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The Nuclear Five-Body Problem*

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Perturbational and variational calculations with single-particle Legendre functions have been made for the nuclear five-body problem of He^5. The symmetric Hamiltonian is employed; the values of the nuclear parameters chosen favor high binding. With a symmetric interaction, Li^5 and He^5 differ only in the Coulomb energy, states of the former lying about one Mev above those of the latter. The calculations show that there is no state of He^5 which is stable against dissociation into an alpha-particle and a neutron. There are two low-lying virtual levels, one of ^2P, the other of ^2S symmetry; according to the Hartree model, the energies of these states are -11.7 and -6.5 Mev, respectively. Second-order perturbation calculations diminish the interval $^2S-{}^2P$ radically and locate both the states in the neighborhood of -19.0 Mev. The last two results are roughly confirmed by variational calculations with two variation parameters (alpha-particle model), and -20 Mev may be set as a probable lower bound to the position of either state. These estimated values fall some seven Mev short of the experimental energy.

I. INTRODUCTION

ALTHOUGH much attention has been devoted to the theory of the light nuclei, little consideration has been given to the nuclear five-body problems. This is due partially to the fact that He^5 and Li^5 are of only minor experimental importance, and partially to the fact that Li^6 and higher nuclei offer more challenging problems and more stringent tests of theory.

There are reasons, however, why the five-body problem is of interest. There are now available from several investigations data which lead to an experimental energy of He^5. Moreover, for He^4, the first-order (Hartree) approximation is good, and convergence of higher terms is rapid. For Li^6, the Hartree model is poor, and higher terms converge slowly. This makes He^5 mathematically interesting as an intermediate case.[1]

II. ASSUMPTIONS AND PRELIMINARIES

The analysis is based upon the work of Margenau and Carroll on Li^6. Care has been taken to use the same notation as far as possible, and the reader is referred to their paper[2] for an explanation of points which are omitted here.

The symmetric Hamiltonian is assumed; the potential between pairs of nuclear particles is taken to be

$$V_{ij}=-A\exp(-r^2_{ij}/a^2)\times(w+mP_{ij}+bQ_{ij}+hP_{ij}Q_{ij}). \quad (1)$$

For the parameters, the following values are employed:

$w=-2/15$; $m=14/15$; $b=7/15$; $h=-4/15$; $A=35.60$ Mev; $a=2.25\times10^{-13}$ cm.[3]

The values of w, m, b, h are derived from the Breit-Feenberg and Kemmer inequalities[4] with equality signs taken, a choice corresponding to maximum binding in the Hartree approximation for light nuclei.

Since individual particle coordinates are used, the Hamiltonian for the five-body problem is[5]

$$H=-\frac{4}{5}\frac{\hbar^2}{2M}\Sigma_k\nabla_k{}^2+\frac{2}{5}\frac{\hbar^2}{2M}\Sigma_{k>l}\nabla_k\cdot\nabla_l+\Sigma_{k>l}V_{kl}$$
$$+\text{Coulomb interaction.}$$

The $\nabla_k\cdot\nabla_l$ terms and the numerical factors in the kinetic energy operators arise from the transformation to the center of mass of the system.

The single-particle functions employed are

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[1] Preliminary results for the ^2P state of He^5 have been presented at the New York meeting of the American Physical Society. Cf. Phys. Rev. **55**, 678 (1939).

[2] H. Margenau and K. G. Carroll, Phys. Rev. **54**, 705 (1938).

[3] It has recently been suggested that an interaction of shorter range and greater depth is needed to agree with proton-proton scattering data. Cf. G. Breit, H. M. Thaxton and L. Eisenbud, Phys. Rev. **55**, 603 (1939).

[4] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); N. Kemmer, Nature **140**, 192 (1937).

[5] H. Margenau and D. T. Warren, Phys. Rev. **52**, 790 (1937).

solutions of the Schrödinger equation for the isotropic three-dimensional harmonic oscillator. Thus, they are products of a radial Gaussian exponential, an associated Legendre function, and an azimuthal exponential, and they can be designated by the usual spectroscopic notation. The functions contain a common parameter q (q or k, for the two-parameter scheme), to be adjusted later for minimum energy. All functions are symmetrized by writing them in determinantal form. Generalized spin functions are used:

$\eta_1 \sim$ proton, $s_z = -\frac{1}{2}$; $\eta_3 \sim$ neutron, $s_z = -\frac{1}{2}$;
$\eta_2 \sim$ proton, $s_z = +\frac{1}{2}$; $\eta_4 \sim$ neutron, $s_z = +\frac{1}{2}$.

For a detailed description of the method for evaluating any matrix element H_{ij}, reference 2 may be consulted.

