β-Decay and Spin of Light Nuclei[1]

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The β-decay of the nuclei He^6, Be^7, N^{13}, O^{15} and F^{17} is investigated from a point of view of the Gamow-Teller modification of the Fermi-theory. It is assumed that the lowest state of each nucleus forms a spin-multiplet. The various transition probabilities are given as functions of the total angular momentum I of the various states of the nuclei. An accurate knowledge of the β-spectrum would lead to a definite prediction of the spins of the nuclei. For the time-constant in the β-theory a value of $3\cdot 10^3$ sec. is obtained as compared to the previous value of $11\cdot 10^3$ sec.

An investigation of the shape of the β-spectrum of the light nuclei[2] seems to lead to a better agreement with the original theory of Fermi than with the modified theory of Uhlenbeck and Konopinski.[3] The observed asymmetry of the electron spectrum is assumed to be due to the fact that the β-decay leads to several states of the final nucleus, from which a transition to the ground state with the emission of γ-rays occurs. In fact such a γ-radiation has been observed by Richardson,[4] who by the process $N^{13}\rightarrow C^{13}+\beta^++\nu_0$ found a γ-radiation of the energy 285±15 kev with an intensity of 0.8 quanta per disintegration. This figure has lately been corrected by Richardson[5] and Lyman[6] who found the figures 0.4 and 0.25 quanta per decay for the intensity, the energy remaining unchanged by 280 kev. It seems thus to be proved that the decay leads to two states of C^{13}, the ground state and an excited state, with an energy difference of 280 kev, though the ratio between the transition probabilities is only inaccurately known.[7]

The small energy difference predicts (considering that the regarded nucleus is light) that the levels most probably are levels of the same multiplet. The same can be concluded from the processes $O^{15}\rightarrow N^{15}+\beta^++\nu_0$ (energy difference =500 kev), $F^{17}\rightarrow O^{17}+\beta^++\nu_0$(+900 kev) and $Be^7\rightarrow Li^7-\beta^-+\nu_0$ where a K electron is captured into two states with an energy difference of 550 kev, whereas in the other processes, investigated by Bethe, Hoyle and Peierls ($B^{12}\rightarrow C^{12}+\beta^++\nu_0$, $F^{20}\rightarrow Ne^{20}+\beta^++\nu_0$) the large energy to be expected of the γ-rays (5 and 2 Mev, respectively) seems to predict two entirely different states.

The original theory of Fermi for the interaction between the heavy particles and the field of the light particles would not yield any transitions between states of different spins. A direct evidence that such transitions can occur is however given by the large probability of the process $He^6\rightarrow Li^6+\beta^++\nu_0$, which is probably a $^1S-{}^3S$ transition.[8] We will therefore use the spin-dependent modification of the Fermi theory as given by Gamow and Teller,[9, 10] using for the Hamiltonian of the interaction the expression

$$H=g(Q\sigma^H\psi^*\sigma^L\varphi+Q^*\sigma^{H*}\psi\sigma^{L*}\varphi^*), \qquad (1)$$

where g is the normal Fermi constant, σ^H and σ^L are the Pauli spin-operators, acting on the heavy and light particles, respectively. Q is an operator, which converts a neutron into a proton, Q^* is the inverse operator. ψ and φ are wave functions for the electron and the neutrino, respectively, taken as plane waves. The relativistic theory is here used only for the light particles.

This gives for the transition probability per time-unit the expression:[11]

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[1] The subject of this paper was first presented at the Washington Meeting, 1939.

[2] H. A. Bethe, F. Hoyle and R. Peierls, Nature **143**, 200 (1939).

[3] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 107 (1935).

[4] J. R. Richardson, Phys. Rev. **53**, 610 (1936).

[5] J. R. Richardson, Phys. Rev. **55**, 609 (1939).

[6] E. Lyman, Washington Meeting, 1939.

[7] S. Kikuchi, Y. Watase, J. Itoh, E. Takeda, S. Yamaguchi, Proc. Phys. Math. Soc. of Japan **21**, 41 (1939).

