obtained by this method of plotting identifies the current between the electrodes before breakdown as due to field emission. This curve is reversible if the voltage is not raised so high that a bridge is formed.

Since the radii of curvature of all the electrodes used in these measurements are large in comparison with their separation the electrostatic forces involved can be calculated by means of the following parallel plate equation:

$$F=4.42\times10^{-7}\epsilon(V/d)^2A,$$

where F is the force in dynes, V the voltage, d the separation in centimeters, A the area in square centimeters, and ϵ is the dielectric constant which is 1 for air. For the gold spheres this force is 1.1×10^8 dynes per square centimeter at the time the bridge is formed. The tensile strength of gold is about 2.5×10^9 dynes per square centimeter which is larger by a factor of about 20. For the steel electrodes the calculated electrostatic force and tensile strength are, respectively, 1×10^7 and 2×10^{10} dynes per square centimeter. Differences of this order of magnitude are to be expected since local increases of stress are known to exist because of surface roughness[12] and local heating of the field currents produce softening of the metal electrodes.

It is reasonable to conclude, therefore, that the bridges formed in separated contacts at voltages below the minimum sparking potential are the result of pulling metal from the electrodes by electrostatic force.[13]

In conclusion I wish to thank Dr. J. B. Johnson and Dr. F. S. Goucher for many helpful discussions during the preparation of this paper.

[12] Data of other workers studying field emission from tungsten and molybdenum wires in vacuum indicate that local fields due to roughness may be from 20 to 200 times the average: A. J. Ahearn, Phys. Rev. **44**, 277 (1933).

[13] A somewhat similar type of rupture of the electrodes was observed by A. J. Ahearn (Phys. Rev. **50**, 238 (1936)), while studying field emission from fine wires in a vacuum. In explanation of large increases of current at critical values of applied voltage he suggested a rupturing of the surface where, because of roughnesses the electrostatic force exceeded the tensile strength of the material.

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Turbulence in Convection in Gases Between Concentric Vertical Cylinders

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In a thermal diffusion apparatus consisting of two concentric vertical metal cylinders the temperature of the inner hot cylinder is measured for constant power input in the heater as a function of the pressure of the contained gas (N_2 or CO_2). The change from lamellar convection flow to turbulence produces sharply at a certain pressure a cooling which increases with further rise in the gas density. For this critical density the ratio of the remixing by convection to that by diffusion as well as the Reynolds numbers are computed. The latter are around 150; i.e., about $\frac{1}{10}$ the values found for flow through pipes. With increasing temperature difference a minimum value of the pressure for onset of turbulence is found. In order to insure lamellar convection flow in isotope separation by thermal diffusion, conditions should be adjusted so that the ratio of the two remixing effects is not more than $\frac{1}{10}$ these critical values.

In the discussion of experiments on the separation of isotopes by thermal diffusion, such as those reported by Clusius and Dickel[1] and by Brewer and Bramley,[2] it has been considered that the convection flow in the gas is lamellar.[3] The gas is confined in the gap space a few millimeters in width between two vertical surfaces, one of which is hot. For such a case, as shown in reference 3, the Reynolds number is

[1] K. Clusius and G. Dickel, Naturwiss. **26**, 546 (1938); **27**, 148 (1939).

[2] A. K. Brewer and A. Bramley, Phys. Rev. **55**, 590 (A) (1939).

[3] W. H. Furry, R. C. Jones and L. Onsager, Phys. Rev. **55**, 1083 (1939).

very much smaller than the values at which turbulence occurs in flow through pipes. However, Groth[4] has recently found that with a Clusius hot-wire apparatus having a tube diameter of 12 mm the isotope separation for xenon gas at atmospheric pressure was less when the wire surface was at 1650° than when it was at 1000°. Such a drop in the separation factor would indicate the existence of turbulence in the gas, for the theory indicates that with lamellar convection flow the efficiency of the isotope separation process should always increase with greater temperature difference between the walls. Also Brewer and Bramley have mentioned in a discussion of this method that in their apparatus consisting of concentric glass tubes, the inner one heated, they observed turbulence upon introduction of smoke into the gas.

Since the performance of thermal diffusion apparatus in separation of isotopes should be considerably better with lamellar convection flow existing in the gas, an investigation of the conditions under which the flow becomes turbulent is of value. Now if the convection is lamellar the heat transport of the gas should be independent of the pressure since the mean free path is much smaller than the gap space. But since the Reynolds number increases as the square of the density of the gas,[5] with increasing pressure spontaneous turbulence must always set in at some rather definite pressure even though at low pressures the flow is lamellar. And the heat transport of the turbulent gas should increase with further increase in gas pressure. To observe the onset of turbulence, then, one need simply measure the temperature of the hot wall, with constant power input in the heater, for various gas pressures. At a certain critical pressure the temperature of the wall should begin to drop, and this temperature should fall steadily with further increase in gas pressure.

