On the Production of the Hard Component of the Cosmic Radiation

II. Protons or Neutral Particles as Primaries

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The general requirements for understanding the hardening of the cosmic radiation underground together with a terrestrial origin of the hard component are discussed. It is shown that the hypotheses of photons or protons as primaries for mesons lead to the consequence that the inversion of the production processes (i.e., absorption of mesons by production of photons or protons accompanied by multiplication of the mesons) must have cross sections smaller by a factor of order 10 compared to the production processes. As a possibility to avoid this difference, the hypothesis of neutral mesons as additional primaries is discussed.

1. GENERAL REQUIREMENTS FOR EXPLAINING THE ORIGIN OF THE HARD COMPONENT

NE of the principal problems which has to be solved in accounting for the terrestrial origin of the hard component of the cosmic radiation is the simultaneous explanation of the following two facts. Firstly, the occurence of penetrating particles of very high energy (100 Bev and over) and secondly the enormous penetrating power and the hardening of the hard component underground. That this hardening is actually due to the behavior of the ionizing particles themselves is shown by the near equality of the absorption by matter brought between or above the counters of a telescope.¹

In the preceding paper² (quoted henceforth as I) it has been pointed out that the possibility of creating an energetic meson leads by necessity to a mechanism of absorption by the inversion of the production process. Roughly speaking one would expect that a particle will have a range only of the order of magnitude of the free path of the primary with regard to the production process. Any acceptable hypothesis regarding the terrestrial origin of the mesons has therefore to satisfy the condition that the production mechanism does not entail too strong an absorption or multiplication at the same time.

It was shown in (I) that it would be somewhat difficult to account for the mesons with photons (or electrons) as sole primaries. For these the meson production would be in competition with the normal radiative processes and would be crowded out, unless uncomfortably large production cross sections were assumed at high energies.

If we admit other primaries than photons, we have unfortunately much less information to go on since both the initial distribution of such incoming primaries as well as the laws of production of secondaries are unknown. A discussion of this subject has therefore to be of a very preliminary nature, and one can only try to account qualitatively for the most typical features of cosmic-ray effects. However, some general conclusions seem possible.

The hardening of the cosmic radiation underground can hardly be explained otherwise than by ascribing a larger average range to rays of higher energy. Therefore, in case the rays responsible for the hardening suffer, in addition to an energy loss independent of energy, a discontinuous absorption, the absorption coefficient has to decrease for an increasing energy; and the same must be true for the production cross section. In such a picture the meson production would have to extend over a considerable depth range, the more energetic primaries being effective at greater depths. We shall see in the next section, that it is possible to account for a power absorption law and a hardening in this way, even if it is assumed that in the primary process mesons of all energies

¹ W. M. Nielsen and K. Z. Morgan, Phys. Rev. **54**, 245 (1938); V. C. Wilson, Phys. Rev. **53**, 908 (1938). Though these measurements have been made only down to a depth of 60 m H_2O equivalent, the smooth continuation of the absorption curve to considerably greater depths leads to rays will continue downwards. ² L. W. Nordheim and H. M. Hebb, Phys. Rev. 56, 494 (1939).

from the primary energy down to very low energies are produced.

It remains, however, difficult to explain the equality of absorption with an absorber between or above the counters of a cosmic-ray telescope, which equality means that the number of particles produced in the absorber is much less than the number which is actually absorbed. It is quite possible to make the ratio between both absorption coefficients a constant. In order to have a ratio near unity, however, the average range either of the primaries or of the secondaries has to be much larger than that of the others for the same energy. In the first case (the range of the primaries is the longer) the absorption will in the main be determined by the primaries, in the second case it will be determined by secondaries.

We shall illustrate these points in Section 2 on a simple model. As has already been emphasized, the details of this model should not be taken too literally. In the remaining sections we shall then attempt to give a discussion of the most plausible mechanisms for the production of mesons.

