Theory of Magnetic Effects in the Plasma of an Arc

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(Received May 29, 1939)

The equations governing electron drift in the presence of a magnetic field are applied to the low pressure uniform positive column plasma. (1) An exponential variation of electron concentration with distance across an arc in a transverse magnetic field, found experimentally, is accounted for quantitatively. (2) A longitudinal magnetic field leaves the point-to-point concentration of electrons unchanged and does not alter the relative potentials in the cross section although transverse potential differences in the plasma decrease everywhere in proportion as the magnetic field increases. The transverse plasma fields vanish or even reverse slightly for large enough fields. (3) The plasma exhibits a diamagnetic susceptibility for longitudinal fields, which is proportional to the electron current density to the tube walls. With nonconducting walls electron wall current is automatically adjusted to the ion wall current. The magnetic polarization then increases oppositely to the magnetic field at first, reaches a maximum and then decreases hyperbolically to zero, so that beyond the maximum the plasma is paramagnetic for small variations in the field. Qualitatively this is the same behavior predicted by a previous erroneous theory which was itself checked only qualitatively by experiment. The present theory predicts that the diamagnetic polarization shall be directly proportional to the electron current to the tube wall and the magnetic field. (4) The magnetic field of the arc itself has a concentrating pinch effect which would cause the axial concentration of electrons to become infinite at a finite arc current if other limitations did not intervene. This may be the cause of rapidly increasing arc gradients in the neighborhood of certain critical currents in large mercury arc rectifiers. Larger diameter arc columns and higher pressures favor the magnetic pinch effect as a cause of current limitation over a blowout effect due to the outward-moving positive ions driving the gas before them.

I. Introduction

The fundamental equations describing the motion of electrons in a gas under the simultaneous action of electric and magnetic fields and concentration gradients have been known for some time[1–3] but they have not been systematically applied to phenomena occurring in the plasma of an arc. In the present article this application will be made in two cases which have already been investigated experimentally, and also to two cases in which the direct experimental evidence is more meager but on which the theory can shed light. These cases are, in order of treatment:

(1) Killian's measurements[4] on the effect of a transverse field on a positive column in low pressure mercury vapor.

(2) The effect of a longitudinal magnetic field on a uniform positive column.

(3) Steenbeck's determination[5] of the diamagnetic susceptibility of a positive column.

(4) The constrictive effect of the arc's own magnetic field.[6]

For a magnetic field H_z in the positive z direction, the components of electron drift velocity ξ and η in the x and y directions, respectively, are, according to the paper by Tonks and Allis[3]

$$\xi=\tfrac{1}{3}l_e\bar{c}(\alpha b_x+\beta b_y), \quad (1)$$

$$\eta=\tfrac{1}{3}l_e\bar{c}(\alpha b_y-\beta b_x), \quad (2)$$

where l_e is the electron mean free path; $\bar{c}$ the mean electron speed; α and β are constants which depend on l_e, the electron temperature T_e, and H_z. The quantities b_x and b_y are space derivatives of what may be called the "Boltzmann velocity potential function" [3]

$$B=-(eV/kT_e+\ln n_e). \quad (3)$$

It will often be convenient later to use the diffusion coefficient of electrons, D_e, to eliminate the electron mean free path by

$$D_e=l_e\bar{c}/3. \quad (4)$$

It will also be convenient to introduce

$$\omega_H=eH_z/(m_ec)=-1.778\times10^7H_z, \quad (5)$$

[1] F. B. Pidduck, Proc. London Math. Soc. **15**, 89 (1915) and Phys. Rev. **53**, 197 (1938).
[2] R. Gans, Ann. d. Physik **20**, 293 (1906).
[3] L. Tonks and W. P. Allis, Phys. Rev. **52**, 710 (1937).
[4] Unpublished work done in this laboratory.
[5] M. Steenbeck, Wissen. Veröff. Siemens **15** (2) 1 (1936).
[6] A brief statement of the result of this analysis has been given: L. Tonks, Trans. Electrochem. Soc. **72**, 167 (1937).

the angular momentum corresponding to the magnetic field. The negative sign is a consequence of the negative charge on the electron. Another useful quantity will be h', which is related to the h of reference 3 and is defined by

$$h' = \omega_H l_e/\bar{c} = (eH/m_e c)(3D_e/\bar{c}^2) = 0.886h. \quad (6)$$

The exact equations for α and β are inconveniently complicated. But we can circumvent this difficulty by putting

$$\alpha = \gamma_\alpha \alpha_a, \quad \alpha_a = (1/(1+h'^2), \quad (7)$$

$$\beta = \gamma_\beta \beta_a, \quad \beta_a = h'/(1+h'^2), \quad (8)$$

where γ_α and γ_β are correction factors which do not differ greatly from unity in the whole range of h' (or h) from zero to infinity, as can be seen from Fig. 1.

It has been noted that the actual drift velocity is only slightly less than the product of the b for the drift direction and the normal nonmagnetic mobility.[3] In effect the magnetic field determines simply that the electrons shall drift at an angle (whose tangent is $\gamma_\beta h'/\gamma_\alpha$) to the direction which they would take under the influence of the existing potential and concentration gradients alone. Since the electrons are constrained to move at this angle (almost as if by an inclined plane) only the components of the gradients in this direction are effective, but these components have almost the full effectiveness (between 88.3 and 100 percent) which they would have were the magnetic field absent. Thus

$$u_d = \gamma_r D_e b_d, \quad (8.5)$$

where b_d is the b in the drift direction, u_d is the drift velocity, and γ_r is the correction factor given by

$$\gamma_r = \alpha + \beta^2/\alpha. \quad (9)$$

This also is plotted in Fig. 1. It varies between unity at zero h and 0.883 at infinite h. Fig. 1 contains, besides, a plot of

$$\gamma = (\beta/\alpha)/(\beta_a/\alpha_a) = \beta/(\alpha h') = \gamma_\beta/\gamma_\alpha. \quad (10)$$

These curves furnish an idea of the numerical magnitude of any error which arises from treating the γ's as if they were constants.

II. Effect of a Transverse Magnetic Field on a Positive Column

In 1928 in these laboratories T. J. Killian made some measurements on a low pressure mercury arc subjected to a uniform transverse magnetic field (in the z direction). The arc tube was the same one that served for experiments already described.[7] It suffices here to note that the vapor pressure in his 6.2-cm diameter tube was under control and that a fine wire probe parallel to the tube axis (y direction, positive toward anode) could be moved along a tube diameter which was perpendicular to the magnetic field (x direction). By analyzing probe characteristics, the electron temperatures and concentrations were determined from one wall to the other. Fig. 2 shows the results of such measurements at the saturated mercury vapor temperature 38.6°C, corresponding to 7.1 bars. This temperature has been chosen because at lower vapor pressures the electron mean free paths become so long relative to arc tube diameter that diffusion and mobility relations do not strictly apply.

