with
$$n_0 \backsimeq (T_p/T_e)n_a. \tag{76}$$

Accordingly

$$\frac{i_{p \text{ crit}}}{i_{H \text{ crit}}} = 7.98\frac{\gamma a^2 e^2 D_e^2 \xi^2 T_p n_a}{c^2 k T_e^2}. \tag{77}$$

Now $D_e\xi$ is practically independent of n_a, but ξ varies oppositely from a though not as strongly

as a^{-1}. Thus it appears that for larger radii and higher pressures the critical condition for blowing the gas out of the arc column lies at a higher value of current than does the critical condition for 100 percent ionization due to pinch effect. Putting in numerical values and using p for the gas pressure in bars, we have

$$i_{p \text{ crit}}/i_{H \text{ crit}} = 11a^2 D_e^2 X^2 p/T_e^4 \text{ (practical)}. \tag{78}$$

Investigations of Ferromagnetic Impurities. I

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A method has been developed whereby a permanent magnetic moment as small as 2×10^{-7} per cc may be measured in any small solid specimen. Various materials were tested by this method for ferromagnetic impurities. To remove surface impurities it was necessary to dissolve away part of the specimen. Most metals then showed some volume impurity. In aluminum, however, the volume impurity was nonmagnetic but could be dissolved out and deposited on the surface in a ferromagnetic state. Measurements were also made on the magnetic hardness and hysteresis curves of the impurities; part of the impurity was found to be very "hard," with saturation incomplete at several thousand gauss and a coercive force$\sim$100–200, and part quite "soft," with a relatively large initial susceptibility. Other properties of these impurities are being studied. The method was also extended to single crystals of dia- or paramagnetic materials whose magnetic anisotropy could be easily detected and measured.

A S previously reported,[1] an experimental method has been developed whereby very minute ferromagnetic impurities may be detected in various materials and some of the properties of these small impurities studied. The results of such studies should be of interest for several reasons. First, by such a method one may quickly detect and measure the amount of ferromagnetic impurity present in materials which are generally regarded as nonmagnetic and, in some cases, possibly estimate the kind and amount of impurity present. Then from a study of the properties of such impurities one may find how best to render a slightly magnetic material, brass, for example, less magnetic. Finally such studies should furnish further information as to the fundamental nature of ferromagnetism.

EXPERIMENTAL METHOD

The method is quite simple. The specimen to be tested may have any shape, even be powdered if contained in a capsule, but small lumps or rods of about one gram were generally used. The specimen is first placed in a magnetic field of several thousand gauss, furnished by an electromagnet. This serves to magnetize all ferromagnetic impurities present in a common direction. Upon removing the specimen from this strong field such impurities will remain partially magnetized in this direction because of their remanent magnetization. The specimen is then placed in a light stirrup and suspended by a fairly strong quartz fiber in the center of a pair of Helmholtz coils, the direction of the remanent magnetization being perpendicular to the axis of the coils. A weak field of about 10 gauss is then applied and the resulting rotation measured by

[1] F. W. Constant and J. M. Formwalt, Phys. Rev. 53, 432 (1938).

means of a mirror, attached to the fiber, and a telescope and scale. The suspension was mounted inside a brass case with a glass window and was controlled by a torsion head at the top. The fiber was calibrated by measuring its period and was found to have a torsion constant of 0.0275 dyne centimeter per radian; it could support at least ten grams. The magnetic moment per cc is given by the formula $I = k\theta/VH$, where k is the torsion constant, θ the twist in a field H, and V the volume of the specimen. With the fiber used values of I as small as 2×10^{-7} could be measured, and with a finer suspension still greater sensitivity would be possible. Comparing this with the remanent magnetization of ordinary iron, about 500, shows that very minute impurities of the order of a millionth of one percent may be detected by this method provided they are ferromagnetic.