[8] M. Goldhaber, unpublished.

[9] G. Gamow and E. Teller, Phys. Rev. **49**, 895 (1936).

[10] The fact that the transition $Be^7\rightarrow Li^7$ speaks in favor of the Gamow-Teller theory has already been pointed out by G. Breit and J. Knipp, Phys. Rev. **54**, 652 (1938).

[11] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 193 (1936). (Integrating over ϵ in the formula 210 according to the formula 213 on the page 193.)

$$w=\frac{1}{\tau_0}f(W)|G|^2. \tag{2}$$

τ_0 is here the decay-constant, and $f(W)$ is given as:

$$f(W)=(W^2-1)^{\frac{1}{2}}\left(\frac{W^4}{30}-\frac{3}{20}W^2-\frac{2}{15}\right)+\frac{W}{4}\log\left[W+(W^2-1)^{\frac{1}{2}}\right], \tag{3}$$

W being the energy including the rest mass of the electron, in units of mc^2. (The original Fermi theory leads to the same expression.) A graphic representation of the function $f(W)$ is given in Fig. 1. One has

$$|G|^2=\frac{1}{3}\left\{\left|\int u_n{}^*\sigma_x v_m d\tau\right|^2+\left|\int u_n{}^*\sigma_y v_m d\tau\right|^2+\left|\int u_n{}^*\sigma_z v_m d\tau\right|^2\right\}, \tag{4}$$

where u_n and v_m are the wave functions of the heavy particle. We will make the assumption that u_n and v_m approximately belong to the same set of orthogonal functions. This assumption seems to be justified because the Hamiltonians, corresponding to the systems of functions denoted by u and v, respectively, differ only by a Coulomb term, and thus the neglected effect is only one of the second order in the Coulomb energy.

$$\mathrm{He}^6\rightarrow\mathrm{Li}^6+\beta^++\nu_0.$$

The β-spectrum has lately been measured by Bjerge and Brostrøm.[12] The half-lifetime has been found to be 0.8±0.1 sec., the upper limit of the spectrum was found at 3.5 Mev. If we write the wave function of the initial state of He^6 in the form:

$$\psi=n(1)n(2)\cdot\frac{\alpha(1)\beta(2)-\alpha(2)\beta(1)}{2^{\frac{1}{2}}}u_{\mathrm{space}}$$

and apply on this function the operator $Q_1\sigma_{z1}+Q_2\sigma_{z2}$ (corresponding to the fact that we have two terms like (1) in the complete interaction, one for particle 1 and one for particle 2), we obtain:

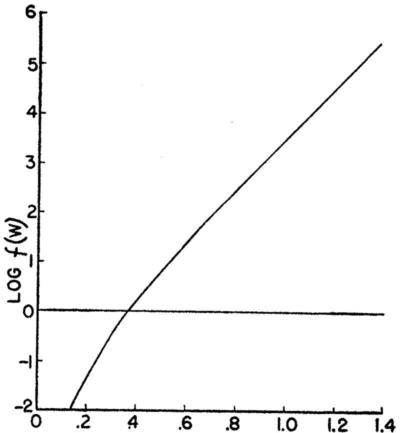

FIG. 1. Value of $f(W)$ *vs.* log W (Eq. (3)).

$$2^{\frac{1}{2}}\cdot\frac{n(1)p(2)-n(2)p(1)}{2^{\frac{1}{2}}}\cdot\frac{\alpha(1)\beta(2)+\alpha(2)\beta(1)}{2^{\frac{1}{2}}}u_{\mathrm{space}}=2^{\frac{1}{2}}\varphi_{\mathrm{charge}}\chi_{\mathrm{spin}}u_{\mathrm{space}}.$$

This yields[13] $|G|^2=2$.