The question of interpreting probe characteristics in the presence of a magnetic field naturally arises. In the z direction electrons could reach the probe unhampered by H_z. But the

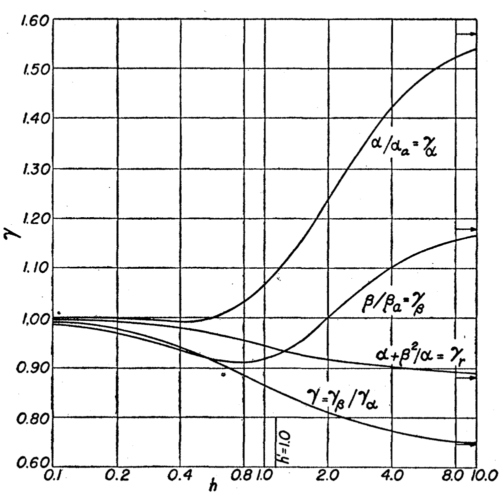

FIG. 1. Correction factors for approximate electron drift formulas. The h' scale is slightly displaced from the h scale, but this is only of minor importance in evaluating the γ's.

[7] T. J. Killian, Phys. Rev. 35, 1238 (1930).

decrease of mobility in the x direction leads to the expectation of smaller maximum probe currents with the magnetic field than without.

This affects the possibility of calculating absolute values of electron concentration from the characteristics but in no way changes the proportionality of the probe currents to the electron concentrations. Thus the probe current where the characteristic ceases to be exponential is still a measure of relative electron concentration and the potential at this current is the space potential.

Finally, it must be certain that the probe characteristics give the electron temperature. In general a perfectly reflecting negative probe would establish electron concentrations and current densities in its vicinity in accord with the Boltzmann equation irrespective of the presence of a magnetic field. The electrons very close to the surface of the probe which are moving toward it and would, therefore, strike it can be thought of as consisting of two classes, C and F. Class C consists of electrons whose last atom collision lay close to the probe. Class F consists of those which last collided at some distance from the probe. Of the class C electrons, an appreciable fraction (class C_r) will have just previously been reflected from the probe. For Class F geometrical considerations make this fraction negligible. Hence, if the probe becomes nonreflecting and actually collects the electrons striking it, the concentration of C electrons is depleted by the loss of the C_r electrons while the F electrons are unaffected. If the concentration of C_r electrons is negligible relative to that of the F electrons or the C electrons, or their sum, the actual flow of current to the probe causes no departure from the ideal characteristic. Unless the magnetic field sufficiently increases the number of Class C_r electrons relative to Class F or Class C, it will not distort the probe characteristic.

Such distortion was certainly not present in Killian's work. The electron mean free path was about 1 cm, the probe radius 0.013 cm so that in the absence of magnetic field few electrons leaving the vicinity of the probe would have made their first collision near the probe and the number of C_r electrons would have been negligible.

The presence of the field would, by spiraling the electron paths around the z direction, confine them to helical paths having the average radius

$$r_\lambda = 3.35\, V_e^{\frac{1}{2}}/H_z, \quad V_e = T_e/11{,}600.$$

For $V_e \sim 1.5$ v and $H_z = 12$ oersteds

$$r \backsimeq 0.34 \text{ cm}$$

and this represents the average progress of an electron in the x direction before its next impact. Since the z component of electron motion is unaffected by H_z, the average progress of an electron in the z direction can be found by averaging the z components of mean free paths of random directions. This leads to half the normal mean free path, or 0.5 cm.

We now define as C electrons those whose last collision prior to striking the probe lay within the xz section extending 0.5 cm in the z direction and 0.34 cm in the x direction. Of these, half came from the direction away from the probe. Of the other half only the fraction represented by the ratio of probe thickness, 0.025 cm, to section width, 0.68 cm would have collided with the probe. Thus the ratio of C_r to C electrons is $\frac{1}{2} \times 0.025/0.68 = 0.018$. This rough estimate therefore indicates a correction of less than 2 percent arising from a field of 12 oersteds, which will be neglected.

Since the experimental results can be interpreted to give relative electron concentration, space potential, and electron temperature, the

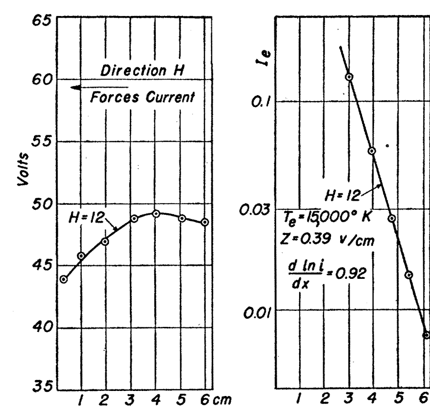

FIG. 2. Space potential and electron current density distributions across a Hg arc positive column in a transverse magnetic field. Tube diameter 6.2 cm. Condensed Hg temperature 38.6°C. Arc current 5 amp. Longitudinal gradient without magnetic field 0.31 v/cm, with field 0.39 v/cm. Data by T. J. Killian, 1928.

theoretical equations can be applied. The ratio of transverse ion current density, and hence transverse electron current density, to arc current is of the order of 1 : 400. Accordingly, in Eqs. (1) and (2)

$$\xi/\eta \sim 1/400$$

so that

$$b_x/b_y = -\frac{\beta/\alpha - 1/400}{1+\beta/(400\alpha)} = -\beta/\alpha = -\gamma h' \quad (10.5)$$

as long as

$$400 \gg \gamma h' \gg 1/400.$$

Since n_e is constant along the plasma axis, Eq. (3) yields eY/kT_e for b_y where $-Y$ is the arc gradient. In the x direction Fig. 2 shows that V is approximately constant from 3 to 6 cm so that for b_x in this region we have $-d \ln n_e/dx$. Making use of Eqs. (10.5) and (6) we can write

$$d \ln n_e/dx = \omega_H l_e e Y\gamma/\bar{c}kT_e.$$

Now

$$l_e/\bar{c} = 1.605 \times 10^{-6} l_e T_e^{-\frac{1}{2}} \quad (11)$$

and

$$eY/kT_e = -1.16 \times 10^4 Y/T_e \text{ (practical)}.$$

Combining and using Eq. (5) we have:

$$d \ln n_e/dx = 3.31 \times 10^5 l_e H_z Y T_e^{-\frac{3}{2}}\gamma. \quad (12)$$

Killian's own data[7] on Hg arcs without a magnetic field yield a mean free path of 1.5 cm for the electrons at 38.6° condensed Hg temperature. From the experimental data and curves: $H_z = 12$, $Y = -0.39$ v/cm, $T_e = 15{,}000$°K. Accordingly, from Eq. (12)

$$d \ln n_e/dx = -1.23\gamma.$$

Now from Eqs. (6), (4) and (11)

$$h = 32.2 H l_e T_e^{-\frac{1}{2}},$$

whence

$$h = 4.74,$$

thus justifying the approximation in Eq. (10.5). From Fig. 1,

$$\gamma = 0.77,$$

so that

$$d \ln n_e/dx = -0.95.$$

This is to be compared with the direct experimental determination of the variation of random electron current with x, shown in Fig. 2. That gives

$$d \ln n_e/dx = -0.92.$$

This is a very satisfactory check.

Unfortunately, probe characteristics taken at 30 oersteds field strength show deviations from the classical shape which make it impossible to compare the theoretical and observed gradient of electron concentration at this field strength.

