We now wish to find the asymptotic behavior of e^λ, e^ν, and τ for large values of t. When t is large we obtain the approximate relation from Eqs. (36) and (27):

$$t\sim -r_0 \ln \{\tfrac{1}{2}[(R/R_b)^2-3] + R_b/r_0(1-3r_0^{\frac{1}{2}}\tau/2R_b^2)^{\frac{2}{3}}\}. \quad (37)$$

From this relation we see that for a fixed value of R as t tends toward infinity, τ tends to a finite limit, which increases with R. After this time τ_0 an observer comoving with the matter would not be able to send a light signal from the star; the cone within which a signal can escape has closed entirely. For a star which has an initial density of one gram per cubic centimeter and a mass of 10^{33} grams this time τ_0 is about a day.

Substituting (27) and (37) into (28) and (29) we find

$$e^{-\lambda}\simeq 1-(R/R_b)^2\{e^{-t/r_0}+\tfrac{1}{2}[3-(R/R_b)^2]\}^{-1}, \quad (38)$$

$$e^{\nu}\simeq e^{\lambda-2t/r_0}\{e^{-t/r_0}+\tfrac{1}{2}[3-(R/R_b)^2]\}. \quad (39)$$

For R less than R_b, e^λ tends to a finite limit as t tends to infinity. For R equal to R_b, e^λ tends to infinity like e^{t/r_0} as t approaches infinity. Where R is less than R_b, e^ν tends to zero like e^{-2t/r_0} and where R is equal to R_b, e^ν tends to zero like e^{-t/r_0}.

This quantitative account of the behavior of e^λ and e^ν can supplement the qualitative discussion given in I. For λ tends to a finite limit for $r<r_0$ as t approaches infinity, and for $r=r_0$ tends to infinity. Also for $r\leqslant r_0$, ν tends to minus infinity. We expect that this behavior will be realized by all collapsing stars which cannot end in a stable stationary state. Of course, actual stars would collapse more slowly than the example which we studied analytically because of the effect of the pressure of matter, of radiation, and of rotation.

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Formation of Ions in the Cyclotron

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Measurements of the initial ionization in a cyclotron, produced by the use of a filament, as a function of the pressure, electron emission, and dee voltage are presented. The amount of ionization is found to be too high to be simply explained by an electron passing between the region between dees only once. A theory is proposed wherein some of the electrons are caught by the changing electric field between the dees and oscillate back and forth many times during a cycle of the dee voltage. Experimental observations which conform to the theory are described.

The manner of formation of the initial ions in a cyclotron is important as it affects the intensity and homogeneity of the high energy beam. Ideally the ions should be formed in a very small region at the center of cyclotron and at the median plane. Livingston, Holloway and Baker[1] have developed a capillary type of ion source with these characteristics. However, extended filament ion sources are more generally used at present because they give larger circulating currents, and this paper will be concerned only with an analysis of this type of ion source.

The usual arrangement is for the filament to be located under a shield in which a wide slot is cut to permit the electrons to pass up along the lines of magnetic force between the dees and so ionize by collision the gas in the central region of the cyclotron. A potential difference, hereafter referred to as the emission voltage, between the filament and shield accelerates the electrons from the shielded region.

One is first interested in the magnitude of the ionization between the dees caused by the narrow beam of electrons. This was measured directly

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[1] M. S. Livingston, M. G. Holloway and C. P. Baker, Rev. Sci. Inst. **10**, 63 (1939).

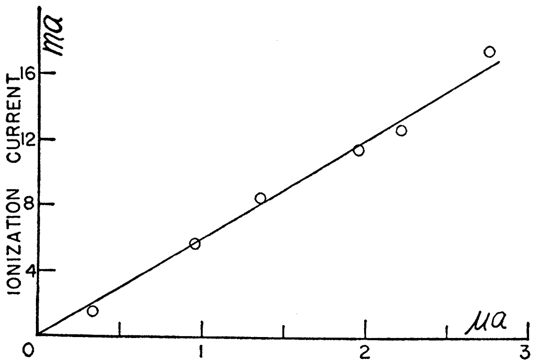

FIG. 1. The total ionization current in milliamperes is plotted as a function of the gas pressure measured in microamperes of positive ion current at an ion gauge. For the data of this curve the emission current was 0.2 of an ampere, and the emission voltage was 600 volts.