The volume of the specimen was either measured, or computed from its mass and density. The field of the Helmholtz coils was calibrated by means of a search coil and checked with the calculated value. The coils contained 400 turns each of No. 14 copper wire, mounted on circular frames 40 cm in diameter and 20 cm apart. The field at the center was then 18 gauss per ampere in the coils and it varied less than one percent within a 5-cm range.
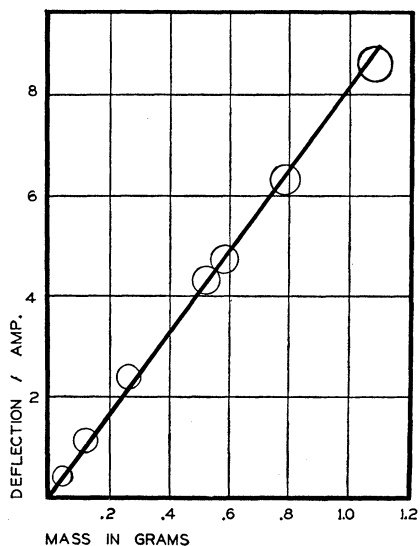

FIG. 1. Magnetic moment *vs.* mass for copper remaining after treatment in aqua regia.

TABLE I. *Volume impurities for four samples of sterling silver.*

SAMPLE	DEFLECTION/G OF SPECIMEN
a	23.05 cm
b	22.80
c	23.10
d	23.05
Mean	23.00 ±0.10

Mean magnetic moment/cc = 1.89×10^{-3}

As explained later dia- or paramagnetic effects were generally too small to be measurable. However, in the case of soft ferromagnetic impurities the deflecting field would slightly magnetize the specimen in its direction and thereby give rise to a torque tending to twist the specimen in the direction of easiest magnetization, i.e., the direction for which the demagnetization factor was a minimum. Such effects were eliminated by reversing the field of the coils and taking the average of the two deflections since such a reversal also reversed the torque due to the permanent but not that due to the induced magnetization.

SURFACE AND VOLUME IMPURITIES

In measuring such minute impurities cleanliness was of the highest importance. Hence surface impurities had to be eliminated. The only effective method was to sandpaper the surface and then dissolve away 25 percent or more of the specimen. For metals nitric acid was generally used, although the choice of dissolving agent usually made little difference. Measurements of the magnetic impurity remaining were made as the specimen was gradually dissolved away in successive steps. It was found that after an abrupt initial decrease the magnetic moment then decreased linearly with the mass of the specimen thus indicating that the impurity was then a true volume one, distributed throughout the specimen. This linear relationship between total magnetic moment and mass in the case of a specimen of copper wire is shown in Fig. 1.

It was also found that different samples taken from the same spool of wire showed nearly the same magnetic moment per cc, again indicating a uniform volume impurity. In Table I are given

TABLE II. *Volume impurities for samples of copper.*

SOURCE	DEFLECTION/G OF SPECIMEN	$I \times 10^5$
No. 12 wire	9.5 cm	
No. 12 wire	11.9	
No. 12 wire	9.7	
Square wire	7.8	
Drawn rod	15.3	
Mean	10.8±2.2	38.
Eimer & Amend, (C.P., 0.000% Fe)		5.
National Bureau of Standards, (oxygen free)		0.6

the results for four samples of sterling silver wire, with the mean and mean deviation.

A sample of "chemically pure" silver foil, on the other hand, had a magnetic moment/cc $=3.3 \times 10^{-4}$.

In Table II are given the results for various samples of copper, the first three specimens being taken from the same spool of wire, the rest from other sources.

In the case of brass the variations from sample to sample were larger.

In Table III are summarized the results of measurements of the volume impurities of various substances, "no impurity" meaning a magnetic moment/cc less than about 10^{-6}.

The surface impurities were sometimes unimportant but often amounted to considerably more than the total volume impurity of the specimen.

In the case of aluminum, however, a different result was encountered. The surface impurity was removed from a cast specimen with HNO_3 and thereafter a very small volume impurity remained (see Table III) which decreased with the mass. When, however, a specimen was dissolved in HCl or aqua regia it rapidly became increasingly ferromagnetic. Apparently the aluminum contained a relatively large volume impurity which, being in solid solution, (microscopic study indicated $FeAl_3$), was not magnetic, but which upon solution became precipitated upon the surface as a ferromagnetic film. It is not surprising that aluminum was the only metal tested which showed this effect since it was the only one above iron in the electromotive series; but it is interesting that the film is ferromagnetic. Evidently the film could not appear when HNO_3 was used but could with HCl. This

is attributed to the fact that HNO_3 is a stronger oxidizing agent. Mason[2] attempted to use such a film, formed by the action of NaOH on aluminum, to determine the percentage of iron present. In the present case a magnetic moment of approximately 0.1 was obtained for each cc of aluminum dissolved so that the aluminum originally must have contained a considerable iron impurity in a nonmagnetic state. However, an accurate estimate could not be made as there were indications that the film was very easily disturbed and so partly lost.