III. Theory of a Positive Column Plasma in a Longitudinal Magnetic Field

The pressure range for which this theory will be developed is that in which the quasi-neutral diffusion relations apply in the absence of magnetic field.

The essential difference of the magnetic from the nonmagnetic[8, 9] case is that we cannot ignore the resistance of the gas to the radial flow of electrons. In a tube with nonconducting walls the radial drift velocities of electrons u_e and ions u_p are everywhere equal, but the ratio of the two can be controlled by the potential of conducting walls so that in general

$$u_e = \mu u_p \quad (13)$$

with any value from zero to several hundred possible for μ.

Choosing cylindrical coordinates with the z axis along the tube axis, we have the following relations left unaffected by the magnetic field:

$$n_e = n_p, \quad (14)$$

$$u_p = -D_p[\partial \ln n_p/\partial r - (e/kT_p)\partial V/\partial r], \quad (15)$$

$$\partial(n_p u_p r)/\partial r = r\lambda n_e, \quad (15.5)$$

where D_p is the positive ion diffusion coefficient, T_p is the positive ion temperature, and λ is the number of ions formed per electron per second. Eqs. (1) and (2) applied to the cylindrical case give

$$u_e = D_e(\alpha b_r + \beta b_\varphi), \quad (16)$$

$$v_e = D_e(\alpha b_\varphi - \beta b_r), \quad (17)$$

[8] W. Schottky, Zeits. f. Physik **31**, 163 (1925) and Physik. Zeits. **25**, 342 (1924).

[9] L. Tonks and I. Langmuir, Phys. Rev. **34**, 876 (1929). In referring to this paper it will be called "General Theory."

where v_e is the tangential drift velocity, and

$$b_r = -\frac{e}{kT_e}\frac{\partial V}{\partial r} - \frac{\partial \ln n_e}{\partial r},$$
$$b_\varphi = -\frac{e}{kT_e}\frac{\partial V}{r\partial \varphi} - \frac{\partial \ln n_e}{r\partial \varphi}. \tag{18}$$

Obviously, there is axial symmetry so that $b_\varphi = 0$.

Relating (15) and (16) in accordance with (13) we have

$$\alpha D_e\left(-\frac{e}{kT_e}\frac{\partial V}{\partial r} - \frac{\partial \ln n_e}{\partial r}\right) = -\mu D_p\left(\frac{\partial \ln n_e}{\partial r} - \frac{e}{kT_p}\frac{\partial V}{\partial r}\right),$$

whence $$n_e = n_0 \exp(-eV/kT_e\tau) \tag{19}$$

with $$\tau = \frac{\alpha D_e - \mu D_p}{\alpha D_e + \mu D_p T_e/T_p}. \tag{20}$$

Eq. (19) replaces the Boltzmann equation among those whose solution gives the plasma relations. Since, mathematically Eq. (19) is simply the Boltzmann equation with $T_e\tau$ in place of T_e, and since T_e appears nowhere else among the original equations which are to be solved simultaneously, we can immediately write the equations for the radial distribution of electrons in a column of radius a as modifications of the usual equations:[10]

$$n_e = n_0 J_0(2.40r/a) \tag{21}$$

with $$a = 2.40\left(\frac{D_p(\tau T_e + T_p)}{\lambda T_p}\right)^{\frac{1}{2}} \tag{22}$$

or, by Eq. (20)

$$a = 2.40\left[\frac{(T_e + T_p)D_e D_p \alpha}{\alpha D_e T_p + \mu D_p T_e}\lambda^{-1}\right]^{\frac{1}{2}}. \tag{23}$$

(It must be noted that only in Eq. (22) does the modification of T_e to τT_e appear. In relating D_e to the electron mean free path or in relating λ to the ionization probabilities, the unchanged value of T_e still applies.) Also by Eq. (19)

$$V = (kT_e\tau/e)\ln[J_0(2.40r/a)]. \tag{24}$$

[10] The elimination of u_p between Eqs. (15) and (15.5) and then V by Eq. (19) and n_p by Eq. (14) gives Bessel's equation with the boundary condition $n_e = 0$ at $r = a$.

Several conclusions are now possible. First, Eq. (21) shows that the radial distribution of ions and electrons remains unchanged from the nonmagnetic case. Second, Eq. (20) shows that τ decreases with magnetic field due to the decrease of the numerator, with the consequence that λ in Eq. (22) decreases. Since λ increases rapidly[11] with T_e, this means that T_e decreases with increase of magnetic field. Looked at qualitatively, the decrease in outward flow of electrons and hence ions balances with a lower rate of ionization per electron and hence a lower electron temperature. It is easily observed that an axial magnetic field reduces the longitudinal gradient in an arc. Third, by Eq. (24) the potential distribution remains the same, but the scale of potential variation is reduced. As the transverse mobility of the electrons decreases toward that of the ions, the electric field whose function it is to equalize the ion and electron drifts becomes less.

It would even appear that τ could become zero and reverse in sign, accompanied by a complete flattening of the potential distribution across the arc with a subsequent reversal so that the axis of the arc is negative.

How negative the axis can become depends upon μ. Even if an electron wall current many times the magnitude of the ion current (large μ) is drawn the reversal is relatively small and unimportant. The fact that the fields in the plasma become small is probably of importance, however. For the purpose of discussion we fix our attention on the case where μ is unity, that is ion and electron currents are equal. Zero plasma field occurs when α has decreased to

$$\alpha = D_p/D_e,$$

but even when α is T_e/T_p times this, τ has already reached the value $\frac{1}{2}$ and the plasma fields are halved. In this range

$$\alpha \propto H_z^{-2},$$

so that when H is only $(T_p/T_e)^{\frac{1}{2}}$, roughly 15 percent, of its critical (zero plasma field) value, this marked effect has already appeared.

It seems fairly likely that the approach to the zero field condition may be a contributing

[11] Equation (16) of reference 7 or Eq. (92) of General Theory (reference 9).

factor among those which cause an arc to pull away from the tube walls with sufficiently strong magnetic fields, but it is probable that the action is indirect—that the effect of other factors such as those which control higher pressure arcs are enhanced by the strong field. However, another factor, which has not been sufficiently investigated experimentally, is that the initial concentration of the arc in the converging lines of force where the longitudinal field is increasing is, at least partially, a transition state so that a long enough column in the field might eventually fill the whole tube cross section.

The positive ion current density to the wall follows directly from General Theory

$$I_p = 0.216 a n_0 e \lambda.$$

With the substitution for λ from Eq. (23) this becomes

$$I_p = 1.248 \frac{\alpha D_e D_p (T_e + T_p)}{\alpha D_e T_p + \mu D_p T_e} e n_0 / a. \quad (24.5)$$

At first I_p to a nonconducting tube wall ($\mu = 1$) will not be affected by the decrease of α attendant upon increasing H because $D_e T_p$ is so much greater than $D_p T_e$. Only when α becomes comparable with $D_p T_e / D_e T_p$ will I_p begin to decrease. Under the conditions of measuring positive ion current, however, $\mu \ll 1$, so that the effect would set in at somewhat larger fields. Experimentally this would appear as more difficult positive ion current saturation the stronger the field.