2. On the Mechanism of the Hardening of the Cosmic Radiation

We suppose that there are two kinds of particles, the primaries, whose distribution as function of their energy E and of depth below the top of the atmosphere is P(E, x), and the secondaries (mesons) with a distribution function S(k, x), where k is the energy of the mesons. We assume furthermore a probability $\phi(E, k)dk$, that a primary E creates a secondary in the interval dk per unit path where ϕ is supposed to decrease with increasing E. The number of mesons produced in a layer dx is then

$$R(k, x)dx = dx \int_{k}^{\infty} P(E, x)\phi(E, k)dE.$$
 (1)

As the absorption of the mesons seems to be well represented as a range absorption, we assume for them a linear law of energy loss. The equation determining the distribution function S(k, x) is then

$$\partial S(k, x) / \partial x = \beta \partial S / \partial k + R(k, x),$$
 (2)

which is solved for the boundary condition

S(k, 0) = 0, i.e., no incoming mesons, by

$$S(k, x) = \int_0^x R(k + \beta z; x - z) dz.$$
 (3)

Here β is the specific energy loss assumed to be independent of energy. The total number of mesons with an energy $>k_0$ at depth x will be then

$$S(x) = \int_{k_0}^{\infty} S(k, x) dk$$
$$= \int_{0}^{x} dz \int_{\beta z + k_0}^{\infty} R(k; x - z) dk, \quad (4)$$

i.e., all mesons created with an energy $k > k_0 + \beta z$ at a distance z above the place of observation can reach x. The true absorption is determined by the number of mesons which actually stick in a layer dx, i.e., those which are created at z with a k between $k_0 + \beta z$ and $k_0 + \beta z + \beta dx$. It is then

$$A(x)dx = dx \int_0^x R(k_0 + \beta z; x - z)dz$$
$$= dx\beta S(k_0, x), \quad (5)$$

where k_0 is the sticking energy, i.e., the energy at which the meson can be considered as absorbed. It will be of the order of the rest energy. The total change of S with depth, the number G(x) of generated mesons and the number A(x) of absorbed mesons are connected by the relation

$$\frac{dS(x)}{dx} = \int_{k_0}^{\infty} R(k, x) dx - \beta S(k, x)$$

= $G(x) - A(x).$ (6)

These formulas will remain approximately correct even if we assume, in addition to a linear energy loss β_0 , a true absorption with an absorption coefficient $\kappa = \beta_1/k$, i.e., inversely proportional to the energy, as such an absorption will make the average range again proportional to the initial energy; the constant β will stand then for the sum $\beta = \beta_0 + \beta_1$.

For the primaries we similarly assume a linear energy loss and a true absorption, which latter will be due, in first instance, to the production of absorbed per unit layer

the mesons. We neglect the reproduction³ of primaries by secondaries. The equation for P(E, x) will then be

$$\frac{\partial P(E, x)}{\partial x} = \beta \frac{\partial P(E, x)}{\partial x} - \kappa(E) P(E, x), \quad (7)$$

where $\kappa(E)$ is the absorption coefficient. The solution of this equation in terms of the initial distribution P(E, 0) is

$$P(E, x) = P(E + \beta x; 0)$$
$$\times \exp\left[-\int_{0}^{x} \kappa(E + \beta z) dz\right]. \quad (8)$$

If we neglect the ionization loss β this reduces to

$$P(E, x) = P(E, 0) \exp\left[-\kappa(E)x\right].$$
(8a)

We give two examples of the application of these formulas. In the first one it is assumed that a primary of energy E can create r secondaries of energy k=E/r with a probability α/E . The latter quantity also gives the absorption coefficient for the primaries. For the initial distribution we take a power law

$$Z(E, 0) = C/E^{s+1},$$

$$Z(E, x) = \frac{C}{E^3} \cdot e^{-\alpha x/E}$$
(9)

and we specify for simplicity s=2, which makes all necessary integrations elementary ones. We then obtain the following results for a depth such that $k_0 < \alpha x$ and $k_0 < \beta x$. The number of particles created per unit layer becomes

$$R(k, x) = \frac{\alpha C}{r^2 k^4} \exp\left[-\frac{\alpha x}{r k}\right], \qquad (10a)$$

the total number of primaries and secondaries

$$P(x) = \frac{C}{(\alpha x)^2}; \quad S(x) = \frac{C}{(\alpha x)^2} \left[\frac{1}{2r} \left(\frac{\alpha}{\beta} \right)^2 + \frac{\alpha}{\beta} \right] \quad (10b)$$

and the number of secondaries generated and

$$G(x) = \frac{C}{(\alpha x)^2} \frac{2}{x};$$

$$A(x) = \frac{C}{(\alpha x)^2} \frac{2}{x} \left[r + \frac{\alpha}{\beta} + \frac{1}{2r} \left(\frac{\alpha}{\beta} \right)^2 \right].$$
(10c)