In the design of thermal diffusion apparatus it is important to know the ratio of the remixing effect due to diffusion to that due to convection. This ratio, K_d/K, is given by Eq. (32) of reference 3:

$$\frac{K_d}{K}=\frac{1890(T_1^2+T_1T_2+T_2^2)}{(T_2-T_1)^2}\times\left(\frac{D\eta}{w^3\rho g}\right)^2, \quad (1)$$

where T_1 and T_2 are the absolute temperatures of the cold and hot walls, respectively, D is the coefficient of self-diffusion of the gas, η its coefficient of viscosity and ρ its density, $2w$ is the distance between the walls and g is the acceleration of gravity. Values of D, η and ρ corresponding to the mean temperature between T_1 and T_2 are to be used. Knowing the density of the gas when turbulence sets in, we may compute the ratio K_d/K for this critical condition. The Reynolds number R should be proportional to $(K/K_d)^{\frac{1}{4}}$. To show this we compute the velocity of flow $v(T)$ from Eqs. (18), (28) and (29) of reference 3, using in the differentiation the fact that $\rho D/\lambda\sim$const. Maximizing this expression for $v(T)$, one gets

$$v_{\max}=\frac{(2w)^2}{18\sqrt{3}}\frac{g\rho}{\eta}\left(\frac{T_2-T_1}{T_2+T_1}\right). \quad (2)$$

Now $v_{\max}=(R\cdot\eta)/(\rho\cdot l)$ so that

$$R=\frac{\rho}{\eta}(2w)v_{\max}=\frac{(2w)^3}{18\sqrt{3}}g\left(\frac{\rho}{\eta}\right)^2\left(\frac{T_2-T_1}{T_2+T_1}\right). \quad (3)$$

Comparison of (1) and (3) shows that $R\cong(K/K_d)^{\frac{1}{2}}$.

Experimental Procedure

The apparatus consisted of a pair of concentric metal cylinders 82 cm long mounted vertically. The inner copper cylinder was of $\frac{1}{2}$ inch diameter heated indirectly by a G. E. Kalrod heater held coaxially inside it by insulating bushings top and bottom. Thermocouple wires were silver-soldered into this copper tube at the mid-point, and were brought out through the space between the heater and the inner surface of the copper tube in protecting glass sheaths. Surrounding this copper cylinder was one of brass giving in one case an interval between the hot and cold walls of 6 mm, in another case 12.5 mm. The outer brass cylinders were in turn encased with a water condenser through which a good flow of water uniformly at 22°C was constantly maintained. Connecting the inner and outer cylinders at the lower end was a sylphon bellows to take up the differential expansion.

A single small-bore pipe led from the top of the apparatus to the cylinder of gas, either nitrogen

[4] W. Groth, Naturwiss. **27**, 260 (1939).
[5] Cf. Eq. (5), reference 3.

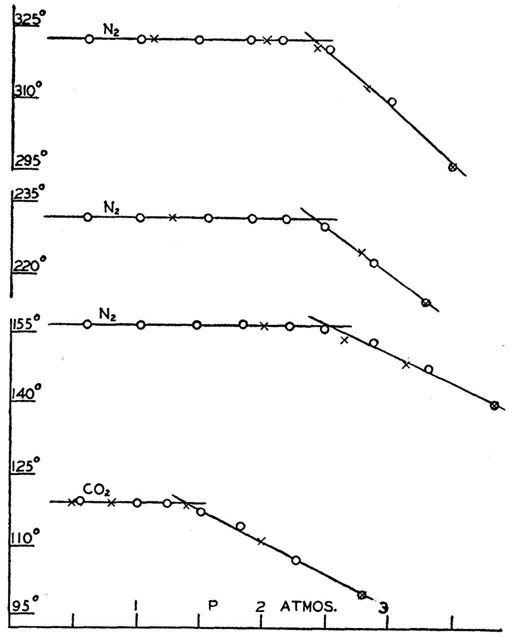

FIG. 1. Plots of the temperature of the hot wall as a function of the gas pressure in each case for constant power input in the heater. 6 mm interval between the hot and the cold surface. Circles indicate observations with pressure increasing, crosses for pressure decreasing. The break in the curves indicates the setting in of turbulence.

or carbon dioxide, with a side connection leading to a pressure gauge reading to 150 lb. and another leading through a valve to a vacuum pump and an open-arm Hg manometer. The heater current was supplied by a large storage battery to insure its constancy. Changes in gas pressure were made slowly and a sufficient time was allowed to elapse after a change for equilibrium to be established before reading the temperature. For each setting of the heater current the temperature of the hot surface was read not only for increasing gas pressure but also as a check for several pressures returning to the initial lowest value.

RESULTS

Typical curves showing the variation of the temperature of the hot cylinder with gas pressure are plotted in Figs. 1 and 2. It is to be noted that the setting in of turbulence indicated by a decreasing wall temperature with increasing gas pressure is abrupt. The critical pressure has been determined for each set of data from the intersection of the two approximately straight lines that may be drawn through the observed points. There is a small but definite variation of this critical pressure with temperature difference between the two walls, other factors remaining constant, and a marked lowering in this pressure for carbon dioxide as compared to nitrogen.

