clearly seen that concentrations of charge between atoms lead to strong attractive forces, and hence, are properly called valence bonds.

Van der Waals' forces can also be interpreted as arising from charge distributions with higher concentration between the nuclei. The Schrödinger perturbation theory for two interacting atoms at a separation R, large compared to the radii of the atoms, leads to the result that the charge distribution of each is distorted from central symmetry, a dipole moment of order $1/R^7$ being induced in each atom. The negative charge distribution of each atom has its center of gravity moved slightly toward the other. It is not the interaction of these dipoles which leads to van der Waals' force, but rather the attraction of each nucleus for the distorted charge distribution of its *own* electrons that gives the attractive $1/R^7$ force.

The author wishes to express his gratitude to Professor J. C. Slater who, by his advice and helpful suggestions, aided greatly in this work. He would also like to thank Dr. W. C. Herring for the latter's excellent criticisms.

AUGUST 15, 1939 PHYSICAL REVIEW VOLUME 56

Intercrystalline Thermal Currents as a Source of Internal Friction*

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(Received June 12, 1939)

An experiment has been designed to detect the contribution of intercrystalline thermal currents to the internal friction of polycrystalline metals. In accordance with a theory developed by one of the writers (C.Z.), the internal friction is a maximum when the vibration is partly isothermal and partly adiabatic with respect to adjacent grains. By passing in small steps from the nearly isothermal case of very small grain size through maximum internal friction to the nearly adiabatic case of large grain size, one can detect the relative importance of the intercrystalline thermal currents. Such an experiment has been performed on single phase 69–31 brass, with mean grain size ranging in small steps from 0.0006 cm to 0.4 cm, and with frequencies of 6000, 12,000 and 36,000 cycles per second. Not only was a maximum obtained with the anticipated grain size, but the maximum is of a larger order of magnitude than the background upon which it is superimposed. The internal friction in the extreme isothermal case ($Q>300{,}000$) was lower than has ever been observed for metals; in the extreme adiabatic case it approached the low values obtained for single crystals. This experiment indicates that in annealed nonferromagnetic metals at room temperature, intercrystalline thermal currents are the dominant cause of internal friction measured at small strains, aside from possible macroscopic thermal currents.

§1. INTRODUCTION

THE term internal friction refers to the capacity of a solid to transform its ordered energy of vibration into disordered internal energy. One of the authors (C. Z.) has recently made a start at understanding the mechanism of this transformation.[1, 2, 3] His basic idea was that the direct coupling between the macroscopic and the internal coordinates may be treated by examining the thermoelastic effects which accompany vibration. Thus the increase in internal energy per cycle is equated to the temperature times the increase in entropy per cycle. The increase in entropy per cycle is obtained by studying the thermal currents which flow back and forth during vibration between stress inhomogeneities.

If these thermal currents are able nearly to maintain temperature equilibrium between the stress inhomogeneities, the vibration proceeds isothermally with little internal dissipation of energy. In the other extreme case of adiabatic vibration, the internal dissipation of energy is

* This research was supported by a grant from the Penrose Fund of the American Philosophical Society, and by a grant from the Rumford Fund of the American Academy of Arts and Sciences.

[1] C. Zener, Phys. Rev. **52**, 230 (1937).
[2] C. Zener, Phys. Rev. **53**, 90 (1938).
[3] C. Zener, Proc. Phys. Soc. (in print).

also very small. It reaches a maximum when the vibration is partly isothermal and partly adiabatic.

The condition for isothermal and for adiabatic vibration may be obtained from dimensional considerations. The degree of adiabaticity of the vibration can be a function only of some measure L of the linear dimension of the stress inhomogeneity, of the frequency of vibration ν, and finally of the thermal diffusion coefficient D. Since D has dimensions of (length)^2/time, the degree of adiabaticity can depend upon these factors only in the dimensionless combination $\nu L^2/D$. As this combination changes from a number much less than unity to a number much greater than unity, the vibration passes from the nearly isothermal to the nearly adiabatic case. The thermoelastic internal friction may thus be detected experimentally by varying this parameter $\nu L^2/D$ over a wide range. The internal friction will be at a maximum when this parameter has a value comparable to unity, and will fall to low values when this parameter becomes either very small or very large.