The quasi-neutral diffusion theory of the positive column exhibits two important weaknesses. One is that the λ calculated from the observed ion currents to the wall is two to ten times larger than the λ calculated from the electron temperature and the known probability of ionization by single impact of electrons. Although the discrepancy increases with arc current, an explanation based on two-stage ionization is not without difficulties. The other discrepancy appears when the value of λ from the ion current is substituted in the plasma balance equation, (22), to find D_p. The ionic mean free path corresponding to the diffusion coefficient found is only a fraction of the expected value and varies too slowly with gas pressure.

This is not the place to discuss this question at length, but these difficulties in the theory of the nonmagnetic arc mean that for the present it is not wise to pursue the theory of the magnetic arc beyond the point of showing the modifications required in "accepted" equations and drawing simple conclusions from them. An application of the theory which is justified, however, particularly in view of theoretical and experimental work already done, is the calculation of the magnetic susceptibility of the positive column.

IV. The Diamagnetic Susceptibility of a Plasma

M. Steenbeck[5] has determined the diamagnetic susceptibility of a cylindrical plasma for axial magnetic fields with various gas pressures and magnetic field strengths. The positive column which he used was confined within the walls of a glass tube.

To account for his experimental results he developed a theory which, like a theory of diamagnetism in electrical conductors proposed by H. A. Wilson,[12] traces the path of a single electron and seeks to calculate the areas embraced by its path in the course of time. The moving electron thus constitutes a current flowing about the peripheries of such areas and contributes a magnetic moment to the plasma. Thus the magnetic moment of the plasma would seem to depend on the details of each electron path, and hence on what happens to the electrons in the course of time. Actually, of course, the moment depends only on the instantaneous motions of the electrons, and we should not sum areas but rather sum the instantaneous angular velocities of the electrons about an axis.

Now, in a plasma which is in equilibrium there can be no net drift of electrons through any volume element. Therefore, in such an element all directions of motion are equally probable and the total angular velocity of all the electrons passing through it is zero. It follows that the diamagnetic susceptibility of a plasma in equilibrium is zero.

In this respect the plasma electrons resemble the classical conduction electrons in a metal. Steenbeck himself points out that N. Bohr and

[12] H. A. Wilson, Proc. Roy. Soc. **97**, 321 (1920).

H. A. Lorentz have shown (in references not easily accessible) that free electrons in a conductor would as a whole have zero diamagnetic susceptibility by virtue of a surface current whose moment would exactly cancel the moment of the amperian currents in the body of the metal. The metal is bounded by a sharp potential barrier which concentrates this current in a single layer. The plasma potential on the other hand decreases continuously from the axis outward to a point near the tube walls, and the compensating current is, consequently, distributed through this region. The sharpness of the concept of the reversed surface current is lost, but an examination of the details of the electron paths as in Steenbeck's Fig. 6 shows how contributions of both positive (paramagnetic) and negative (diamagnetic) magnetic moment are made by many electrons in the plasma. While Steenbeck recognized this factor he was unable to calculate the relative contributions and erroneously judged that the paramagnetic moments were negligible compared to the diamagnetic, instead of being exactly equal to them.

It follows that the experimentally observed susceptibility must be accounted for by the disequilibrium in the plasma, which consists chiefly of the outward radial drift of electrons toward the walls. This possible origin of the magnetic response was suggested by Steenbeck but not examined more closely than to observe that it would give a result of the proper sign. The theory of Section III furnishes the means of calculating the plasma susceptibility on this basis.

The tangential component v_e of the electron drift velocity is the significant one for calculating magnetic moment M, per axial cm of the plasma, because

$$M=(\pi e/c)\int_0^a n_e v_e r^2 dr. \tag{25}$$

From Eqs. (17), (18) and (19)

$$v_e=(1-\tau)\beta D_e(\partial n_e/n_e\partial r).$$

From Eq. (21)

$$v_e=-\frac{2.4n_0}{an_e}(1-\tau)\beta D_e J_1(2.4r/a).$$

Substitution in Eq. (25) and integration gives

$$M=-\frac{1.04\pi a^2 e}{2.40c}n_0(1-\tau)\beta D_e.$$

With Eq. (20) this gives

$$M=-(0.432\pi a^2 n_0 e/c)\frac{(T_e+T_p)D_p D_e\mu\beta}{\alpha D_e T_p+\mu D_p T_e}. \tag{26}$$

Comparison with Eq. (23) shows that

$$M=-(0.0746\pi a^4 n_0 e/c)\mu\lambda\beta/\alpha.$$

Now the ion current density I_p (intrinsically negative to conform to experimental convention) at the tube wall is related[13] to λ:

$$2\pi a I_p=0.432\pi a^2 e n_e\lambda, \tag{27}$$

so that, by eliminating λ and using Eq. (10)

$$\begin{aligned} M&=-(0.346\pi a^3 I_p/c)\gamma h'\mu \\ &=-0.1086a^3 I_p\gamma h'\mu \text{ (practical)}, \end{aligned} \tag{28}$$

a form convenient for experimental test when the electron current density to the wall, which is $-\mu I_p$, is under direct control. Under this condition M is seen to be directly proportional to electron wall current and magnetic field. A change in field will, of course, profoundly affect the wall current at any fixed wall potential.

When, however, the wall is nonconducting so that $\mu=1$ we shall have to determine in effect the behavior of $I_p h'$ with increasing H. It is this problem which concerns us from now on.

We note that the right-hand member of Eq. (26) divides naturally into two factors, the first, included within the parentheses giving the total electronic charge Q per axial centimeter in e.m.u. and the second giving its average areal velocity, A. Thus,

$$\left.\begin{aligned} Q&=0.432\pi a^2 n_0 e/c \\ &=-2.17\times10^{-20}a^2 n_0 \quad \text{(e.m.u.)}, \end{aligned}\right\} \tag{29}$$

$$A=-\frac{(T_e+T_p)D_p D_e\mu\beta}{\alpha D_e T_p+\mu D_p T_e}, \tag{30}$$

$$M=QA \tag{31}$$

and, per electron,

$$M_1=Ae/c=-1.6\times10^{-20}A. \tag{32}$$

[13] See, for instance, Eq. (51) of General Theory (reference 9).

Let us neglect T_p in comparison with T_e in the numerator in Eq. (30), set μ equal to unity, then

$$A=\frac{-T_eD_pD_eh'\gamma_\beta}{\gamma_\alpha D_eT_p+(1+h'^2)D_pT_e}. \quad (33)$$

As h' increases negatively from zero with the impressed magnetic field, γ_α and γ_β vary in accordance with Fig. 1 and D_e and T_e change as well. The changes in the γ's will be taken into account later, those in D_e and T_e will be entirely neglected. Watching the change in A from h' alone then, we see that initially it increases linearly with h' and that finally it decreases as $1/h'$, with a maximum between. This is the behavior found by Steenbeck in his experiment, where μ was unity.