Taking for W the value $W=7.9$ and for the half-lifetime the value 0.8 sec. we thus obtain for the decay constant the value $\tau_0=2.2\cdot 10^3$ sec.

$$\mathrm{Be}^7\rightarrow\mathrm{Li}^7-\beta^-+\nu_0.$$

This process means a capture of a K electron. The final nucleus Li^7 has an excited level at 450 kev. The K-electron capture leading to this state has been observed by means of the γ-rays emitted in the subsequent transition of Li^7 from the excited state to the ground state. From the intensity of these γ-rays as compared to the number of neutrons emitted in the original formation of Be^7 it has been concluded that roughly 10 percent lead to the excited state of Li^7. The total energy involved in the K-electron capture can best be obtained from the threshold of the reaction $\mathrm{Li}^7+\mathrm{H}^1\rightarrow\mathrm{Be}^7+n^1$.[14] This threshold has been observed by Hill and Valley[15] to be 1.75 Mev. Subtracting 0.75 Mev

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[12] T. Bjerge and K. J. Brostrøm, Det. Kgl. Danske Videnskabernes Med. **16**, no. 8 (1938).

[13] Compare, however, G. Breit and J. Knipp, reference 10.

[14] R. B. Roberts, N. P. Heydenburg and G. L. Locher, Phys. Rev. **53**, 1016 (1938). L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **54**, 657 (1938).

[15] J. E. Hill and G. E. Valley, New York Meeting, 1939.

from the difference between neutron and proton we find 1.00 Mev for the energy of the neutrino emitted when the K-electron capture leads to the ground state of Li^7. When the transition leads to the excited state the neutrino energy will accordingly be 0.55 Mev.—The observed half-lifetime of Be^7 is 43 days.[16]

The probability per time unit of an electron capture is:[17]

$$w=(\pi^2\epsilon^2\rho/\tau_0)(\hbar/mc)^3|G|^2. \tag{5}$$

ϵ means the energy of the outgoing neutrino, ρ is the density of the two K electrons in the nucleus:

$$\rho=2Z_{\mathrm{eff}}^3/\pi a_0^3 \quad (a_0=\text{Bohr radius}). \tag{6}$$

The ground state of Be^7 is in the Hartree model given as a ${}^2P_{3/2}$ state,[18] which is in agreement with the experiments.[19] We shall here, as well as with the other nuclei, consider the ground state of the formed nucleus to be the same as for the initial nucleus, which is equivalent to the assumption that the splitting is not magnetic, but is due to the Thomas force.[20] The excited state involved in the process is assumed to be a ${}^2P_{1/2}$ state.

If we write the Pauli eigenfunctions for the hole in the p shell in the form:[21]

$$\begin{aligned} u_{nl,\ j=l+1/2,\ m} &= \frac{1}{(2l+1)^{\frac{1}{2}}}\begin{pmatrix} k_1 Y_{l,\ m-1/2}(\vartheta,\ \varphi) \\ -k_2 Y_{l,\ m+1/2}(\vartheta,\ \varphi)\end{pmatrix} R_{nl}(r), \\ u_{nl,\ j=l-1/2,\ m} &= \frac{1}{(2l+1)^{\frac{1}{2}}}\begin{pmatrix} k_2 Y_{l,\ m-1/2}(\vartheta,\ \varphi) \\ k_1 Y_{l,\ m+1/2}(\vartheta,\ \varphi)\end{pmatrix} R_{nl}(r), \end{aligned} \tag{7}$$

where $k_1=(l+m+\frac{1}{2})^{\frac{1}{2}}$, $k_2=(l-m+\frac{1}{2})^{\frac{1}{2}}$ we obtain (averaging over m in the initial state and summing over m in the final state):

$$\begin{aligned} |G({}^2P_{3/2}\rightarrow{}^2P_{3/2})|^2 &= \tfrac{1}{4}\sum_{m=-3/2}^{m=+3/2}|k_1^2-k_2^2|^2=5/9 \\ |G({}^2P_{3/2}\rightarrow{}^2P_{1/2})|^2 &= \tfrac{1}{4}\sum_{m=-1/2}^{m=+1/2}|k_1k_2|^2=4/9 \\ |G({}^2P_{1/2}\rightarrow{}^2P_{1/2})|^2 &= \tfrac{1}{2}\sum_{m=-1/2}^{m=+1/2}|k_1^2-k_2^2|^2=1/9 \\ |G({}^2P_{1/2}\rightarrow{}^2P_{3/2})|^2 &= \tfrac{1}{2}\sum_{m=-1/2}^{m=+1/2}|k_1k_2|^2=8/9. \end{aligned} \tag{8}$$

