Deviation From the Coulomb Law for a Proton*

Fröhlich, Heitler and Kahn[1] have recently discussed the possibility of a short range deviation from Coulomb's law between an electron and a proton, on the basis of a quantized wave field theory of the mesotron type. They considered the interaction of the electron with the charge distribution about the proton. This is just the influence of the electric field of the electron on the self-energy that the proton has due to its interaction with the mesotron field. They concluded that it was possible to obtain a proton-electron force which became repulsive at short distances, and hoped thereby to account for some recently noted[2] anomalies in the fine structure of hydrogen. They emphasize, moreover, that it is for this essential to obtain a repulsion and not just a reduction of the Coulomb attraction as would, for example, result from merely spreading the charge of the proton over a small volume. I wish to point out, however, that despite the indications of the above calculations, no mesotron theory which is not radically different than those considered at present can possibly give such a short range repulsion between a proton and an electron.

According to such theories, the charge density about a proton has the form

$$\rho(\mathbf{r})=e(1-\alpha)\delta(\mathbf{r})+e\rho_1(\mathbf{r}), \tag{1}$$

where

$$\alpha=\int \rho_1(\mathbf{r})d\mathbf{r} \tag{2}$$

plays the role of the probability[3] that the proton is found dissociated into a neutron and a positive mesotron distributed about it with a charge density $e\rho_1(\mathbf{r})$. The proton-electron potential[4] is then given by

$$V(\mathbf{R})=-\frac{e^2}{R}(1-\alpha)-e\int d\mathbf{r}\rho_1(\mathbf{r})/(|\mathbf{R}-\mathbf{r}|), \tag{3}$$

so that, if as is the case with present field theoretic calculations, $\rho_1(\mathbf{r})$ is everywhere positive, one has the inequality

$$V(\mathbf{R})<-\frac{e^2}{R}(1-\alpha). \tag{4}$$

Thus $V(\mathbf{R})$ may become repulsive only if $\alpha>1$, an occurrence which is not physically sensible since the proton cannot be a neutron more than 100 percent of the time or give more of its charge to mesotrons than it has initially.

Although with their theory, α was divergent, the authors were able to obtain convergent integrals for $V(\mathbf{R})$, which could apparently be positive. To show how this is related to the value of α, we write

$$V(R)=-\frac{e^2}{R}+e^2\int_R^\infty 4\pi r^2\left\{\frac{1}{R}-\frac{1}{r}\right\}\rho_1(r)dr, \tag{5}$$

which follows from Eqs. (3) and (2) in the case of spherical symmetry. The integral converges for $R\neq 0$ if $\rho_1(r)$ is finite for $r\neq 0$. According to the scalar mesotron theory,[5]

$$\rho_1(r)\sim c/r^3 \quad \text{for} \quad r\rightarrow 0,$$

so that for $R\rightarrow 0$,

$$V(R)\sim -\frac{e^2}{R}+\frac{4\pi ce^2}{R}\log(b/R)-4\pi e^2c\left(\frac{1}{R}-\frac{1}{a}\right),$$

where a, b, c are positive constants. The logarithmic term seems to make $V(R)$ positive for small R. As we have seen above, however, this must mean that $\alpha>1$, in fact α is infinite, since $\rho_1(r)$ is not integrable. A similar result is obtained in the vector mesotron theory, where the divergence of α is stronger.

It must be noted, however, that even if the field theory were not divergent, one could erroneously obtain a value of α larger than unity in a perturbation calculation which stopped at a finite order (as the present calculations necessarily do!). One would simply need to make the coupling of the field with the heavy particles large enough.

One sees then how the apparent result that there may be a repulsion in the electron-proton interaction on the basis of current mesotron theories is based, not so much on the inherent inconsistencies of field theories, as on the nonconvergence of the perturbation methods used.

W. E. Lamb, Jr.

Columbia University,
New York, New York,
July 24, 1939.

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

1 H. Fröhlich, W. Heitler and B. Kahn, Proc. Roy. Soc. **171**, 269 (1939).

2 See S. Pasternack, Phys. Rev. **54**, 1113 (1938).

3 While the formalism does permit one to use the concept of a dissociation probability to the extent that it is in the text, it does not permit one to use this concept to deduce more complicated properties of the heavy particles such as magnetic moments. Several erroneous estimates involving α and the magnetic moment of a free mesotron have appeared in the literature.

4 According to the field theories, Eq. (3) is valid if a polarization of the mesotron cloud by the electron is neglected, since no exchange terms enter due to the distinguishability of electron and mesotron.

5 W. E. Lamb, Jr., and L. I. Schiff, Phys. Rev. **53**, 651 (1938).

Virtual State of He^5 and Meson Forces

Experiments by Staub and Stephens[1] on the breakup of He^5 indicate a virtual level of this nucleus at about 0.8 Mev. If, as seems probable, it is the ground state of He^5 that is involved here, and if one assumes that the general features of the Hartree method remain applicable, then the added neutron may be said to be in a P state. The states of the system that may result are a $^2P_{1/2}$ or a $^2P_{3/2}$.