The thermoelastic internal friction accompanying transverse vibrations has been studied in this manner.[4, 5] In such vibrations the stress varies across the specimen. Hence the dimension of the stress inhomogeneities may be taken as the transverse width a of the specimen. The parameter $\nu a^2/D$ was varied by taking measurements at various frequencies. The frequency range from where the internal friction is at half its maximum value on the isothermal side, to where it is at half its maximum on the adiabatic side covers nearly four octaves. Over this wide frequency range the internal friction due to the transverse thermal currents has been found to be of a larger order of magnitude than that due to all other causes.

All types of elastic vibrations in polycrystalline specimens are accompanied by stress inhomogeneities between adjacent crystallites. These microscopic stress inhomogeneities are caused by the elastic anisotropy and at least partial random orientation of the individual crystallites.[2] They give rise to microscopic thermal currents. The present paper describes an experiment which investigates the internal friction arising from these intercrystalline thermal currents.

The diameter d of the average grain may here be taken as the linear dimension of the stress inhomogeneity. In order to obtain a wide variation of the parameter $\nu d^2/D$, measurements were made not only at various frequencies, but also on specimens with a wide range of average grain diameter. These specimens were prepared by C. S. Smith and E. W. Palmer of the American Brass Company. The material, 69-31 brass, was chosen in preference to a pure metal in order to avoid possible energy dissipation by slipping. With this variation of both ν and d, measurements were made at values of $\nu d^2/D$ ranging from 0.005 to 16,000.

In the present investigation it was desirable to avoid damping arising from macroscopic thermal currents. For this reason longitudinal vibrations were used in preference to transverse vibrations. Here the only macroscopic stress inhomogeneity has dimensions of the wave-length of the vibration. The corresponding parameter $\nu\lambda^2/D$ is very large for ordinary frequencies. Longitudinal vibrations are thus macroscopically adiabatic.

§2. Preparation of Material*

The alpha-brass obtained for this investigation was normal high grade commercial material of the following analysis:

Copper	68.74 percent.
Zinc (diff.)	31.23 "
Lead	0.02 "
Iron	0.008 "

It had been cold rolled to 0.315 inch following standard practice, annealed at about 600°C to produce a grain size of 0.06 mm, then cold rolled to 0.125 inch thickness (60 percent reduction), slit to $\frac{1}{2}$ inch width and straightened. Slight curvature at the edges resulted from the slitting rolls, but had no noticeable effect upon the internal friction measurements. The cold-rolled

[4] C. Zener, W. Otis and R. Nuckolls, Phys. Rev. **53**, 100 (1938).

[5] K. Bennewitz and H. Rötger, Zeits. f. tech. Physik **19**, 521 (1938).

* This material and the following description of the method of preparing it was furnished through the generosity of C. S. Smith and E. W. Palmer of the American Brass Company, whose assistance the authors gratefully acknowledge. Without their assistance this research would have been impossible.

TABLE I. *Grain size data.*

ANNEALING TEMP. (IN °C)	DIAMETER AVE. GRAIN BY COMPARISON	TWINS PER GRAIN CROSSED BY LINE
350	0.006 mm	
400	0.009	0.6
450	0.015	0.7
500	0.025	1.0
550	0.037	1.0
600	0.055	1.1
650	0.080	1.4
700	0.11	1.1
750	0.15	1.6
800	(0.25)	1.7
850	(0.5)	1.7
900	(1.0)	1.4

strips, cut to 12-inch lengths and numbered, were annealed in the laboratory at temperatures between 50 and 900°C in 50° steps. Anneals were for 30 minutes, followed by air cooling. Separate samples were given identical treatment alongside the 12-inch test pieces to provide material for grain size measurement. The specimens were supported on edge in a jig during annealing and great care was taken at all times to avoid distortion. The largest grain size obtainable by straight annealing this material was about 1 millimeter. A still larger grain size, 0.04 mm, was obtained by stretching a piece briefly annealed at 900°C to about one percent elongation, and then reheating it at 900°C for 30 minutes.