In Eq. (33) we can neglect unity in comparison with h'^2 without error, because D_pT_e is small in any case relative to $\gamma_\alpha D_eT_p$. Then A can be written

$$A=\frac{-\gamma h'D_pT_e/T_p}{1+h'^2D_pT_e/(\gamma_\alpha D_eT_p)}. \quad (34)$$

For variation of h' (or H) the maximum value of A is then found to be

$$A_{\max}=(\gamma\gamma_\alpha^{\frac{1}{2}}/2)(D_eD_pT_e/T_p)^{\frac{1}{2}}, \quad (35)$$

occurring at

$$h'_{\max}=-[\gamma_\alpha D_eT_p/(D_pT_e)]^{\frac{1}{2}}. \quad (36)$$

The various quantities appearing in the above equations have now to be related to measurable quantities in the arc.

For h', Eq. (6) and

$$\bar{c}=[8kT/(\pi m_e)]^{\frac{1}{2}} \quad (37)$$

give

$$h'=(3\pi/8)(e/kc)H_zD_e/T_e = -1.375\times10^{-4}H_zD_e/T_e. \quad (38)$$

Comparison with Eq. (36) gives

$$H_{\max}=7.27\times10^3[\gamma_\alpha T_eT_p/(D_pD_e)]^{\frac{1}{2}}. \quad (39)$$

It is to be noted that the behavior of A with increasing H_z is qualitatively the same as that found both theoretically and experimentally by Steenbeck. The occurrence of a maximum magnetic moment has just been pointed out. For small values of H_z Eqs. (34) and (38) show that[14]

$$A=1.375\times10^{-4}(\gamma D_eD_p/T_p)H_z, \quad \chi=1.375\times10^{-4}Q\gamma D_eD_p/T_p \quad (40)$$

so that the magnetic moment is proportional to the field and the susceptibility is constant. Considerably beyond the maximum in M (or A)

$$A=7.27\times10^3\gamma\gamma_\alpha T_e/H_z, \quad =8.57\times10^3T_e/H_z, \quad \text{(e.m.u.)} \quad \chi=QdA/dH_z=-8.57\times10^3QT_e/H_z^2. \quad (41)$$

It is interesting that in this range Steenbeck's Eq. (10) leads to the identical expression. Otherwise, however, the occurrence of ion temperature and diffusion coefficient in the present equations emphasize the essential difference of the present treatment from the other, even though the qualitative behavior of M is the same.

Unfortunately Steenbeck has given sufficient arc data to permit a comparison of these formulas with his experimental results. Accordingly it seems best to apply the present theory to a case in which more complete data are available in order to obtain some idea as to the magnetic magnitudes to be expected.

Excellent measurements on a Hg arc in a 3.2-cm diameter tube have recently been published by Klarfeld.[15] He has analyzed his data even up to 7.5×10^{-3} mm of pressure in terms of the ion-free-fall case. There is some reason to expect the quasi-neutral diffusion theory to apply here and at his higher pressure of 20×10^{-3} mm. For the present purpose we shall analyze the data for the 0.3-amp. arc at these pressures in this way. The small correction for wall sheath thickness has not been made.

Equation (28) requires that the ion wall current be measured in the presence of a magnetic field, so that it cannot be applied now. We must rather determine the necessary quantities in Eqs. (33), (35), (36) and (39).

[14] The γ has been included in this formula because, even though γ approaches unity as h' approaches zero, it will appear later that Eq. (40) may still be a good approximation for values of h' so large that γ is appreciably less than unity.

[15] B. Klarfeld, Zeits. f. Tech. Physik U. S. S. R. **5**, 913 (1938).

Eliminating λ between Eqs. (27) and (22) (with $\mu=1$ and T_p neglected relative to T_e) gives

$$D_p/T_p=5.02\times10^{18}aI_p/(T_e n_0) \qquad (42)$$

in practical units.

We obtain D_e from the arc current and longitudinal gradient Z,

$$i_z=0.432\pi a^2 n_0 e^2 D_e Z/(kT_e),$$

whence

$$D_e=3.96\times10^{14} i_z T_e/(Za^2 n_0) \qquad (43)$$

in practical units.

Table I contains in its first six rows Klarfeld's data on a 0.3-amp. arc in a 3.2-cm diameter arc tube. The following rows contain the magnetic quantities already defined, each accompanied by the number of the equation by which it was calculated. First $h'_{\max}/\gamma_\alpha^{\frac{1}{2}}$ was found from Eq. (36). Since this can differ from $h'_{\max}$ by 20 percent at the most, sufficiently accurate values of the corresponding α and γ can be found from Fig. 1. These have been used in Eq. (36) itself to determine h' and also in all other equations. In the present example $h'_{\max}$ is so great that γ_α and γ take on their extreme values of $\pi/2$ and $\frac{3}{4}$, respectively. The calculation of the remaining quantities, maximum magnetic moment per axial cm $M_{\max}$ and the magnetic field $H_{\max}$ at which this occurs, also the low and high field susceptibilities follow in straightforward fashion.

The values of $H_{\max}$ calculated here are higher than the 50 oersteds shown by Steenbeck in his Fig. 12. The present values of $M_{\max}$ have to be divided by $\pi a^2=8.05$ cm² to yield the average moment per cm³ which is, presumably the ordinate in that same figure. Since Steenbeck shows 1.3×10^{-3} as the value of that quantity, the present values would seem to be some ten times smaller.

If the γ's were not present in the formulas relating M and H_z, this relationship could be represented by a single universal curve. This is still true if we are content to relate $M\gamma^{-1}$ to $H_z\gamma_\alpha^{-\frac{1}{2}}$. By plotting $\log(-M\gamma^{-1})$ against $\log(H_z\gamma_\alpha^{-\frac{1}{2}})$ we obtain the curve of Fig. 3. The two asymptotes are then 45° straight lines, the curve lies symmetrically with respect to them, and it resembles a hyperbola. The value of $-M_{\max}\gamma^{-1}$ lies log 2 below the intersection of

TABLE I. *Calculation of magnetic properties of a positive column. Arc current = 0.3 amp., tube radius = 1.6 cm*[14].

QUANTITY	UNITS	EQUATION	NUMERICAL VALUES		
P	mm		7.5	20	$\times10^{-3}$
T_{gas}	°K		315	329	$\times1$
T_{em}	°K		1.86	1.31	$\times10^{4}$
I_p	amp. cm^{-2}		0.105	0.155	$\times10^{-3}$
I_{em}	amp. cm^{-2}		82.5	175	$\times10^{-3}$
Z	v cm^{-1}		0.478	0.538	$\times1$
n_0	cm^{-3}		2.44	6.16	$\times10^{10}$
D_p/T_p		42	1.86	1.54	$\times1$
D_e		43	7.4	1.83	$\times10^{7}$
$-Q$		29	1.36	3.42	$\times10^{-9}$
$\gamma_\alpha^{-\frac{1}{2}}h'_{\max}$		36	46.2	30.1	$\times1$
$\gamma_\alpha^{\frac{1}{2}}$ (at max.)		Fig. 1	1.253	1.253	$\times1$
γ (at max.)		Fig. 1	0.75	0.75	$\times1$
$\gamma\gamma_\alpha^{\frac{1}{2}}$ (at max.)			0.94	0.94	$\times1$
$h'_{\max}$		36	57.9	37.7	$\times1$
$A_{\max}$		35	7.52	2.86	$\times10^{5}$
$-M_{\max}$	oersted cm²	31	1.026	0.978	$\times10^{-3}$
$H_{\max}$	oersted	39	106	196	$\times1$
$-\chi/\gamma$ (small H)		40	2.57	1.33	$\times10^{-5}$
χH^2 (large H)		41	0.217	0.384	$\times1$
H_0	oersted	45	800	1240	$\times1$

the asymptotes. Any plot of $\log(-M\gamma^{-1})$ against $\log(H\gamma_\alpha^{-\frac{1}{2}})$ is simply the fundamental curve and its asymptotes translated with respect to the coordinate system. Roughly speaking, pressure change translates with respect to H_z and arc current change with respect to M.