Chief among the forces which might be expected to contribute to a splitting of this pair of states is the spin orbit force

$$V(r^{ij})(\boldsymbol{\sigma}^i\cdot\mathbf{r}^{ij})(\boldsymbol{\sigma}^j\cdot\mathbf{r}^{ij}), \tag{1}$$

which arises in the meson theory of nuclear forces; the observed value of the quadrupole moment of the deuteron indicates that terms such as (1) are an important part of the nuclear Hamiltonian.

A perturbation calculation was undertaken in order to estimate the amount of separation that this type of force might bring about. The use of perturbation theory in this apparently unfavorable case will be justified because higher order perturbations will be shown to fall off even though the perturbing potential is large.

The diagonal matrix elements of the term (1) were found to be equal for the states $^2P_{1/2}$ and $^2P_{3/2}$ without introducing explicitly the radial dependance of V and of the wave functions; hence to first order the splitting vanishes. The result was found to follow directly from the fact that the configuration being considered consists of a single particle outside a closed shell.

The treatment of the second-order splitting is simplified by the fact that there are no low lying states which combine with the two being considered. The configuration which gives rise to the $^2P_{1/2}$ and $^2P_{3/2}$ may be written S^4P. Other possible configurations are

(a) S^4D or S^4S', (b) S^4F,
(c) S^3P^2, (d) S^3DP or $S^3S'P$.
(e) S^2PD^2.

Here S' represents a neutron or proton in an excited S state. Of these possible intermediate states (a) and (c) are excluded on grounds of parity; (b) has a total angular momentum of 7/2 or 5/2 and is hence noncombining. We are therefore restricted to such states as (d), (e), etc., which have odd parity and whose reduction yields components with angular momentum 3/2 and 1/2. These configurations involve the breaking up of the alpha-particle core and therefore lie perhaps 15 or 20 Mev higher than the ground state. Considering then the magnitude of the energy denominators and the probable nonoverlapping of the wave functions, it seems certain that second and higher order perturbations will be greatly reduced.

Under these circumstances an important factor in the splitting of the two states will be the relativistic (Thomas) splitting described by Inglis and estimated by him for the case of Li^7. From the order of magnitude of his result, as well as from a rough evaluation, one would put the Thomas splitting for the ^2P state of He^5 at one or two hundred thousand volts. The second-order meson splitting should not exceed this magnitude. The sign of the resultant splitting will be more difficult to predict, since the Thomas doublet is inverted.[2]

I am indebted to Dr. Gaerttner for the information that new work at Pasadena on the structure of the He^5 level reveals it to be a doublet with a splitting of about 150–175 kev, in satisfactory agreement with theoretical expectations.

Thanks are due Professor J. R. Oppenheimer and Dr. L. I. Schiff for valuable suggestions and criticism.

S. M. DANCOFF

Department of Physics,
University of California,
Berkeley, California,
July 31, 1939.

[1] W. E. Stephens and H. Staub, Phys. Rev. **54**, 237 (1938).
[2] D. R. Inglis, Phys. Rev. **50**, 783 (1936).

Erratum: The Interaction of Configurations: $sd-p^2$

(Phys. Rev. **43**, 264 (1933)

An error was made in the evaluation of the radial integral R_c. Using the Rydberg constant ($R_\infty = 109{,}737$) and correcting this error we obtain $R_c = 21{,}387$ cm^{-1} in place of 25,620 cm^{-1}. The $sd\ ^1D$ is now calculated to be at -4272 cm^{-1} (observed -3592 cm^{-1}) from the center of sd configuration and $p^2\ ^1D$ at 21,353 cm^{-1}. The $sd\ ^1D$ appears below the ^3D (as before) due to the inclusion of the interconfiguration matrix elements, but the numerical agreement between theory and experiment is markedly improved. The $sd\ ^1D$ is now calculated to be 2234 cm^{-1} below ^3D (previously 4164 cm^{-1}) whereas it is observed at 1554 cm^{-1} below ^3D.

The writer is greatly indebted to Dr. J. P. Vinti for directing his attention to this integral.

R. F. BACHER

Cornell University,
Ithaca, New York,
July 21, 1939.

Erratum: The Paschen-Back Effect. VI. The Spectrum of Neon

(Phys. Rev. **56**, 54 (1939))

In the article with the above title the upper halves of Figs. 2*C* and 3*B* have been interchanged.

J. B. GREEN

Ohio State University,
Columbus, Ohio,

J. A. PEOPLES, JR.

Lehigh University,
Bethlehem, Pennsylvania,
July 17, 1939.

Velocity of Radio Waves in Air

(Phys. Rev. **55**, 1100 (1939))

In the discussion "Velocity of Radio Waves in Air" by G. H. Brown, which appeared in the "Letter to the Editor" section in the *Physical Review*, we note that the captions of Figs. 1 and 2 have been interchanged. Fig. 2 should read "Intensity pattern when the velocity is 80 percent of that of light."

G. H. BROWN

RCA Manufacturing Company,
Camden, New Jersey,
July 31, 1939.