All the data with the resulting values of K/K_d and R computed with the aid of Eqs. (1) and (3) are included in Table I. In place of the unknown coefficients of self-diffusion for N_2 and CO_2 we have had to use the values of D for

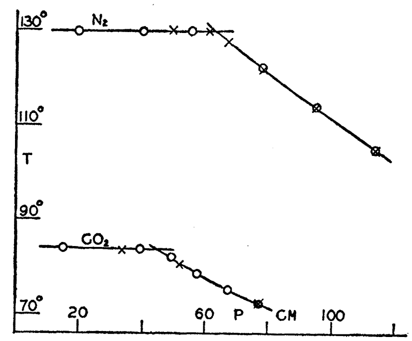

FIG. 2. Same conditions as are listed for Fig. 1 except the space between the hot and the cold surface is 12.5 mm.

O_2-N_2 and for N_2O-CO_2, respectively. The known D, η and ρ for 0°C were in every case extrapolated with the usual formulas to give the values for the mean of the wall temperatures. In agreement with the theory are the large drop in the density at which turbulence begins with increase in the gap space between the cylinders and the indication apparent in the data for CO_2 with the 6-mm gap and for N_2 with the 12.5-mm gap that a minimum value of this density occurs

TABLE I. *Critical pressures at which turbulence sets in, together with values of the ratio K/K_d of remixing by convection to that by diffusion and of the Reynolds number R for these critical densities.*

N_2				CO_2			
t(HOT WALL)	CRIT. P	K/K_d	R	t(HOT WALL)	CRIT. P	K/K_d	R
6 mm between surfaces							
78°C	2.66 atmos.	86.9	138.5	118°C	1.39 atmos.	113.2	160.0
115°	2.57 atmos.	127.2	167.9	214°	1.32 atmos.	100.0	152.7
157°	2.52 atmos.	141.5	177.7	330°	1.46 atmos.	97.7	154.6
232°	2.39 atmos.	131.6	161.1				
322°	2.37 atmos.	86.1	141.4				
12.5 mm between surfaces							
74°C	75 cm	122.9	165	83°	44.5 cm	200.9	211.9
129°	63.5 cm	125.6	167	143°	34 cm	112.0	159.5
177°	57.5 cm	97.0	147.2				
231°	52 cm	62.5	119.0				
282°	55 cm	69.9	126.6				

for a certain temperature difference. According to Eq. (3), with constant Reynolds number the squares of the densities should vary inversely as $(2w)^3$. Such a variation is roughly indicated by our data.

Discussion

The magnitude of these Reynolds numbers is of interest as an experiment in hydrodynamics. They are of the order of one-tenth the values obtained for pure pressure flow through pipes or between plates, and should be around these same values for all gases. This relative ease with which the convection flow changes from the lamellar to the turbulent type is in agreement with the indications of others as mentioned in the introduction above.

These values of the ratio K/K_d should serve as a guide in the determination of operating conditions for the separation of isotopes by thermal diffusion. To insure that the convection flow be lamellar this ratio should be no larger than one-tenth the values we find for the onset of turbulence. In the numerical example for methane gas treated by Furry, Jones and Onsager[3] this condition is strongly satisfied.

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Wide-Angle Interferences and the Nature of the Elementary Light Sources

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It was proved, in connection with the purely theoretical papers by Halpern and Doermann that a wide-angle optical interference phenomenon can be realized only if the light source is two-dimensional with its third dimension of less than 1/10,000 mm. Observations of the author made in 1911 and 1938 with a simple optical arrangement which fulfills this condition have proved that fluorescence light possesses all the properties of the simple dipole radiation.

In a paper entitled "Theoretical Evaluation of a Wide-Angle Interference Experiment"[1] F. W. Doermann and O. Halpern discuss my recent experimental investigations on wide-angle interferences.[2] They summarize their results in saying "that the source of fluorescence light has been shown to consist of electric dipoles." Stating this they not only omit mentioning that the same conclusion has already been drawn from my observations by myself, but the manner of their discussion gives the impression, that this result can be attained and understood only by a rather complicated theoretical treatment. In reality, the experiment in question is a simple one that can be performed in half a day at the utmost, and it can also be clearly explained in a few lines and without any mathematical formulae.

[1] F. W. Doermann and O. Halpern, Phys. Rev. **55**, 486 (1939).

[2] P. Selényi, Zeits. f. Physik **108**, 401 (1938); **111**, 791 (1939).

Figure 1 shows the simple arrangement used by the author. Between the glass prism *Pr* and the thin mica sheet *M* is a film of gelatin, fluorescein, the thickness of which is small in comparison with the wave-length of light. By a concentrated beam of light a small spot of the film is excited to fluorescence and the fringes due to the interference of the rays I and II are observed through a hand-spectroscope, not shown on the figure. Spectroscopic observation is necessary, because the fluorescence light is far from being monochromatic. The experiment was carried out in this form in 1911[3] and it proved the possibility of wide-angle interferences and also the coherence of the emitted light rays within a range of divergence of at least 100°. Repeating this experiment in 1938 under more

[3] P. Selényi, Ann. d. Physik (4) **35**, 444 (1911). For some similar Röntgen-interferences of W. Kossel and his collaborators see Ergebn. d. exakt. Naturwiss. **16**, 295 (1937).