obtains good wave functions even if one neglects all forces involving ordinary or isotopic spin coordinates, App. 2 that one can neglect only forces depending on isotopic spin coordinates. App. 3

TABLE I. *Dependence of transition probabilities under various conditions.*

		App. 1	App. 2	App. 3 or 4
$n_p-n_n=\pm1$	a	r	r	s
	b	r	s	s
$n_p-n_n=0,\ \pm2$	a	r	s	s
	b	r	s	s

refers to neglecting only ordinary spin forces and App. 4 that no such approximations are permissible. The letters a and b refer to the operators 5(a) and 5(b), the letter r means that the transition probabilities depend regularly on the energy, s means that the transition probabilities are small and their dependence on the disintegration energy shows no marked regularity.

5

Table II contains a summary of the experimental information on nuclei with a difference of one in the number of protons and neutrons ($(\frac{1}{2}\frac{1}{2}\pm\frac{1}{2})$ multiplet). The first column contains the atomic mass, the second the symbol of the lowest term, as assumed at present. The third contains in the first row the observed upper limit of the position spectrum in Mev, in the second row the calculated values for this quantity, assuming a Coulomb energy of[22] $\frac{1}{2}\times 2.35 n_p(n_p-1)A^{-\frac{1}{3}}$ mc^2. From this, two electron masses and the neutron-proton energy difference, i.e., 1.78 Mev, were subtracted in order to obtain the upper limit of the positron spectrum. One sees that the agreement is quite good, it is within 7 percent for the whole Coulomb energy. This shows that for the nuclei in question at any rate there is no appreciable blowing up by the electrostatic repulsion of the protons. The column I gives the values of I, given in (1a), calculated for both values of ϵ_l. The next column gives the observed half-life t in seconds, the following one 10^{-3} $It/\ln 2$ which should be a constant on assumption (3a) for the matrix element. The last column gives the product of the previous one and f_1 taken from Fig. 4(b), for the lowest state of both initial and final nucleus. This number should be a constant on assumption (3b) if there were only transitions to the lowest state of the final nucleus. Actually, a certain average of the last two columns should be taken,

[22] The constant of this equation was assumed to be 2.4 in the second paper of reference 12 and 2.32 in reference 11. More accurate calculations on the Coulomb energy were made by E. Feenberg and J. Knipp, Phys. Rev. **48**, 906 (1935). S. Share, Phys. Rev. **50**, 488 (1936). E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937). H. A. Bethe, Phys. Rev. **54**, 436 (1938). H. Brown and D. Inglis, Phys. Rev. **55**, 1182 (1939). The fact that ϵ_l is given by the Coulomb energy alone was already pointed out by W. A. Fowler, L. A. Delsasso and C. C. Lauritsen, Phys. Rev. **49**, 561 (1936).

TABLE II. *Summary of experimental information on nuclei with a difference of one in the number of protons and neutrons.*

A	TERM	ϵ_l	I	t	It/l	$It f_1/l$
7	$P_{\frac{3}{2}}$	<0.09		$3.72\cdot 10^6$		
		0.05?	7.10^{-5}?			
9	$P_{\frac{3}{2}}$					
		0.46?	0.21?			
11	$P_{\frac{3}{2}}$?	1.05	5.23	1230	9.3	3.90
		.87	2.49		4.45	1.85
13	$P_{\frac{3}{2}}$	1.21	9.30	630	8.45	3.50
		1.24	10.3		9.35	3.90
15	$P_{\frac{1}{2}}$	1.72	38.6	125	6.95	0.60
		1.58	28.6		5.15	0.43
17	$S_{\frac{1}{2}}$?	1.93	65.6	64?	6.05	4.55
		1.91	63.7		5.90	4.40
19	$S_{\frac{1}{2}}$	2.20	123	20.3	3.50	2.60
		2.23	123		3.60	2.70
21						
		2.49	198			
23		2.82	348	11.6	5.85	
		2.82	348		5.85	
25		2.99?				
		3.10	523			

$A=7$. Cf. reference 4. Hill and Valley, Phys. Rev. **55**, 678 (1939).
$A=11$. Bonner and Brubaker, Phys. Rev. **50**, 308 (1936).
$A=13$. Kikuchi, Watase, Itoh, Takeda, Yamaguchi reference 6. E. M. Lyman, Phys. Rev. **55**, 234 (1939).
$A=15$. Fowler, Delsasso and Lauritsen, Phys. Rev. **49**, 561 (1936).
$A=17$. Kurie, Richardson and Paxton, Phys. Rev. **49**, 368 (1936).
$A=19$, 23 and 25. White, Delsasso, Fox and Creutz, Phys. Rev., preceding paper.