We see that the features previously discussed are well verified. We have a power law for both the primary and the secondary intensity and a hardening as shown by a more rapid decrease of G(x) and A(x) with depth than the total intensity. Also the ratio A/G is constant, which leads to a constant ratio between the absorption coefficient with matter between or above counters. In order to make the latter ratio of the order unity one has either to make the secondary intensity very small compared to the primary one, i.e., $\alpha \ll \beta$, or one has to make the secondaries much less absorbable than the primaries, i.e., $G \ll A$, $\alpha \gg \beta$. In either case the absorption coefficient for matter between counters will approach the over-all value 2/x corresponding to the power law. From the inversion argument, however, a near equality of α and β would be expected.

As the second example we take a case where the energies of the secondaries produced are continuously distributed with preference to lower energies. To obtain formulas which can be handled easily we choose for the cross section and primary distribution

$$\phi(E, k)dk = \frac{\alpha}{\lg (E/k_0)} \frac{dk}{Ek}; \quad k > k_0, \quad (11a)$$

$$Z(E, 0) = (C/E^3) \log (E/k_0).$$
 (11b)

The expression (11a) gives again a total absorption coefficient $\kappa(E) = \alpha/E$ and we obtain for the number of mesons generated at x

$$R(k, x) = \frac{1}{k} \int_{k}^{\infty} \frac{\alpha C}{E^4} e^{-\alpha x/E} dE$$

= $\frac{\alpha C}{k(\alpha x)^3} \left[2 - e^{\alpha x/k} \left\{ 2 + \frac{2\alpha x}{k} + \left(\frac{\alpha x}{k}\right)^2 \right\} \right].$ (12)

For the further integration we approximate this by the asymptotic expressions for large and

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³ The consideration of this effect would lead to an integral equation similar to the one used in the theory of showers. The neglect of the reproduction is here less serious than in the shower theory as the cross sections decrease with increasing energy.

small $\alpha x/k$

$$R(k, x) = \begin{cases} \frac{2\alpha C}{k(\alpha x)^3} & \text{for } k \leq \alpha x, \\ \frac{\alpha C}{3k^4} & \text{for } k > \alpha x. \end{cases}$$
(12a)

The remaining integrations are then elementary (with some attention to the integration limits) and give the result

$$P(x) = \frac{C}{(\alpha x)^2} \lg \frac{\alpha x}{3k_0};$$

$$S(x) = \frac{C}{(\alpha x)^2} \left\{ \frac{\alpha}{\beta} \cdot \frac{13}{6} + \left(\frac{\alpha}{\beta}\right)^2 \frac{2}{3} \right\};$$

$$G(x) = \frac{C}{(\alpha x)^2} \cdot \frac{2}{x} \left[\lg \frac{\alpha x}{k_0} + \frac{1}{18} \right];$$

$$A(x) = G(x) + \frac{2}{x} S(x).$$
(13a)
(13b)

These results are very similar to those of the first example, and the same remarks apply.

It is also possible to admit only primaries with an energy larger than a critical energy E_0 , in order to take the geomagnetic effect into account. We quote only one result for later discussion. In the second example, the geomagnetic effect will be small for $\alpha x > E_0$ as then the missing primaries of lower energy will be ineffective. For smaller depths the ratio of the number of primaries to secondaries will become

$$\frac{P(x)}{S(x)} = \frac{3E_0 \lg (E_0/k_0)}{2\alpha x [\lg (E_0/\beta x) + (4/3)]}, \quad \alpha x < E_0,$$

i.e., the number of secondaries remains smaller than the number of primaries for any choice of the constants.