Table I shows the annealing temperatures and the average grain diameters obtained. The grain size (diameter of average grain) of each specimen was obtained by comparison of the projected image of a polished and etched section parallel to the rolled surface, at a magnification of 75, with the standard micrographs of the A. S. T. M. (A. S. T. M. Standard E2-36). These standards were established by counting the total number of individual grains visible in a large typical field of known area, and express the grain size as the square root of the average area of grain. A constant factor would convert this "grain size" to the true average diameter of the grains.

Figure 1, A, B, C, D shows the appearance of typical samples at a magnification of 75 diameters, and Fig. 1, E, F the appearance of a large and a small grain size, respectively, at magnifications at which they compare with a medium grain size, thereby showing the differences in grain characteristics. All samples were etched with ammonium and hydrogen peroxide.

For the present measurements it is possible that the significant structural unit should be based on twin dimensions as well as grain size. The average number of twins in a grain is greater in large than in small grains, but the increase appears to be a gradual one. This is shown in Table I, where the third column gives the ratio of the number of twins to the number of grains crossed by a line normal to the rolling direction. It should also be noted that the grain size distribution varies somewhat with the grain size, at least at the lower annealing temperatures where recrystallization is barely complete.

§3. EXPERIMENTAL PROCEDURE

Several measures of internal friction have been used in the literature. One is to measure, under forced oscillations, the frequency width $\Delta\nu$ of a resonance curve for an amplitude half that at resonance. The ratio of $\Delta\nu$ to the resonance frequency ν is a measure of the internal dissipation. Another measure is the logarithmic decrement δ. Then from the electrical analogy there is the ratio of the effective electrical resistance to the effective electrical reactance, $R/2\pi L=1/Q$. The relation between these measures is

$$1/Q=3^{-\frac{1}{2}}\Delta\nu/\nu=\delta/\pi.$$

The general method of measurement was similar to that used by Wegel and Walther.[6] Fig. 2 shows the arrangement. It was found possible to use an electromagnetic drive without attaching pole pieces. The driver and detector were air core solenoids whose impedances were matched to the oscillator and amplifier, respectively. An alternating current of variable frequency was supplied to the driving coil from a General Radio type 377 oscillator. The eddy currents induced in the reed reacted with the nonaxial components of a steady external magnetic field (furnished by permanent magnets) to produce longitudinal vibrations. At the pick-up end a small alternating potential was induced in a reciprocal manner, which was fed into the input circuit of a linear amplifier. The latter consisted of four pentode stages, giving a maximum voltage gain of over 10^6, the last two plate circuits containing tuned coupling elements. The "Q" of the reed was at least 100 times that of the over-all amplifier

[6] R. L. Wegel and H. Walther, Physics **6**, 141 (1935).