by inserting a milliammeter in the ground lead from the center tap of the inductance which was connected between the dees. With the high frequency voltage on, the ions formed are pulled over and caught by one of the dees and give rise to the measured current while the electrons, because of their smaller mass, are constrained to travel along the lines of magnetic forces in tight cycloidal motions and are not caught by the dees. Of course, the large high frequency current must be by-passed by means of a condenser and choke coil. The variation of this ionization current with the pressure of deuterium gas is shown in Fig. 1. The pressure was measured by means of an ion gauge located on the tube leading to the vacuum pumps, and all pressures are given here in units of microamperes of positive ion current in this ion gauge. As determined by a McLeod gauge, one microampere of current at the ion gauge corresponded to a pressure of about $2.7 \cdot 10^{-4}$ mm Hg in the accelerating chamber. This determination depended upon the pumping speed which was kept constant during the measurements. At the optimum operating values of pressure and electron emission, the ionization current was about 15 milliamperes. From Bleakney's[2] measurements of the relative yields of atomic and molecular ions, we would infer that roughly one-tenth of the measured total ionization was comprised of deuterons. Thus, there would be about 1.5 milliamperes of deuterons available for acceleration. It is interesting to note that in the 37-inch cyclotron usually about five percent of this can be accelerated to 8 Mev and deflected to the target chamber, while about half of it can be picked up on a probe. Hydrogen and helium gas yield similar total ionization currents, but, as judged from the relative beams, only about one-half of one percent of the helium ions are doubly charged.

How the ionization current varies with electron emission depends on the position of the filament with respect to the dees, the manner in which the filament is shielded, and whether the emission current is varied by changing the filament temperature or the emission voltage. Fig. 2 indicates a typical variation of ionization current as the emission was increased by changing the filament temperature. While increasing the emission voltage always increases the emission, there is not always a corresponding increase in ionization current, and usually there exists a value of emission voltage corresponding to a maximum amount of ionization and a maximum beam. This optimum voltage is extremely dependent on position of the filament and the shielding. Fig. 3 shows that the ionization current is almost independent of the voltage between the dees—the initial increase of ionization with dee voltage being attributed to the increased ability of the dees to catch the ions.

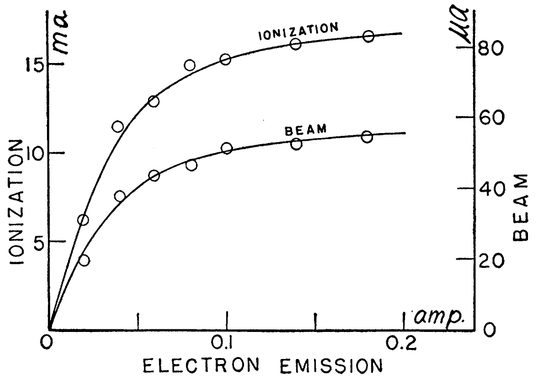

FIG. 2. A typical curve of the total ionization current in milliamperes is plotted as a function of the electron emission current in amperes. The beam current, which is also plotted, is seen to vary proportionally to the total ionization. This is for a single line filament; but if a hair pin or spiral filament is used, the beam does not always vary proportionally to the total ionization. The gas pressure was 3.0 microamperes, and the emission voltage was 600 volts when the above data were taken.

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[2] W. Bleakney, Phys. Rev. **35**, 1180 (1930).

If the filament were very narrow and were located electrically midway between the dees, there would be no vertical component of the dee field along the electron's path and its vertical motion would not be affected by the dee voltage. The amount of ionization expected could then be calculated from the data of Bleakney[2] or Compton and Van Voorhis.[3] Thus, for an emission current of 0.3 ampere, emission voltage of 600 volts, and pressure at $6 \cdot 10^{-4}$ mm Hg, only about one milliampere of total ionization would be expected, whereas more than ten times this amount has always been observed. As the electrons are in a crossed electric and magnetic field when traveling between the dees, their horizontal path is cycloidal, and one might expect this to lengthen the path. However, calculations show the cycloids to be extremely small—the diameter of the generating circle is about $2 \cdot 10^{-4}$ cm at most—and the calculated increase in path length is negligible.

To explain the large currents observed, we must remember that the peak voltage between dees is of the order of 100,000 volts and that the chance of the filament being electrically centered between dees is very small. After an electron leaves the shielded region, its subsequent path is conditioned almost entirely by the large dee field. In general the filament will be closer to one dee than the other, and the electrons will only be able to leave the filament region when the voltage on the near dee is positive. When the near dee is negative, the electrons will travel out a short distance until they lose the energy given to them by the emission voltage. If the phase is such that the voltage of the near dee is becoming increasingly negative, the electric field will be greater when an electron is being accelerated back toward the filament so that the electron will return to the filament with a gain in energy and will cause heating of the filament. On the other hand, when the voltage of the near dee is becoming less negative, the electron will return with less energy and cannot repenetrate the emission field to reach the filament as the magnetic field restricts the electrons from moving sideways—they will probably build a space charge which prevents other electrons from leaving the filament during this part of the voltage phase.