3. The Proton Hypothesis

The most natural choice of primaries other than electrons consists, of course, in the admission of protons, a hypothesis which has already been discussed occasionally in connection with the hard component before the discovery of the

meson.⁴ It has been advocated strongly by Johnson⁵ for reason of the east-west effect as discussed in I. Also a mechanism for the production of mesons by protons follows quite naturally from Yukawa's theory.6 It seems, however, to be rather unlikely that protons could be made entirely responsible for all of the hard component. The smallness of the geomagnetic effect demands that protons of an energy below the critical energy at the equator are either comparatively small in number, or else so highly absorbable (through the process of meson production) that their effect near sea level remains small. It is rather certain, furthermore, that only a minor fraction of the penetrating rays at sea level can be protons. This follows from the near equality of the numbers of positives and negatives, the failure to find slow protons,⁷ and the considerable number of energetic electron secondaries, produced in direct collisions.8 The percentage of protons according to these observations can hardly be as large as 20 percent, but might be considerably less. It is then necessary to assume that a single proton of an energy above the geomagnetic threshold at the equator, produces quite a large number of mesons which come down to sea level.9 The free path of a proton of 15 to 20 Bev energy therefore must be considerably less than one atmosphere. The application of the models discussed in Section 2 leads to results in keeping with the general argument above. In order to make the geomagnetic effect small and the number of secondaries large, the constant α in the absorption coefficient $\kappa = \alpha/E$ for the protons has to be made so large that $\alpha x > E_0$. With $E_0 \sim 17$ Bev (threshold energy for protons at the equator) and $x = 10 \text{ m H}_2\text{O}$ one obtains $\alpha > 1.7$ Bev/m H₂O. This would correspond to a cross section of $0.3 \times 10^{-25} (10^9 \text{ ev}/E)$

⁴ A. H. Compton and H. Bethe, Nature **134**, 734 (1934); W. Nordheim, Annales Inst. Poincaré (1936).

^b V. Nordheim, Annales Inst. Poincaré (1936).
^b T. H. Johnson, J. Frank. Inst. 227, 37 (1939); T. H. Johnson and J. G. Barry, Phys. Rev. 55, 504 (1939).
^c L. W. Nordheim and G. Nordheim, Phys. Rev. 54, 254 (1938).
⁷ C. G. Montgomery, D. D. Montgomery, W. E. Ramsey and W. F. G. Swann, Phys. Rev. 50, 403 (1936).
⁸ J. I. Hopkins, W. M. Nielsen and L. W. Nordheim, Phys. Rev. 55, 233 (A) (1939).
⁹ The same conclusion follows from the argument in I

The same conclusion follows from the argument in Ithat only a small fraction of the field-insensitive primaries can be protons while the number of hard rays at sea level is nearly equal to the total number of all field-insensitive primaries.

per nuclear particle. It is then again difficult to see how a meson of 15 to 20 Bev energy could have a range of perhaps 50 to $100 \text{ m H}_2\text{O}$ as seems to follow from Blackett's and Clay's10 comparison of the energy spectrum of the mesons and their absorption underground. The range of the mesons would have to be made greater than the range of protons of the same energy by a factor of order 10. In addition, special assumptions for protons of smaller energies would have to be made (i.e., that they either are very small in number or can produce only mesons of rather low energy), but the situation is better than for the photon hypothesis since the discrepancy does not increase with depth.

It seems difficult, otherwise, to decide between the two hypotheses. The cross sections have to be of the same order of magnitude in both cases. For protons, however, the competition of the meson production process with the radiative processes is absent, and therefore a smaller number of primaries would be required. As practically all of the protonic energy would be given over to mesons, the protons would have to bring in only about 10 percent of the total cosmic-ray energy, an amount which is certainly compatible with our knowledge regarding the multiplication of the primary distribution (compare I).

The assumption of a combined photonic and protonic origin of the mesons would not improve the situation very much. If we assume that the cross section for photons of high energies are of the same order as those for medium energies (7 to 18 Bev) as determined in I, then we can account only for 10 to 20 percent of all the mesons at sea level and the situation regarding the protonic part is not much changed.

4. NEUTRAL PARTICLES AS PRIMARIES TO THE HARD COMPONENT

The seriousness of the inversion difficulty might be a matter for debate. It seems, however, that one is justified to go to some length to avoid it. It will evidently remain as long as charged particles are assumed as primaries for the mesons, since the geomagnetic effect makes it necessary that primaries above 18 Bev are the most effective ones. It is suggestive, therefore, to attempt to make a neutral primary radiation responsible for at least part of the hard component. It is probably more than a coincidence that the recent theories of nuclear forces, based on Yukawa's ideas, lead guite naturally to a suitable primary of this kind and a production mechanism for charged particles.