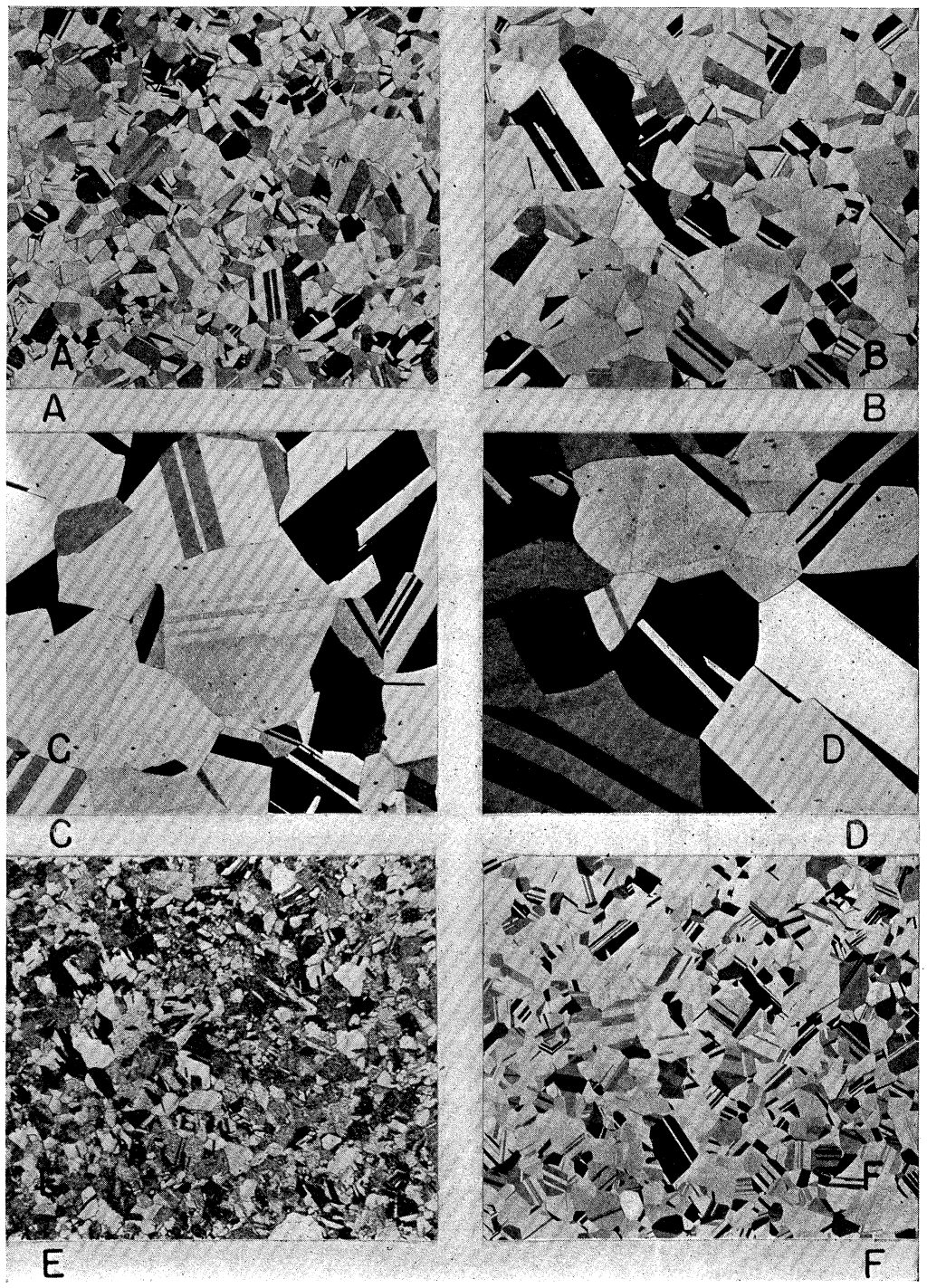

FIG. 1. Micrographs of brass specimens with different grain size. The horizontal axis is parallel to the rolling direction. Micrographs A, B, C, D have the same magnification of 75. The corresponding diameters of average grains are 0.037, 0.080, 0.15, 0.5 mm, respectively. Micrographs E and F compare a small and a large grain size specimen ($d=0.009$, 0.15 mm). Their magnifications ($\times 300$, $\times 17$) are such as to give the same apparent grain size. Annealing temperatures are, respectively, 550°C, 650°C, 750°C, 850°C, 400°C and 750°C.

characteristic, making the response linear to frequency over the resonance curve of the reed. Tuning the amplifier very greatly increased the useful amplification by cutting down background noise. Final voltage amplitudes were read on a d.c. microammeter in a diode rectifier circuit.

In order to eliminate direct inductive feed over between driver and pick-up, which occurred despite the use of shielding, a counter magnetic field was supplied with a coil similar to the driver connected to the oscillator.