Before leaving this question it should be pointed out that the attempt has been made to apply the electron drift theory over a distance, the positive column radius, which is comparable with, if greater than, the electron mean free path. For magnetic fields so great that the electron orbit diameter is small compared to that of the column, the application is justifiable, but for lower fields some question can be raised and some modification of the theory may be necessary.

Two other points should be recalled. The first is that the quasi-neutral diffusion theory on which the present treatment is based is itself unsatisfactory. Secondly, the relatively small changes in T_e and D_e, and possibly D_p/T_p with H_z have been omitted from consideration. Any complete experimental test of the theory would have to include the measurement of T_e, I_{em}, I_p and Z in the presence of the magnetic field from which to calculate all the necessary quantities for each field condition.

While a numerical example is before us it is interesting to calculate the magnetic field H_0 at which the plasma field vanishes, according to Eqs. (20) and (24) for μ equal to unity. We have, to a sufficient degree of approximation:

$$0=D_e-D_p/\alpha=D_e-h'^2D_p\gamma_\alpha^{-1},$$

whence

$$h'=(\gamma_\alpha D_e/D_p)^{\frac{1}{2}}. \quad (44)$$

Combining with Eqs. (38) and (39) gives for H_0,

$$H_0=H_{\max}(T_e/T_p)^{\frac{1}{2}}. \quad (45)$$

The values of H_0 from this formula are included in Table I.

V. Constriction of Arc Under Its Own Magnetic Field—Pinch Effect

The electron drift equations can be applied in quite another direction. In arcs of large cross section and carrying large currents the magnetic field of the arc itself has an influence on the plasma relations. This factor, as well as others which become important, has been discussed in the *Transactions of the Electrochemical Society* article,[6] under the heading Pinch Effect, but only in general outline. Here the theory will be developed more completely. In doing this, the other factors, such as multiple stage ionization, transverse pressure differences in the column and the longitudinal pressure gradient will be neglected, partly because of theoretical complications and partly because they probably do not change the essential mechanism and effect of the self-magnetic fields although they undoubtedly modify such effects.

The magnetic field which arises from the arc current itself is circular and has no radial component. Let us adopt a cylindrical coordinate system in which x lies along the tube axis and is directed toward the anode, r is radial, and the angular coordinate is φ so that x, r, φ form a right-handed system. Thus r takes the place of y and φ of z in the usual formulation (see Eqs. (1) and (2)) of the drift equations.

The field influences the radial and longitudinal drift of electrons in accordance with the equations

$$u_x=D_e(\alpha b_x+\beta b_r),$$

$$u_r=D_e(\alpha b_r-\beta b_x).$$

Using Eq. (3) and noting that $\partial n_e/\partial x$ is zero, these become

$$u_x=-D_e[\alpha\xi+\beta(\partial\eta/\partial r+\partial \ln n_e/\partial r)], \quad (46)$$

$$u_r=-D_e[\alpha(\partial\eta/\partial r+\partial \ln n_e/\partial r)-\beta\xi], \quad (47)$$

where

$$\eta=eV/kT_e \quad (47.5)$$

and

$$\xi=\partial\eta/\partial x.$$

(In terms of the longitudinal gradient X of the arc

$$\xi=-(e/kT_e)X. \quad (48)$$

Both X and ξ are intrinsically negative.)

The continuity equation for the ions, Eq. (15.5) and the ion drift equation Eq. (15) remain unchanged. If, in addition, we retain Eq. (13) corresponding to a nonconducting tube wall, the resulting equations become unmanageable. Great simplification results if the wall is supposed to be conducting and held so negative that there is no radial flow of electrons. It appears highly unlikely that changes in the small radial flow would make appreciable differences in the main arc

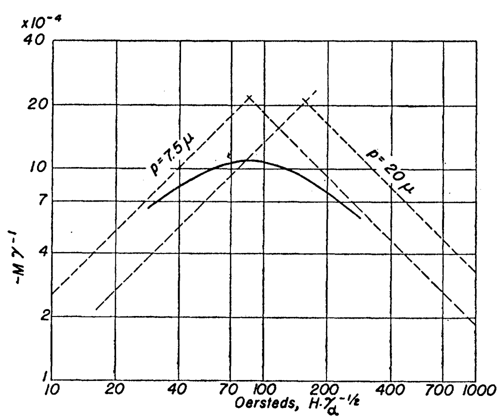

FIG. 3. Calculated diamagnetic moment of positive column, per cm length, *vs.* longitudinal magnetic field strength, uncorrected for γ's. Arc data from Klarfeld (reference 15).

flow. Accordingly we abandon Eq. (13) and set $u_r=0$ in Eq. (47) so that

$$\partial\eta/\partial r+\partial \ln n_e/\partial r-\gamma\xi h'=0. \quad (49)$$

Elimination of u_p and η between this and Eqs. (15) and (15.5) (remembering Eqs. (14)

and (47.5)) leads to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn_e}{dr}\right)+\frac{\lambda T_p}{(T_e+T_p)D_p}n_e-\frac{T_e}{T_e+T_p}\frac{d(\gamma h' n_e r)}{rdr}=0. \quad (50)$$

This equation contains not only n_e but also h' (with γ) as dependent variables. The second equation which is required for a solution is furnished by the relation of the magnetic field H_φ to the current density i_x in the arc:

$$r^{-1}d(rH_\varphi)/dr=4\pi i_x/c.$$

Since the cross drift u_r is zero the effective longitudinal mobility (see Eq. (8.5)) is $\gamma_r D_e e/kT_e$ so that

$$i_x=-e\xi\gamma_r D_e n_e. \quad (51)$$

Combining the two we have

$$r^{-1}d(rH_\varphi)/dr=-4\pi e\xi\gamma_r D_e n_e/c, \quad (52)$$

which is the second of the two necessary equations.

From this point it will be necessary to treat γ and γ_r as constants, and since the interest lies in large fields, their values for infinite h' will be used. Also, a set of substitutions simplifies Eqs. (50) and (52) greatly and finally permits the elimination of H_φ (h') between them. The substitutions with an explanation of each are:

(1) $$s=\frac{T_p\lambda r^2}{4(T_e+T_p)D_p}, \quad (53)$$

so that

$$s_0/a^2=\frac{T_p\lambda}{4(T_e+T_p)D_p}, \quad (54)$$

where a is as usual the tube radius and s_0 is the value of s at which n_e falls to zero. We can also write

(2) $$r^2=(a^2/s_0)s, \quad (55)$$

$$\theta=n_e/n_0, \quad (56)$$

where n_0 is the axial electron concentration so that θ ranges from unity at the tube axis to zero at the wall.