III. FIRST-ORDER CALCULATIONS FOR THE TWO LOWEST STATES

In this section, the Coulomb energy will be omitted. In the Hartree approximation, the lowest state of He^5 and Li^5 has ^2P symmetry and corresponds to the configuration $(1s)^4 2p$. Therefore, for He^5,

$$ {}_p\psi_0 = (1/5\,!)^{\frac{1}{2}} \left| s\eta_1 \cdot s\eta_2 \cdot s\eta_3 \cdot s\eta_4 \cdot 2p_1\eta_4 \right| \qquad (2)$$

can be taken as the lowest state function. The "fifth position" in ${}_p\psi_0$ may be filled equally well by the product of $2p_1$, $2p_0$, or $2p_{-1}$ and either η_3 or η_4; the Hamiltonian is diagonal for these six functions, and each corresponds to the same total spin, total angular momentum, and energy.

For Li^5, the fifth position must contain η_1 or η_2.

In terms of one variation parameter $\sigma = qa^2$, ${}_p\psi_0$ gives the first-order energy

$$ {}_pH_{00} = 3.5T\sigma - \frac{36}{15}\mathrm{A}\left(2 + \frac{1}{\sigma+2}\right)\left(\frac{\sigma}{\sigma+2}\right)^{\frac{3}{2}}, \qquad (3)$$

where $T \equiv \hbar^2/Ma^2$. The expression (3) has a minimum of -12.3 Mev at $\sigma = 1.6$.

In the same approximation, the next highest state is of ^2S symmetry, and is represented by the configurations $(1s)^4 2s$ and $(1s)^3(2p)^2$. In the former configuration there is but one function:

$$ {}_s\psi_0 = (1/5\,!)^{\frac{1}{2}} \left| s\eta_1 \cdot s\eta_2 \cdot s\eta_3 \cdot s\eta_4 \cdot 2s\eta_4 \right|. \qquad (4)$$

There are 10 distinct functions in $(1s)^3(2p)^2$; each of these is not alone an acceptable wave function, for the spin and isotopic spin matrices are not properly diagonal. It is necessary to combine all 10 functions linearly into a function, say ${}_s\psi_1$, which is suitably diagonalized. The method of performing the combination is well known from the theory of atomic spectra. ${}_s\psi_0$ and ${}_s\psi_1$ are degenerate and may be combined linearly to give an energy lower than that from either function alone. Solution of the secular equation corresponding to ${}_s\psi_0$ and ${}_s\psi_1$ shows, however, that this lowest energy differs so little from ${}_sH_{00}$ that the configuration $(1s)^3(2p)^2$ may safely be neglected. The excellence of (4) might be expected from the fact that, of the available 11 functions, it alone comprises the alpha-particle group (closed s shell).

In terms of σ, (4) yields the first-order energy

$$ {}_sH_{00} = 3.8T\sigma - \frac{36}{15}A\left[2 + \frac{2.5}{(\sigma+2)^2}\right]\left(\frac{\sigma}{\sigma+2}\right)^{\frac{3}{2}}. \qquad (5)$$

This has a minimum of -7.1 Mev at $\sigma = 1.2$.

The function (2) or (4) represents He^5 as an alpha-particle group and a p neutron or s neutron, respectively. This suggests the use of two variation parameters, one ($\sigma = qa^2$) for the four $1s$ particles, and one ($\kappa = ka^2$) for the $2p$ or $2s$ neutron. The functions (2) and (4) then lead to the first-order energies:[6]

$$ \begin{aligned} {}_pH_{00} = {} & (2.4\sigma + \kappa)T \\ & + 3.2T\sigma^{7/2}\kappa^{5/2}\left(\frac{1}{\sigma+\kappa}\right)^5 - \frac{72}{15}A\left(\frac{\sigma}{\sigma+2}\right)^{3/2} \\ & - \frac{1152}{15}A\sigma^{3/2}\left(\frac{\kappa}{\sigma+\kappa}\right)^{5/2}\left(\frac{1}{\sigma+\kappa+4}\right)^{5/2}, \end{aligned} \qquad (6)$$

$$ \begin{aligned} {}_sH_{00} = {} & \frac{1}{1-\Delta^2}\left\{\frac{2}{5}T\left[\frac{9}{2}\sigma\left(\frac{4}{3} - \Delta^2\right) + \frac{7}{2}\kappa \right.\right. \\ & \left. + 4\sqrt{3}\Delta(\sigma\kappa)^{7/4}\frac{7\sigma - 3\kappa}{(\sigma+\kappa)^{7/2}}\right] \\ & - \frac{36}{15}A\left[\left(\frac{\sigma}{\sigma+2}\right)^{3/2}(2-\Delta^2)\right. \\ & - \sqrt{6}\Delta\sigma^{9/4}\kappa^{3/4}\frac{\frac{1}{2}(\sigma+1)(\kappa-\sigma) - \sigma}{\{(\sigma+1)[\frac{1}{2}(\sigma+\kappa)+1]-1\}^{5/2}} \\ & \left.\left. + \frac{1}{2}(\sigma\beta)^{3/2}(5\beta^2 - 3\beta\sigma - 6\beta + 3)\right]\right\}, \end{aligned} \qquad (7)$$

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[6] Cf. Section VI of this paper in regard to the calculation of $\nabla_i \cdot \nabla_j$ integrals with two parameter functions.