as has been shown by Grönblom. The calculation has not been carried out for $A=7$ since K-capture plays an important role in this case.[4] The fluctuations of the last two columns are probably within the limits of experimental error as they are very sensitive to small errors in ϵ_l. The very small value of $It f_1/l$ for $A=15$ could be explained by assuming that the $^2P_{3/2}$ level is very near to the ground state $^2P_{1/2}$ in this case. Certainly, the figures of the last column do not increase strongly with increasing A as would be expected by assuming a breakdown of the underlying formulas for the larger A. The corresponding numbers, calculated for all transitions not considered in this paper (such as $F^{20}\rightarrow Ne^{20}$) are a hundred times greater. We must infer that *either* approximation 1 is valid for these elements, *or* Fermi's original matrix element (3a) is responsible for the major part of the decay constant and approximation 2 holds sufficiently well. In the latter case, most of the transitions would go from the lowest state of the parent element to the lowest state of the product nucleus.

We now go over to the nuclei with a mass number $A=4k+2$. The first three of these are

TABLE III. *Summary of experimental information about nuclei with mass number 4k+2.*

A		I	t	It/l	Itf_1/l
6	3.6	1000	0.8	1.15	1.75
10	?				
14	?				
18	0.55	0.43	6700	4.15	2.10
22	0.55	0.43	9.5×10^7	5.9×10^4	2.9×10^4
26	?		7		
30	2.6	241	170	59.2	29.5
34	2.6	240	1970	680	340
38	2.2	116	460	77	39

$A=6$. Bjerge and Broström, quoted from reference 5.
$A=18$. Snell, Phys. Rev. **51**, 142 (1937), DuBridge, Barnes and Buck, Phys. Rev. **51**, 995 (1937), Jasaki and Watanabe, Nature **141**, 787 (1938).
$A=22$. F. Oppenheimer and Tomlinson, Am. Phys. Soc. Stanford Meeting (1939).
$A=26$. The experiments on the limit of the β-ray spectrum widely disagree. If the values of R. O. Frisch (Nature **133**, 721 (1934)) or Brandt (Zeits. f. Physik **108**, 726 (1938)) of 1.8 or 1.5 Mev should prove correct the present considerations became untenable. Almost equal difficulties would be encountered if Magnan's value of 4.6 Mev (Comptes rendus **205**, 1147 (1937)) were correct.
$A=30$. The experiments greatly disagree in this case also. The value of Meye (Zeits. f. Physik **105**, 232 (1937)) appears most reliable at present.
$A=34$. Sagane, Phys. Rev. **50**, 1141 (1936).
$A=38$. Hurst and Walke, Phys. Rev. **51**, 1033 (1936), Ridenour and W. J. Henderson, Phys. Rev. **52**, 889 (1936).

electron emitters, the others positron emitters. Unfortunately, the experimental information, summarized in Table III, is much less conclusive than that for the odd elements.

Unlike the constancy of the It/l column of Table II, the constancy of the It/l column of Table III cannot be brought into connection with the validity of Fermi's original assumption (3*a*) since (3*a*) would give 0 for the probability of the β-disintegration of these elements.

Disregarding $A=22$ for the time being, Table III allows a decision to be made between the two alternatives left open at the consideration of Table II. Assuming only the validity of the second approximation it seems to be very difficult to account for the small values of Itf_1/l particularly for that of F^{18}. The short lifetime of Al^{26} points in the same direction. The energy of this disintegration must be much greater than the values given by Frisch and by Brandt. On the other hand, the calculated value for the difference of the Coulomb energies of Al^{26} and Mg^{26} is 4.85 Mev. This gives 3.05 for the disintegration energy of the state of Al^{26} which corresponds in the second approximation to the normal state of Mg^{26}. This state of Al^{26} is a singlet state and should lie above the normal state of Al^{26}. Hence $\epsilon_l \leq 3.05$ Mev in this case; $\epsilon_l=2.80$ Mev would give $Itf_1/l=1.75$. All this speaks strongly in favor of the assumption that the first approximation is valid for the normal states up to about $A\sim26$ and g_1 is at least of the same order of magnitude as g_0 and even allows Grönblom's assumption of $g_0=0$.

On the other hand, the increase of Itf_1/l beginning at about $A=30$ indicates that approximation 1 becomes inaccurate from this point on. The values of ϵ_l for the elements in question are not very well known but it is unlikely that they are in error by an amount sufficiently large to bring the last column down to about 2.3.