MAGNETIC CHARACTERISTICS OF THE IMPURITIES

As it was realized that only the remanent magnetization of the impurities was being measured an attempt was made to find out whether or not this was large and at the same time to study further the magnetic properties of such minute ferromagnetic impurities.

First it was observed that large fields (several thousand gauss) were necessary to reverse completely the remanent magnetization. To reduce it to zero the following fields in the opposite direction had to be applied: 90 gauss for silver, 200 for copper, 300 for bismuth and 500 for brass. This indicated that at least part of the ferromagnetic impurity was in a very "hard," prob-

TABLE III. *Volume impurities for various materials.*

MATERIAL	$I \times 10^3$
Brass	82.
Silver—sterling	1.9
—C.P.	0.33
Copper—magnet wire	0.38
—C.P., (0.000% Fe)	0.05
—National Bureau of Standards, (oxygen free)	0.006
Bismuth—extruded rod	0.14
—crystallized, C.P. 0.00% Fe)	0.01
Cadmium—C.P., (0.003% Fe)	0.014
Tin—C.P., (0.003% Fe)	0.002
Solder	0.07
Aluminum—drawn rod	0.003
—cast	0.001
Molybdenum	No impurity
Platinum	No impurity
Tungsten	No impurity
Potassium hydroxide	No impurity
Amber and paraffin	No impurity
Soft and Pyrex glass	No impurity

[2] R. B. Mason, Trans. Am. Electrochem. Soc. **56**, 45 (1929).

ably strained, state. The presence of such "hard" impurities assured sufficient remanent magnetization to make the present method practical.
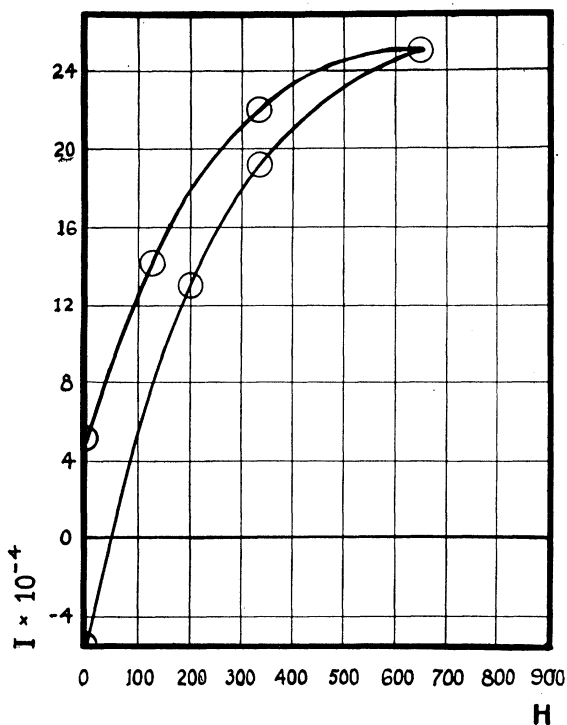

FIG. 2. Hysteresis curve for copper impurities.

Since the remanent magnetization might represent only a small fraction of the saturation magnetization and very "soft" impurities would not be detected at all by the above method the apparatus was modified so as to measure the magnetic moment of a specimen when the specimen was actually in a magnetizing field. For this purpose a second pair of coils of about 1700 turns each of No. 12 wire was mounted with its axis at right angles to that of the deflecting coils. The procedure was to mount the specimen with its length parallel to the axis of the large coils so that when their field was on the torsion fiber suffered no twist. Then the deflecting field was applied as before. Since, however, this latter field rotated the specimen out of line with the strong magnetizing field it was necessary to bring it back into its original position by twisting the torsion head. The strong magnetizing field then exerted no torque and the torque of the deflecting field, which was balanced by that

given the fiber, was found by simultaneously cutting off both fields and noting the resultant rotation of the specimen as the fiber returned to an unstrained position. In this manner the magnetic moment of a specimen in fields up to 1000 gauss could be measured.