(3) $$J=\int_0^s \theta ds, \quad (57)$$

so that J is a measure of the current within a certain radial distance. This allows Eq. (52) to be simplified to

$$H_\varphi=-s^{-\frac{1}{2}}J[\pi e\xi\gamma_r D_e n_0 a/(s_0^{\frac{1}{2}}c)], \quad (58)$$

(4) $$L=\frac{3\pi^2 D_e^2 e^2\xi^2 a^2\gamma_r\gamma n_0}{8c^2k(T_e+T_p)s_0}, \quad (59)$$

which assembles all of the quantities appearing in the coefficient of the third term of Eq. (50) after the previous substitutions have been made and H_φ has been eliminated with h' by Eq. (6). L is the quantity denoted by A in reference 6.

As a result of these substitutions Eq. 50 assumes the form

$$\frac{d}{ds}\left(s\frac{d^2J}{ds^2}\right)+\frac{dJ}{ds}+L\frac{d}{ds}\left(J\frac{dJ}{ds}\right)=0,$$

which is directly integrable to

$$sd^2J/ds^2+J(1+LdJ/ds)=0. \quad (60)$$

That the constant of integration is zero follows from Eqs. (56) and (57).

Near the origin

$$J=s-\frac{1+L}{2}s^2\left[1-\frac{1+3L}{6}s+\frac{1+13L+18L^2}{72}+\cdots\right]. \quad (61)$$

Equation (60) has been integrated by numerical quadrature for a succession of values of L. In each case the process was carried to the value s_0 of s at which dJ/ds passed through zero because the relation

$$\theta=dJ/ds \quad (62)$$

shows that there is where the tube wall lies. The value J_0 of J at this point is directly related to the average electron concentration $\bar{n}_e$ in the cross section:

$$\bar{n}_e=a^{-2}\int_0^a 2n_e r dr=(n_0/s_0)\int_0^{s_0}\theta ds=n_0 J_0/s_0. \quad (63)$$

Referring back to Eq. (51) the total arc current is now seen to be

$$i_x = \pi a^2 e \xi \gamma_r D_e n_0 J_0 / s_0. \tag{64}$$

By eliminating n_0 with Eq. (59) we find

$$\left.\begin{aligned} i_x &= \frac{8c^2k(T_e+T_p)}{3\pi e\gamma\xi D_e} J_0 L, \\ i_x &= 3.32\frac{(T_e+T_p)T_e}{XD_e} J_0 L \text{ (practical)} \end{aligned}\right\} \tag{65}$$

and rearranging Eq. (59)

$$\left.\begin{aligned} n_0 &= \frac{8c^2k(T_e+T_p)}{3\pi^2e^2\gamma\gamma_r D_e^2\xi^2a^2} Ls_0, \\ n_0 &= 9.2\times10^{15}\frac{(T_e+T_p)T_e^2}{X^2D_e^2a^2} Ls_0 \text{ (practical).} \end{aligned}\right\} \tag{66}$$

Since s_0 and J_0 are functions of L alone, these equations express n_0 as a function of i_x insofar as T_e, ξ, and D_e are constant.

Physically, the independent variable is the arc current. The corresponding dimensionless quantity in the mathematical theory is seen to be LJ_0 by Eq. (65). This quantity will, therefore be treated as the independent variable or parameter in the various plots below. It is the quantity which is denoted by G_2 in reference 6. Similarly Ls_0 corresponds to n_0.

It is of interest to find the variation of potential across the arc. This is most readily calculated and plotted in the form $\epsilon^{-\eta}$ which shows the charge distribution which would exist in the actual field if no magnetic forces were present. Elimination of u_p between Eqs. (15) and (15.5) gives (from the substitutions (54), (55) and (56))

$$\frac{d}{ds}\left[s\theta\frac{d\eta}{ds} - (T_p/T_e)s\frac{d\theta}{ds}\right] = \frac{T_e+T_p}{T_e}\theta. \tag{67}$$

Integrating once gives the function J in the right member. But by Eq. (60)

$$J = -\frac{sd\theta/ds}{1+L\theta}, \tag{68}$$

so that the integral of Eq. (67) becomes

$$d\eta/ds = (T_p/T_e)d\ln\theta/ds - \frac{T_e+T_p}{T_e}\frac{d\theta/ds}{\theta(1+L\theta)}.$$

A final integration and rearrangement gives

$$\epsilon^{-\eta} = \theta\left(\frac{1+L}{1+L\theta}\right)^{(T_e+T_p)/T_e} \tag{69}$$

The T_p in the numerator of the exponent can easily be neglected, and this we shall do.

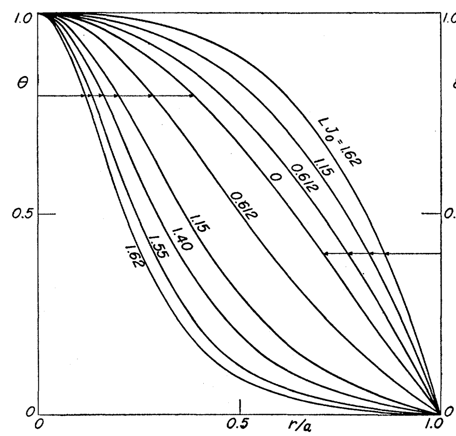

FIG. 4. Radial distributions of electrons (and ions) and potential in self-magnetic pinch effect. Lower left curves show relative electron concentrations with increasing arc current (LJ_0). Upper right curves show the distribution of the Boltzmann factor, exp $(-eV/kT_e)$.

The single quantity which is of primary importance in the magnetic effects is h'. Combining Eqs. (58) and (6) and comparing with Eq. (59) shows that

$$\begin{aligned} h' &= \frac{T_e+T_p}{T_e\xi\gamma a} Ls_0^{\frac{1}{2}} J s^{-\frac{1}{2}}, \\ h' &= 1.15\times10^{-4}[(T_e+T_p)/(aX)] Ls_0^{\frac{1}{2}} J s^{-\frac{1}{2}}. \end{aligned} \tag{70}$$

Thus the quantity corresponding to h' is

$$LJ(s/s_0)^{\frac{1}{2}}.$$

It follows directly from Eq. (63) that the positive ion current density to the wall is

$$I_p = \tfrac{1}{2}ae\lambda n_0 J_0/s_0$$

or by Eq. (54)

$$I_p = (2eD_p(T_e+T_p)/T_p)n_0J_0/a. \tag{71}$$

The calculations involved in the numerical solution of Eq. (60) were carried out by Mr. R. Beresford.

The calculated distributions of electron concentration are shown in Fig. 4 as plots of θ against fractional tube radius. The $LJ_0=0$ curve, corresponding to no magnetic effect, is one of the six curves of this group. The other five all lie below this one. The constriction of the arc is clearly seen. Each curve is labelled with the value of LJ_0 to which it corresponds.