In the theories of nuclear forces the mesons appear as the carrier particles associated with the nuclear force field. The exchange nature of the nuclear forces is then connected with the fact that the mesons are charged. However, the apparent equality of the forces between protons and protons, and between neutrons and protons makes it necessary to assume¹¹ the additonal existence of neutral particles of properties otherwise similar to those of charged mesons. The process of producing charged mesons by neutral ones is simply a generalized scattering process. The Compton effect and other scattering of light can be described as the absorption of a photon by an electron and the immediate re-emission of another photon. Analogously, the scattering of a meson consists in its absorption by a nuclear particle (proton or neutron) and again a reemission. If now the interaction between a nuclear particle and a meson is independent of the charge of the latter, then the scattering cross sections for processes with conservation of charge and with change of charge will be the same. These scattering processes are really the simplest processes involving mesons and nuclei and should be at least as probable as the meson production by photons or protons. The usefulness of an uncharged meson to explain certain cosmic-ray effects and the possibility of change of charge has already been pointed out by Arley and Heitler¹² who proposed the name "neutretto" for the uncharged mesons. We tentatively suggest the neutretto as a possible primary of the cosmic radiation.13

not stable but β -radioactive.

¹⁰ P. M. S. Blackett, Proc. Roy. Soc. A159, 1 (1937); P. H. Clay, A. van Gemert and J. Clay, Physica 6, 184 (1939).

¹¹ N. Kemmer, Proc. Camb. Phil. Soc. 34, 354 (1938);
H. Yukawa and others, Proc. Math. Phys. Soc. Japan 20, 720 (1938);
H. A. Bethe, Phys. Rev. 55, 1130(A) (1939).
¹² N. Arley and W. Heitler, Nature 142, 158 (1938).
¹³ It should be mentioned that energetic neutrons could

be taken equally well as primaries. The objection against this hypothesis is, however, that neutrons are probably

There seems to be no inherent difficulty with regard to this hypothesis. It is, of course, necessary to admit that part of the cosmic-ray mesons are produced by field-sensitive primaries, i.e., electrons (photons) or protons, but the cross section can then be made much smaller, particularly at high energies. The theory leads, furthermore, to the expectation that all the processes mentioned are actually possible and that they have cross sections which are not too different. The occurence of more than one mode of production is therefore quite likely.

Furthermore it seems to be no harder to account for the ultimate origin of the neutrettos than for that of any other kind of cosmic-ray particles. Under conditions which make the production of mesons possible, a mixture of charged and uncharged ones would always be expected. They might then possibly be accelerated by electric fields in the charged state and then suffer a change of charge. Of the mesons which finally escape, the charged ones will decay spontaneously into electrons and neutrinos while the neutrettos remain stable until they collide with matter, as when falling on the surface of the earth.

The neutretto hypothesis evidently could serve to explain the large number of mesons of comparatively low energies which are not influenced by the geomagnetic field. It has to be remarked, however, that one difficulty still remains with this hypothesis, and that is the problem to account for the near equality of the absorption coefficients with matter between and above counters. It would be entirely removed, if the absorption of the neutral mesons were slightly stronger than that for charged mesons. It appears from the second model of Section 2 (which has been chosen with reference to the neutretto hypothesis, as (11a) is similar to the formula for the Compton scattering) that a factor 2 or 3 (i.e., $\alpha \sim 3\beta$) would be sufficient. It would be, of course, premature to base any definite conclusions on such considerations.¹⁴

Summarizing our whole discussion it appears that a final answer to the question of meson production cannot be given at the present time. We hope, at least, to have clarified the problems which have to be solved. We wish to repeat that there is no absolute objection against either the photon or the proton hypothesis or a combination of the two. If the present data on the number and distribution of primaries and mesons and the penetrating power of mesons are correct, one has to admit, however, that the probabilities for the inverse of the production processes have to be smaller by a factor of order 10 to 100 compared to the latter, and this for the same energies of primaries and mesons. If one wishes to avoid this factor, a way out might be offered by the assumption of neutral mesons as primaries.

¹⁴ It might be mentioned, however, that in Bethe's form of the theory of nuclear forces (compare reference 11) the interaction of neutrettos with nuclei is assumed to be much larger than the one of charged mesons and nuclei, and this would make the scattering of a neutretto without change of charge more probable than a conversion process.