Using these figures for the matrix element and the value 3.7 for Z_{eff}, we obtain as the result: 20 percent of the transitions lead to the excited state if the ground state is a ${}^2P_{3/2}$ state. 72 percent of the transitions lead to the excited state if the ground state is a ${}^2P_{1/2}$ state.

A comparison with the experimental figure thus predicts a ${}^2P_{3/2}$ state, in agreement with the known spin of Li^7.

For the decay constant we obtain the value $\tau_0=1.8\cdot10^3$ sec. If the spin of Be^7 were $\frac{1}{2}$, different from the spin of Li^7, the value would be $\tau_0=2.2\cdot10^3$ sec.

$$N^{13}\rightarrow C^{13}+\beta^{+}+\nu_0.$$

The result of the investigation of this process by Richardson and Lyman has been already mentioned. The upper limit of the spectrum lies at 1.20 Mev[22] and the half-lifetime has been found to be 11 minutes. For the intensity of the γ-rays we choose the value found by Lyman from the analysis of his measurements of the β-spectrum, according to which 25 percent of the transitions should lead to the excited state. For the spin of C^{13} the Hartree model predicts the value $\frac{1}{2}$ whereas the α-particle model[23] leads to the value $\frac{3}{2}$. The spin of C^{13} has not been measured.* The calculation of the transition probabilities to the ground state and to the excited state could allow a decision between these values, but since the experimental figure has been subject to considerable change, our

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[16] The figures for the half-lifetimes for Be^7, N^{13}, O^{15} and F^{17} are taken from H. A. Bethe and M. S. Livingston, Rev. Mod. Phys. **9**, 245 (1937).

[17] H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. of Japan **17**, 465 (1935). C. Møller, Phys. Rev. **51**, 84 (1937).

[18] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937).

[19] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 215 (1936).

[20] D. R. Inglis, Phys. Rev. **50**, 783 (1936).

[21] H. A. Bethe, *Handbuch der Physik*, Vol. 24, No. 2 (Leipzig, 1933), p. 309.

[22] E. Lyman, Washington Meeting, 1939.

[23] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 684 (1938). B. O. Grönblom and R. E. Marshak, Phys. Rev. **55**, 229 (1939). R. G. Sachs, Phys. Rev. **55**, 825 (1939).

* *Note added in proof.*—C. H. Townes, Stanford meeting, 1939, finds the value $\frac{1}{2}$ for the spin of C^{13} from measurements on the $C^{13}-C^{13}$ band spectrum.

conclusions should be taken with reservations. This is *a fortiori* true for the nuclei N^{15} and O^{17} where the observations could be still more uncertain.

Using the value 280 kev for the energy difference between the levels and the figures (8) for the matrix elements we obtain: 19 percent of the transitions lead to the excited state if the ground state is a $^2P_{3/2}$ state. 71 percent of the transitions lead to the excited state if the ground state is a $^2P_{1/2}$ state. Hence a spin $\frac{3}{2}$ for the ground state of C^{13} is favored.