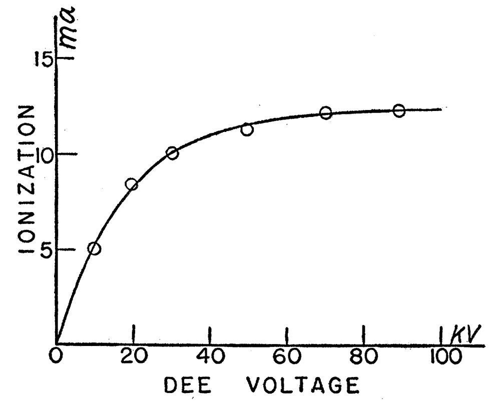

FIG. 3. The total ionization current is plotted as a function of the voltage between the dees. The emission current was 0.2 ampere, the emission voltage was 600 volts, and the pressure was 3.0 microamperes when the data for the above curve were taken.

Now consider when the voltage of the near dee is positive and increasing in magnitude. The electrons are at first strongly accelerated upwards until they pass the level of the bottom edge of the near dee. There the electric field reverses, and the electrons are decelerated until they reach the median plane where the direction of the field again reverses. The electrons are then accelerated upwards until they pass the level of the upper dee edge after which they are strongly decelerated. If the dee voltage had remained constant during the flight of the electron, they would strike the upper cooling plate with a velocity corresponding to the emission voltage. However, during the flight the dee voltage has been increasing and the electrons must do more work against the field in leaving the region between dees than they gained in entering it. Indeed, the difference in energy between entering and leaving the dee region turns out to be greater than the emission voltage so that the electrons cannot strike the cooling plate but must turn back. Nor can the electron, after approximately retraversing its path, return to the filament: rather it is snared and must oscillate up and down until the dee voltage drops to about the value it had when the electron was emitted. Thus, an electron emitted during this phase of the voltage cycle, i.e., near dee becoming increasingly positive, can pass through the region between dees many times and the ionizing power

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[3] K. T. Compton and C. C. Van Voorhis, Phys. Rev. **26**, 436 (1925); **27**, 724 (1926).

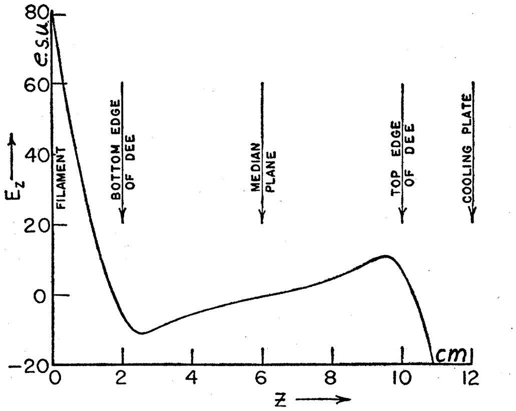

FIG. 4. The vertical component of the electric field along a vertical line situated halfway between the electrical center between dees and one of the dee edges is plotted as a function of Z, the distance above the filament. In the above curve the dimensions are approximately those of the 37-inch Berkeley cyclotron, i.e., distance from filament to cooling plate, 12 cm; dee height, 8 cm; dee separation, 4 cm. The voltage between dees is taken to be 100,000 volts.

thus be increased many fold. Moreover, the electrons are going more slowly and so producing more ions in the neighborhood of the median plane, and they are going faster and producing less ionization in the proximity of the dee edges where ionization might be likely to lead to a voltage breakdown.

In the horizontal plane, the electrons are executing cycloidal motion all the while and walk off in a direction perpendicular to the electric and magnetic field with an average velocity of cE/H. This sidewise motion can amount to about 4 cm for a deuteron magnetic field or about 8 cm for a proton magnetic field in the case of the 37-inch cyclotron. As the length of the filament is from 4 to 6 cm, the region of ionization extends over a distance of about 10 cm. This is disadvantageous as it contributes to the nonhomogeneity of the energy of the beam.

The electrons emitted during the phase when the voltage of the near dee is positive but decreasing in magnitude will gain more energy in entering the region between dees than leaving it and will strike the cooling plate the first time across and with considerable energy. These electrons though will still be speeded up near the dee edges and will go slowest near the median plane so as to tend to give a favorable concentration of ionization. Also during this phase the electrons emitted during the preceding phase can still be oscillating.