Both the method of forced oscillations and the method of free oscillation decay were used. In cases of very low internal friction the method of forced oscillations was found to involve too critical an oscillator adjustment. It was sometimes then possible to observe the time of decay of the free oscillations. Whenever the two methods could both be used, the agreement was within the precision of measurement. The time t to decay to the fraction $1/n$ of the original amplitude is related to δ by the equation: $t=\log_e n/\delta\nu$.

Minimum support dissipation occurred with suspension on silk threads at a pair of displacement nodes. For the fundamental frequency, placing the two supports within about 1/50 of a wave-length of the central nodal point gave negligible dissipation.

The radiation and viscous losses due to the surrounding air were only important in the cases of low damping. Reduction of the surrounding air pressure to several mm of mercury made these losses negligible.

Varying the strength of the steady fields by moving the permanent magnets back and forth produced no noticeable change in the measured internal friction, showing that eddy current losses were a negligible factor.

It was the intention in this experiment to keep well below vibration amplitudes at which plastic flow might occur. No attempt was made to control amplitude and no amplitude effect on the internal friction was observed at any time, indicating that small amplitude conditions prevailed throughout.

§4. Results and Discussion

In Fig. 3 we give the variation of internal friction with grain size and with annealing temperature measured at each of the three frequencies 6000, 12,000 and 36,000 cycles per second. These frequencies were the first, second and sixth harmonics of our specimens. Four specimens of

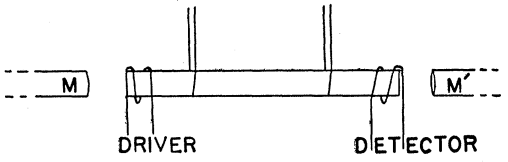

FIG. 2. Method of forcing and of detecting longitudinal vibrations. The specimen is supported by silk threads at nodes of vibration. The driving force is obtained by the reaction upon a permanent magnet M of the eddy currents induced by an alternating current in the driving coil. An alternating electromotive force is induced in a detector coil at the other end by the eddy currents arising from the motion of the specimen in the field of a second permanent magnet M'.

intermediate grain size appeared to have abnormally high internal friction at the highest frequency. In one of these specimens the resonance curve showed a distinct double peak, indicating a near coincidence of two natural frequencies. All four observations were discarded. Accurate measurements at all three frequencies could not be obtained on the specimens of smallest grain size. In these cases the resonance curve was too sharp to follow with our oscillator, while the half-decay time was too short to measure.

Figure 3 is in marked disagreement with the only previously published variation of internal friction with grain size. Förster and Köster[7] have reported the internal friction of 72-28 brass to increase with grain size up to at least a mean diameter of 3.7 mm. Since internal strains raise the internal friction of metals, and since such strains are more readily introduced in large than in small grain size specimens, the explanation of this discrepancy may lie in the presence of strains introduced in handling their specimens of largest grain size.

The initial rise of internal friction, shown in Fig. 3, as the annealing temperature is raised above the recrystallization temperature, has frequently been observed.[6–8] The most extensive measurements on brass, by Förster and Köster,[7]

[7] F. Förster and W. Köster, Zeits. f. Metallkunde **29**, 116 (1937).
[8] W. Köster and K. Rosenthal, Zeits. f. Metallkunde **30**, 345 (1938).

were made up to an annealing temperature of only 800°C. No indication of a maximum was observed. Since they did not report their frequency of measurement, direct comparison with our measurements is impossible. A slight maximum has been observed by Wegel and Walther[6] for copper at the annealing temperature of 300°C.

All the measurements shown in Fig. 3 have been replotted in Fig. 4 against the parameter $\nu d^2/D$. All measurements at the three frequencies lie upon a single curve. This is evidence that the microscopic thermal currents were the dominant cause of internal friction in our specimens. To the left of the maximum the vibration approaches the isothermal case, to the right the adiabatic case.