For the decay constant we obtain the value $6.0 \cdot 10^3$ sec. if the spin of the ground state is $\frac{3}{2}$. If the spin of the ground state were $\frac{1}{2}$ the corresponding figure would be $3.2 \cdot 10^3$ sec.

$$O^{15} \rightarrow N^{15} + \beta^+ + \nu_0.$$

The β-spectrum of O^{15} is measured by Fowler, Delsasso and Lauritsen[24] and investigated from the point of view of the Fermi theory by Bethe, Hoyle and Peierls, who conclude that the transitions lead to two states with an energy difference of 500 kev, 40 percent of the transitions leading to the excited state. The observed half-lifetime is 126 sec. and the upper limit of the spectrum lies at 1.7 Mev. The value $\frac{1}{2}$ for the spin of the ground state of N^{15} was found by Wood and Dieke from the band spectrum of the N^{15} molecule.[25] This value was predicted by Rose and Bethe theoretically.

Using again the formulas (8) we obtain that 63 percent of the transitions should lead to the excited state. Under the false assumption that the ground state has the spin $\frac{3}{2}$ the corresponding figure would be 14 percent.

The value of the decay constant as deduced from this process is $\tau_0 = 2.1 \cdot 10^3$ sec.

$$F^{17} \rightarrow O^{17} + \beta^+ + \nu_0.$$

The half-life of F^{17} is 70 sec. The upper limit of the β-spectrum lies at 2.05 Mev. From the measurements of Fowler, Delsasso and Lauritsen, Bethe, Hoyle and Peierls found that the transition leads again to two states with an energy difference of about 900 kev. It is predicted that 60 percent of the transitions lead to the excited state.

As the p shell is completely filled with O^{16}, the ground state of F^{17} has to be either an $^2S_{1/2}$, a $^2D_{3/2}$ or a $^2D_{5/2}$ state. We will assume that the states involved are ^2D states.

For the matrix elements we obtain, again using the eigenfunctions (7):

$$\begin{aligned} |G(^2D_{5/2} \rightarrow {}^2D_{5/2})|^2 &= 7/15 \\ |G(^2D_{5/2} \rightarrow {}^2D_{3/2})|^2 &= 8/15 \\ |G(^2D_{3/2} \rightarrow {}^2D_{3/2})|^2 &= 1/5 \\ |G(^2D_{3/2} \rightarrow {}^2D_{5/2})|^2 &= 4/5. \end{aligned} \tag{9}$$

If the ground state is a $^2D_{3/2}$ state, 29 percent of the transitions should lead to the excited state as against 5 percent if the ground state is a $^2D_{5/2}$ state. A comparison with the estimated figures of Bethe, Hoyle and Peierls thus decisively favors a spin $\frac{3}{2}$ for the ground state, though a considerable discrepancy still remains. Even a slight change in the predicted value of the γ-ray would change the figure noticeably. Another possibility is that the $^2S_{1/2}$ state is involved in the process, which could be taken as implied by the large energy difference between the levels. A decision can, however, be obtained only by more precise experiments. Even a direct determination of the spins of O^{17} as well as for C^{13}, would be very desirable. This process yields $\tau_0 = 2.5 \cdot 10^3$ sec.

CONCLUSIONS

An accurate knowledge of the shape of the β-spectrum of the light nuclei as well as of the energy and the intensity of the γ-rays emitted by the final nuclei would allow some more precise statements regarding the mechanism of the decay. This would even make possible a prediction of the spin of the nuclei involved.

For the decay constant we obtain values between $1.8 \cdot 10^3$ sec. and $6.0 \cdot 10^3$ sec. The probable value thus seems to lie around $3 \cdot 10^3$ sec. This is to be compared with the previous value of $11 \cdot 10^3$ sec. calculated as an upper limit from the process $N^{13} \rightarrow C^{13} + \beta^+ + \nu_0$ by Bethe and Critchfield.[26]

The author wishes to express his gratitude to Professor H. A. Bethe for the suggestion of this problem as well as for many helpful discussions.

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[24] W. A. Fowler, L. A. Delsasso and C. C. Lauritsen, Phys. Rev. **49**, 569 (1936).

[25] R. H. Wood and G. H. Dieke, J. Chem. Phys. **6**, 908 (1938).

[26] H. A. Bethe and C. L. Critchfield, Phys. Rev. **54**, 248 (1938).