More quantitatively, the vertical component of the electric field along a vertical line situated halfway between the electrical center between dees and one of the dees is plotted as a function of the distance from the filament in Fig. 4. This was obtained by an electrolytic tray method. From it were calculated numerically the electron trajectories plotted in Fig. 5, where the height of an electron is shown as a function of the phase for electrons starting at various initial phases. For the calculations standard operating conditions and dimensions of the 37-inch cyclotron were used, i.e., maximum dee voltage, 100,000 volts; frequency, 11 megacycles; emission voltage, 600 volts; dee height, 8 cm. An electron emitted just as the dee voltage has become positive may be slowed up so much on approaching the median plane that it may not be able to cross it. In this case the electron will oscillate about one of the edges of the dee as indicated. Most electrons oscillate past the median plane, however, and some electrons can make about a dozen crossings. Also interesting is the ionizing power of an electron as a function of height as shown in Fig. 6. This was calculated from the data of Fig. 5 and Bleakney's ionization

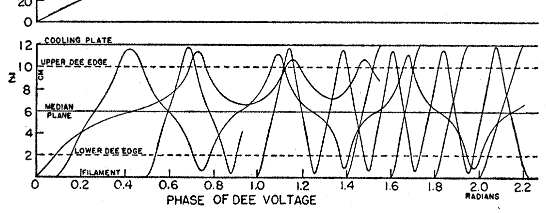

FIG. 5. In the lower curve is plotted the height of an electron above the filament as a function of the phase of the dee voltage when the voltage of the nearest dee to the filament is positive. In the upper curve is plotted the voltage between dees at the phase indicated on the lower graph. An electron starting at zero phase is seen to oscillate about the position of the upper dee edge. An electron which is emitted at an initial phase of 1.0 radian is seen to oscillate back and forth across the median plane and finally return to the filament after the dee voltage has dropped below the value it had when the electron was emitted—in this case after the phase has reached 2.1 radians. Electrons starting after an initial phase of $\pi/2$ radians, i.e., after the near dee voltage starts to become less positive, are seen to cross directly to the cooling plate.

measurements. Curve *a* is for a typical oscillating electron, and curve *b* is for one that just crosses once. It is seen that the ionization will be appreciably greater near the median plane than at the dee edge.

There is considerable experimental evidence that the above actually does occur. Visually, without the dee voltage applied but with the emission voltage on, one sees a sharply defined narrow glow directly above the filament and extending uniformly from the filament to the cooling plate. When the dee voltage is turned on, the glow spreads out and becomes strongest in the vicinity of the median plane and much weaker near the dee edges as Fig. 6 would indicate. When a long probe was put into the 60-inch cyclotron so that it was near the center, a negative current was picked up when the dee voltage was turned on. With an emission of 100 milliamperes and with the probe very near to the filament, a current of about 50 milliamperes was collected which fell off roughly linearly as the probe was moved away from the filament and which reached zero at a distance of about 12 cm from the filament. This was with a proton magnetic field, and considering the dee voltage, etc., the oscillating electrons would be expected to wander out a comparable distance during a half-cycle. Furthermore, as only the electrons emitted during the first half of the positive dee voltage phase will oscillate, one would expect about one-half the emission current to wander off sidewise and be collected by the probe as the above measurement indicates. Also as the magnetic field was increased, the current to the probe fell off more rapidly as the probe was moved away from the filament as would also be expected.

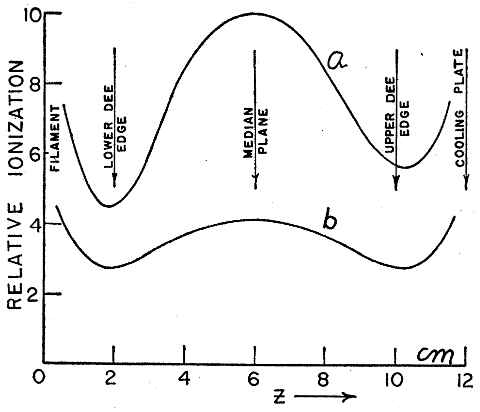

FIG. 6. The relative ionizing power of an electron is plotted as a function of its height above the filament. Curve *a* is for a typical oscillating electron emitted when the phase is about 0.3 radian. Curve *b* is for an electron emitted when the phase is about 2.0 radians, i.e., an electron which crosses directly to the cooling plate. In each case the ionizing power is seen to be increased near the median plane and decreased near the positions of the dee edges.

Another phenomenon can be explained by the above theory. When the dee voltage is applied, the emission current is observed to drop to about half-value and then slowly to increase to considerably higher than half-value. When the dee voltage is turned off, the emission current kicks up to a large value and then slowly decreases to the original reading. The size of the kick increases about linearly with the dee voltage and also becomes greater as the filament is moved closer to either dee—being a minimum when the filament is midway between dees. All this is because some of the oscillating electrons are about as likely to strike the filament when they stop oscillating as to strike the cooling plate. They will also carry off considerable energy with them which will give rise to a heating of the filament. The electrons emitted when the voltage of the near dee is negative and increasing in magnitude can also cause a considerable heating as explained before. The dee voltage suppresses half of the emission current when it is turned on, but the heating due to the returning electrons increases the temperature of the filament slowly and causes the emission to increase. When the voltage is turned off, the emission should double and then fall off to its original value as the filament cools off. Rough calculations show that the heating by the returning electrons is sufficient to account for the observed increase in temperature.

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