Figure 2 shows the hysteresis loop obtained for a specimen of copper wire, and Fig. 3 that for a sample of sterling silver wire. It will be observed that for copper the remanent magnetization is only about 20 percent of the magnetization in a field of 650 gauss and the coercive force, which is 50, is much less than the 200 gauss which was found earlier to be necessary to reduce the remanent magnetization to zero. This would indicate that the copper contains not only a very "hard" impurity whose magnetization does not approach saturation in fields up to 1000 gauss and whose coercive force is 200 or more, but also a considerably larger amount of "softer" impurity which has very little remanence and is magnetized in fields of 50 gauss, perhaps less. As stated above, indications of the "soft" impurity were noted when the deflecting field was reversed and unequal deflections in the two directions resulted. Probably the magnetic impurity actually exists in all degrees of strain and hardness throughout the specimen. In the case of silver the "hard" component appears
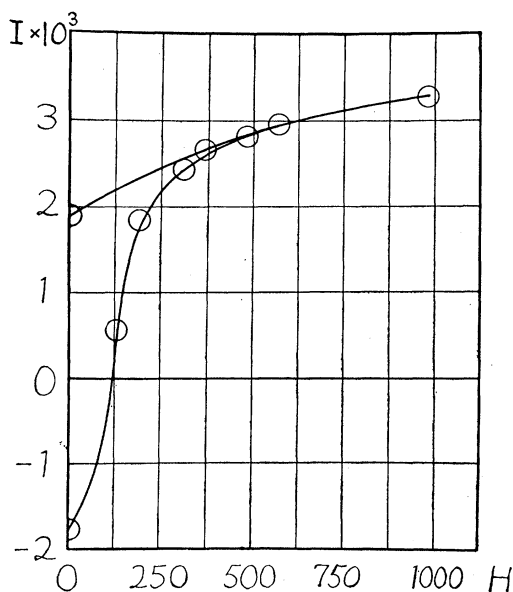

FIG. 3. Hysteresis curve for silver impurities.

more important, but again indications are that saturation is complete only in much larger fields.

The above results are in agreement with those of Bitter and Kaufmann,[3] who examined the magnetic behavior of known amounts of iron added to copper.

In the above measurements the probable error due to the instruments used and the necessary calibrations was estimated to be less than six percent for values of I greater than about 10^{-6}. To test the reproducibility of results a specimen was remagnetized and remounted eight times and the mean deviation in the deflection was less than one percent. However, it was important to remagnetize the specimen in the same direction since even in the field of the electromagnet saturation was not attained and the two sides of the hysteresis loops were not symmetrical.

PARA- OR DIAMAGNETIC EFFECTS

Calculations showed that dia- or paramagnetism of a specimen could cause rotation in the deflecting field only because of (1) different demagnetization factors in different directions when the specimen is not spherical, or (2) different susceptibilities in different directions, as for single crystals.

In the first case the torque per unit volume is given approximately by

$$L = H^2 \sin \theta \cos \theta K^2 (N_1 - N_2), \qquad (1)$$

where θ is the angle between the principal axis of the specimen and the field K is the suscepti-

[3] F. Bitter and A. R. Kaufmann, Phys. Rev. **55**, 1142 (1939).

bility, N_1 and N_2 the demagnetization factors parallel and perpendicular to the principal axis. For the fields used this torque, being proportional to K^2, was negligibly small; the maximum possible deflection due to it was calculated to be about one-hundredth of a millimeter.

In the second case the torque per unit volume is given approximately by

$$L = H^2 \sin \theta \cos \theta \ (K_1 - K_2), \qquad (2)$$

where K_1 and K_2 are the susceptibilities along the two principal horizontal axes. In this case the torque, being proportional to $(K_1 - K_2)$ rather than K^2, is of a much larger order of magnitude. For a single crystal of bismuth a maximum possible deflection of the order of 1 to 10 cm was computed. This torque, being proportional to H^2, would not be reversed upon reversing the field, whereas any torque due to a permanent ferromagnetic moment, M, would be reversed, since it is given by

$$L = MH \sin \theta \qquad (3)$$

and hence is proportional to H.

These calculations were verified experimentally. Platinum foil gave no deflection and an extruded bismuth rod one which was completely reversed with the field. A crystallized specimen of bismuth, however, gave a deflection proportional to H^2, reaching a maximum of 3 cm, which was not reversed with the field. From this it may be seen that we have also a sensitive method for detecting and measuring magnetic anisotropy.

It is planned to give in a subsequent paper the results of further measurements on the properties of ferromagnetic impurities, in particular, the effect of temperature and heat treatment.