Along with the peaking of the concentration goes a decrease of the plasma fields which is qualitatively the same effect as that found in Section III for the longitudinal field case. This is shown by the $\epsilon^{-\eta}$ curves also in Fig. 4. They appear above the $LJ_0=0$ curve. For $L=0$ the θ and $\epsilon^{-\eta}$ curves coincide, but the larger L the further they separate. Where the ion density is most greatly increased by magnetic concentration, the electric field drawing off the ions is most greatly decreased.

Table II shows further results of integrating Eq. (60). The mathematical quantities are listed in the first row, the numbers of the equations relating them to others in the second, their corresponding physical quantities in the third, and numerical values in the subsequent rows. The values of L used in the integrations are shown in the first column, and values of s and J at the tube walls are shown in columns 2 and 3. Column 4 contains the current-like quantity, column 5 the one corresponding to the axial concentration of electrons, and column 6 contains the ratio of axial to average electron concentration.

If there were no progressive concentration of the arc, $n_0/\bar{n}_e$ would be independent of arc current, but the increase of s_0/J_0 with increasing LJ_0 shows that a marked concentration of 4.5-fold takes place in the range of the table. This is shown in Fig. 5 where Ls_0 is plotted against LJ_0. This curve is rather well expressed by the

TABLE II. *Quantities involved in pinch effect.*

L Eq. (59)	s_0 Eq. (54)	J_0 Eq. (62)	LJ_0 Eq. (65) $\propto i$	Ls_0 Eq. (66) $\propto n_0$	s_0/J_0 Eq. (63) $n_0/\bar{n}_e$
0	1.441	0.623	0	0	2.32
2	0.948	0.306	0.612	1.90	3.10
10	0.545	0.1146	1.146	5.45	4.76
25	0.352	0.0558	1.396	8.81	6.31
50	0.258	0.0309	1.55	12.90	8.34
80	0.213	0.0203	1.62	17.0	10.46

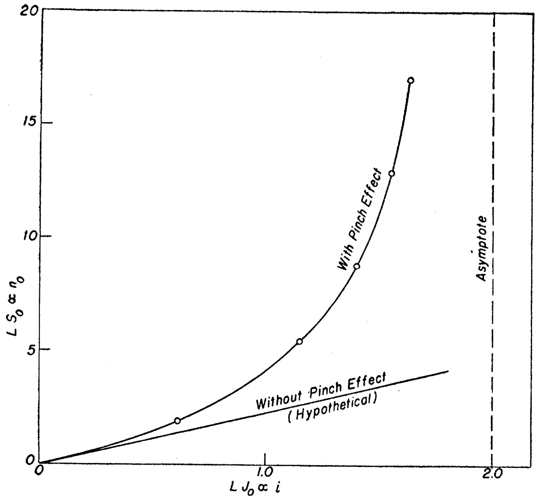

FIG. 5. Axial concentration of electrons (and ions) in self-magnetic pinch effect, as a function of arc current.

empirical equation

$$Ls_0=5LJ_0/(2.16-LJ_0). \qquad (72)$$

It is rising more steeply than this hyperbola at $L=80$. Analysis of Eq. (60) for large values of L shows that 2 is an absolute upper limit for LJ_0 so that actually

$$LJ_0=2 \qquad (73)$$

is the asymptote of the Ls_0 *vs.* LJ_0 curve.

Of course the value of n_0 (corresponding to Ls_0) cannot become infinite. With singly ionized atoms it cannot exceed n_a, the concentration of atoms. Thus this analysis predicts 100 percent ionization at the axis of an arc with currents far below those which, with a normal distribution in the cross section, would effect this. Steep increases in arc drop with current when a large diameter arc is already carrying a large current may well be due to this cause.

It is of interest to compare the critical current for the magnetic constriction with that arising from the transverse pressure effect.[6] For the former, Eqs. (65) and (73) give

$$i_{H\ \mathrm{crit}} \simeq 16c^2kT_e/3\pi e\gamma\xi D_e. \qquad (74)$$

For the latter

$$I_{p\ \mathrm{crit}} \simeq 0.43\pi a^2 eD_e\xi n_0 \qquad (75)$$

with $$n_0 \backsimeq (T_p/T_e)n_a. \quad (76)$$

Accordingly

$$\frac{i_{p\ \mathrm{crit}}}{i_{H\ \mathrm{crit}}} = 7.98\frac{\gamma a^2 e^2 D_e^2 \xi^2 T_p n_a}{c^2 k T_e^2}. \quad (77)$$

Now $D_e\xi$ is practically independent of n_a, but ξ varies oppositely from a though not as strongly as a^{-1}. Thus it appears that for larger radii and higher pressures the critical condition for blowing the gas out of the arc column lies at a higher value of current than does the critical condition for 100 percent ionization due to pinch effect. Putting in numerical values and using p for the gas pressure in bars, we have

$$i_{p\ \mathrm{crit}}/i_{H\ \mathrm{crit}} = 11a^2 D_e^2 X^2 p/T_e^4 \text{ (practical)}. \quad (78)$$

AUGUST 15, 1939 PHYSICAL REVIEW VOLUME 56

Investigations of Ferromagnetic Impurities. I

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(Received June 26, 1939)

A method has been developed whereby a permanent magnetic moment as small as 2×10^{-7} per cc may be measured in any small solid specimen. Various materials were tested by this method for ferromagnetic impurities. To remove surface impurities it was necessary to dissolve away part of the specimen. Most metals then showed some volume impurity. In aluminum, however, the volume impurity was nonmagnetic but could be dissolved out and deposited on the surface in a ferromagnetic state. Measurements were also made on the magnetic hardness and hysteresis curves of the impurities; part of the impurity was found to be very "hard," with saturation incomplete at several thousand gauss and a coercive force$\sim$100–200, and part quite "soft," with a relatively large initial susceptibility. Other properties of these impurities are being studied. The method was also extended to single crystals of dia- or paramagnetic materials whose magnetic anisotropy could be easily detected and measured.

AS previously reported,[1] an experimental method has been developed whereby very minute ferromagnetic impurities may be detected in various materials and some of the properties of these small impurities studied. The results of such studies should be of interest for several reasons. First, by such a method one may quickly detect and measure the amount of ferromagnetic impurity present in materials which are generally regarded as nonmagnetic and, in some cases, possibly estimate the kind and amount of impurity present. Then from a study of the properties of such impurities one may find how best to render a slightly magnetic material, brass, for example, less magnetic. Finally such studies should furnish further information as to the fundamental nature of ferromagnetism.

[1] F. W. Constant and J. M. Formwalt, Phys. Rev. 53, 432 (1938).

EXPERIMENTAL METHOD

The method is quite simple. The specimen to be tested may have any shape, even be powdered if contained in a capsule, but small lumps or rods of about one gram were generally used. The specimen is first placed in a magnetic field of several thousand gauss, furnished by an electromagnet. This serves to magnetize all ferromagnetic impurities present in a common direction. Upon removing the specimen from this strong field such impurities will remain partially magnetized in this direction because of their remanent magnetization. The specimen is then placed in a light stirrup and suspended by a fairly strong quartz fiber in the center of a pair of Helmholtz coils, the direction of the remanent magnetization being perpendicular to the axis of the coils. A weak field of about 10 gauss is then applied and the resulting rotation measured by