To the extreme right the internal friction should approach that of a single crystal. The only published values of single crystals are those of Read[9] for Cu, Pb, Sn. Copper, the metal most like brass, gave $Q^{-1}=1.1\times10^{-5}$. The internal friction shown in Fig. 4 seems to approach an asymptote of this order of magnitude at large grain sizes.

The internal friction on the extreme isothermal

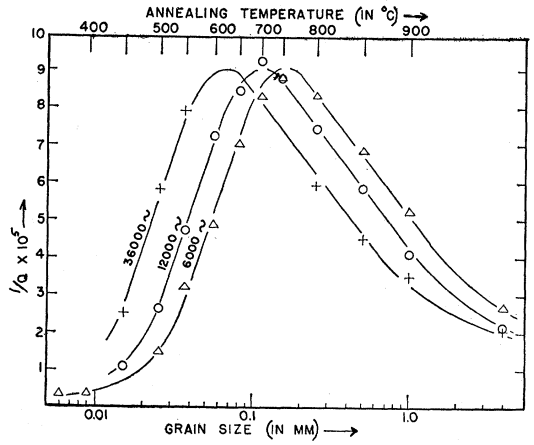

FIG. 3. Internal friction as a function of grain size and of annealing temperature. The triangles, circles, and crosses represent measurements at the frequencies 6000, 12,000 and 36,000 cycles/second. The three curves have been drawn identical save for a horizontal shift.

[9] T. A. Read, Phys. Rev. **54**, 389 (1938).

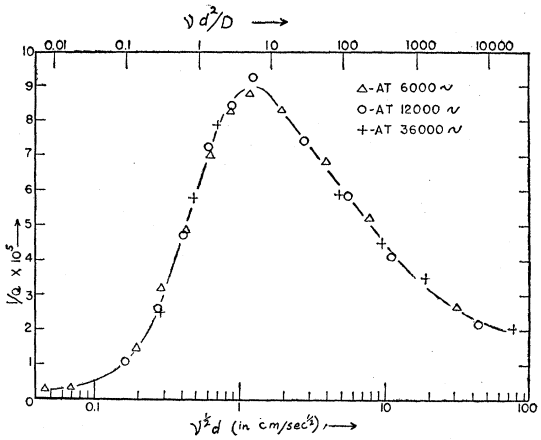

FIG. 4. Internal friction as a function of parameter $\nu d^2/D$. In this figure the three curves of Fig. 3 coincide.

side of the curve is surprisingly small. The lowest value, 0.3×10^{-5}, is three times as low as has ever been reported for a metal. The failure of Köster and Rosenthal[8] to obtain values lower than 2×10^{-5} in their recent extensive work on brass can probably be attributed to excessive external dissipation of energy, either through their supports or to the surrounding air.

In the dimensional argument leading to the prediction that the internal friction be a function only of the parameter $\nu d^2/D$, it was implicitly assumed that all the microscopic dimensions varied linearly with d. The detailed information concerning the specimens given in §2 shows this assumption to be slightly in error. A comparison of the two specimens in Fig. 1, E and F, shows that the specimens with smallest grain size have the greatest distribution in grain size. Further, Table I shows that the average number of twins per grain increases with grain size. Although these factors may have affected the shape of the curve in Fig. 4, they varied sufficiently slowly with d so that the measurements were, within our experimental error, determined solely by the parameter $\nu d^2/D$. The dimensional argument will also be slightly in error at the largest grain sizes where d is comparable to the width of the specimens.