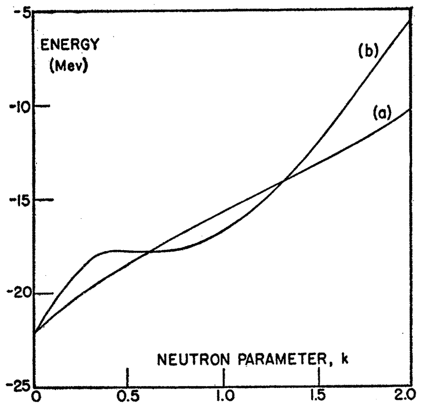

FIG. 1. First-order energies in two parameters. Variation with κ for $\sigma=2.6$. (*a*) ${}_pH_{00}$, Eq. (6). (*b*) ${}_sH_{00}$, Eq. (7).

where

$$\beta\equiv\frac{\kappa}{[\frac{1}{2}(\sigma+\kappa)+1]^2-1},$$

$$\Delta\equiv\int(1s)_\sigma(2s)_\kappa d\tau=2\sqrt{3}(\sigma\kappa)^{3/4}(\kappa-\sigma)\left(\frac{1}{\sigma+\kappa}\right)^{5/2}.$$

Δ is a nonorthogonality integral. The similar integral for the P state vanishes identically on account of a difference in orbital angular momentum.

For the alpha-particle, the ground state function

$$\phi_0=(1/4!)^{\frac{1}{2}}|s\eta_1\cdot s\eta_2\cdot s\eta_3\cdot s\eta_4|$$

gives $\mathcal{H}_{00}=2.25T\sigma-(72/15)A(\sigma/(\sigma+2))^{\frac{3}{2}}$, which has a minimum of -25.2 Mev at $\sigma=2.6$. The quantity $1/\sigma^{\frac{1}{2}}$ measures the separation of the particles within the alpha-particle.

In the plane $\kappa=0$, the energies (6) and (7) have shallow minima at $\sigma=2.4$. Each expression differs little in the two planes $\sigma=2.4$ and 2.6. If the value 2.6 is adopted for σ, ${}_pH_{00}$ and ${}_sH_{00}$ are evaluated quite close to their minima, and, in addition, ${}_p\psi_0$ or ${}_s\psi_0$ represents exactly an alpha-particle in its normal state and a p or s neutron, respectively.

Figure 1 shows the variation of (6) and (7) with κ for the fixed value $\sigma=2.6$. The S state curve has a very shallow minimum at about $\kappa=0.6$; the P state curve has no minimum at all.

The quantity $1/\kappa^{\frac{1}{2}}$ measures the separation between the alpha-particle and the neutron. The point $\kappa=0$ corresponds to the removal of the neutron to infinity. For any value of σ,

$$(H_{00})_{\kappa=0}-\mathcal{H}_{00}=0.15T\sigma.$$

This difference arises because H_{00} and $\mathcal{H}_{00}$ are referred to different centers of mass (c.m.). Analysis shows that, with respect to the c.m. of He^5, the alpha-particle has $0.03T\sigma$ translational energy, and the neutron at infinity has the remaining $0.12T\sigma$. This "spurious" kinetic energy accounts for the shift in the minimum of H_{00} for $\kappa=0$.

IV. THE COULOMB ENERGY

The Coulomb energy (C.E.) of the lowest states of He^5 is due to the single interaction of the two protons in the s shell; for both one- and two-parameter functions, the C.E. of He^5 is given approximately by previous calculations[7] with one parameter for He^4. Since, in first-order approximation, the excited proton of both Li^5 and Li^6 is in a $2p$ state, the Coulomb energies of these two nuclei are approximately equal. From previous work, however, only the C.E. of Li^6 for one-parameter functions is known.

The expressions for the Coulomb term in the first-order energy are

He^5: $E_c=0.51\sigma^{\frac{1}{2}}$ Mev (1 or 2 parameters)
Li^5: $E_c=1.43\sigma^{\frac{1}{2}}$ Mev (1 parameter only).