The case of $A=22$ remains puzzling on every interpretation. It does not appear reasonable to assume that approximation 1 gives a wave function which is contained in the correct one only with a coefficient of about $(1.7/2.9\times10^4)^{\frac{1}{2}}=0.007$. It is much more likely that the states of Na^{22} and Ne^{22} between which the β-transition is observed do not correspond to the same multiplet.

6

If the interpretation of the experimental data as given in the previous section is correct, one would expect that the distinction between allowed and forbidden β-transition becomes increasingly vague at about $A=30$. For a given ϵ_l, the lifetime of the allowed transitions should increase, that of the forbidden ones decrease and the matrix element for both should tend to some average value. When this condition is reached, only the selection rules for J and parity would remain valid ($\Delta J=\pm1$ or 0, no change in parity allowed). The breakdown of one set of selection rules while others remain intact is a familiar phenomenon in atomic spectra.

The test to which the approximation 1 is put by the investigation of the lifetimes of β-transitions is a very severe one. One must expect a breakdown of the formulas derived in Section 4 whenever two $(PP'P'')$ multiplets overlap. Such an overlapping of two multiplets will not greatly influence the validity of the expressions for the energy which were used in references 11 and 12, because the multiplets will differ only in their azimuthal quantum numbers, not in their $(PP'P'')$. If the spin dependent forces are not too strong, they will leave $(PP'P'')$ and in most

cases even S (cf. reference 21) good quantum numbers while L and the grouping of states into multiplets will be more easily destroyed. If approximation 1 makes S a good quantum number it will remain a good quantum number in spite of small perturbations which already affect L. This picture suggests e.g. that the $^2P_{3/2}$ and $^2D_{3/2}$ parts are contained in the normal state of an element with $J=\frac{3}{2}$ to a much greater extent than, say, the $^4S_{3/2}$ part.

It may be worth while to note that according to the considerations presented here the following β-spectra should be simple: He^6, F^{18} and probably Al^{26}, and all elements of Table II in which the ground state is a ^2S state. This is very probable for $A=19$ since the magnetic moment of F^{19} is very nearly equal to the magnetic moment of the proton. These elements should offer a possibility for a simple test of the ideas of reference 6.

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On the Nuclear Magnetic Moments of the Isotopes of Rubidium and Chlorine*

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The molecular beam, magnetic resonance method has been applied to the measurement of the nuclear gyromagnetic ratios of Rb^{87}, Rb^{85}, Cl^{37} and Cl^{35}. The g values are 1.820 ± 0.006, 0.536 ± 0.002, 0.454 ± 0.002 and 0.546 ± 0.002, respectively. The magnetic moments of Rb^{87} and Rb^{85}, obtained from the observed g values and the known nuclear spins, are 2.741 ± 0.009 and 1.345 ± 0.005, respectively. The substantial agreement of the moment ratio μ_{87}/μ_{85}, 2.038, found by this method, with that obtained from atomic beam measurements indicates that any contribution to h.f.s. by a form of interaction between electron and nucleus not electromagnetic in character is small. A nuclear moment of 1.365 ± 0.005 is obtained for Cl^{35} if one takes the band spectra spin value of 5/2 for this nucleus. No information concerning the spin of Cl^{37} is available.

INTRODUCTION

A BRIEF preliminary report[1] of the measurements, by the molecular beam, magnetic resonance method,[2] of the nuclear magnetic moments of the isotopes of rubidium and chlorine has already been published. It is the purpose of this paper to present the results in further detail.

The hyperfine structure of atomic energy states of the rubidium isotopes has been investigated both by spectroscopic methods[3] and by the method of atomic beams.[4] Both methods yield nuclear spins of 3/2 and 5/2 for Rb^{87} and Rb^{85}, respectively. Approximate values of the magnetic moments have been obtained from the h.f.s. of the ground state by use of the Goudsmit, Fermi-Segrè formula. In accordance with our experience with the other alkali nuclei, [2,5] the magnetic moments so calculated may be expected to differ by not more than 10 percent from the values of nuclear moments directly measured by the present method. The differences arise from the fact that exact wave functions for the alkali atoms are not known. On the basis of the assumption that hyperfine structure of atomic energy states is due solely to the magnetic interaction of the nuclear moment with the external electrons, the ratio of the nuclear moments of two isotopes of the same element can be obtained from the ratio of the observed

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[4] S. Millman and M. Fox, Phys. Rev. **50**, 220 (1936).

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