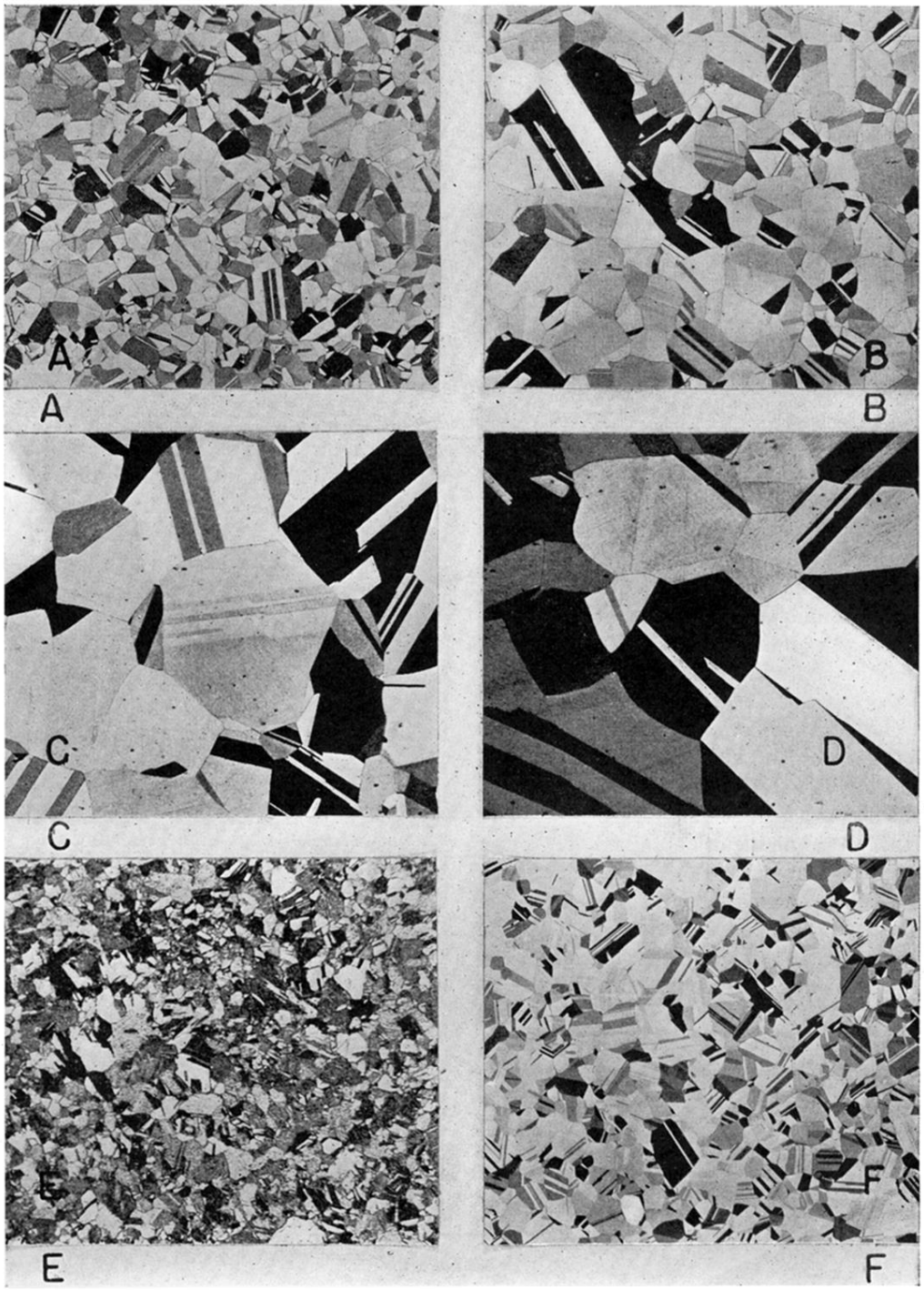

FIG. 1. Micrographs of brass specimens with different grain size. The horizontal axis is parallel to the rolling direction. Micrographs A, B, C, D have the same magnification of 75. The corresponding diameters of average grains are 0.037, 0.080, 0.15, 0.5 mm, respectively. Micrographs E and F compare a small and a large grain size specimen ($d=0.009$, 0.15 mm). Their magnifications ($\times 300$, $\times 17$) are such as to give the same apparent grain size. Annealing temperatures are, respectively, 550°C, 650°C, 750°C, 850°C, 400°C and 750°C.