The inclusion of the C.E. raises the curves of Fig. 1 by $0.51(2.6)^{\frac{1}{2}}=0.8$ Mev. For Li^5 with two-parameter functions, in the region of physical

TABLE I. *List of excited configurations for the ${}^2P$ and ${}^2S$ states of* He^5.

P STATE				*S* STATE	
DOUBLY EXCITED		QUADRUPLY EXCITED		DOUBLY EXCITED	
CONFIGURATION	NO. OF FUNCTIONS	CONFIGURATION	NO. OF FUNCTIONS	CONFIGURATION	NO. OF FUNCTIONS
$(1s)^4 3p$	1	$(1s)^4 4p$	1	$(1s)^4 3s$	1
$(1s)^3 2p2s$	7	$(1s)^3 2p3s$	7	$(1s)^3 2p3p$	21
$(1s)^3 2p3d$	21	$(1s)^3 2p4d$	21	$(1s)^3(2s)^2$	3
$(1s)^2(2p)^3$	15	$(1s)^3 2s3p$	7	$(1s)^3(3d)^2$	17
	—	$(1s)^3 3d3p$	21	$(1s)^2(2p)^2 2s$	18
	44	$(1s)^3 3d4f$	35		—
		$(1s)^2(2p)^2 3p$	33		60
		$(1s)^2(2s)^2 2p$	6		
		$(1s)^2(3d)^2 2p$	30		
			—		
			161		

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[7] W. Heisenberg, Zeits. f. Physik **96**, 473 (1935).

interest ($\kappa<\sigma$), the C.E. will raise the He^5 energy surface less than $(1.43-0.51)(2.6)^{\frac{1}{2}}\leqq 1.5$ Mev. Since the lowest states of Li^5 and He^5 differ only in the Coulomb term and hence by about 1 Mev, the remainder of this paper will be devoted to He^5 alone. Higher order Coulomb corrections will be neglected.

V. Perturbation Calculations with One Parameter

Schrödinger perturbation theory gives the energy

$$E=H_{00}+\sum_i{}'\frac{|H_{0i}|^2}{E_0-E_i}=H_{00}+E^{(2)}.$$

The sum is to be taken over all excited states ψ_i. The denominator is the difference of unperturbed energies:

$$E_0-E_i=-nT\sigma,$$

where n is the degree of excitation of ψ_i relative to ψ_0.

Table I is a list of doubly and quadruply excited configurations for the ^2P state and a list of doubly excited configurations for the ^2S state; all configurations are included whose functions combine with ψ_0. The number of different functions subsumed by each configuration is also given.

Table II gives numerical results for both states. The Coulomb energy, the first-order energy, and the second-order energy (corresponding to the pertinent functions of Table I) are listed for significant values of σ.

It is desirable to find the convergence limit of the second-order energy:

$$E^{(2)}=\sum{}'\frac{|H_{0i}|^2}{E_0-E_i}=E_D{}^{(2)}+G_Q.$$

G_Q is the second-order energy due to all states higher than doubly excited; the smallest denominator in $-G_Q$ is $4T\sigma$. No kinetic energy terms appear in G_Q on account of selection rules. From matrix algebra

$$-G_Q<(1/4T\sigma)\{(V^2)_{00}-V_{00}{}^2-\Sigma_D V_{0i}{}^2\}.$$

On the right-hand side, the only unknown is $(V^2)_{00}$. The calculation of this quantity with the general potential (1) is straightforward though lengthy; the only new feature is the necessity of investigating all double spin products $Q_{ij}Q_{jk}$ and $Q_{ij}Q_{kl}$.

Values of G_Q and of the sum $H_{00}+E_C+E_D{}^{(2)}+G_Q$ are listed in Table II for both states. Since the second-order energy $E_Q{}^{(2)}$ due to quadruply excited states has been explicitly computed for the P state, better bounds G_S may be calculated for this state:

Table II. *Second-order perturbational calculations for the ^2P and ^2S states of* He^5. *Summary of numerical results.*

	^2P State					^2S State			
σ	1.0	1.2	1.4	1.6	1.8	0.8	1.0	1.2	1.4
H_{00}	−10.0	−11.4	−12.1	−12.3	−12.0	−5.7	−6.7	−7.1	−6.9
E_c	+0.5	+0.6	+0.6	+0.6	+0.7	+0.5	+0.5	+0.6	+0.6
$E_D{}^{(2)}$	−4.0	−3.9	−3.8	−3.8	−3.8	−6.2	−6.9	−7.4	
G_Q	−6.0	−4.6	−3.9			−9.7	−8.6	−7.6	
Total	−19.5	−19.3	−19.2			−21.1	−21.7	−21.5	
$E_Q{}^{(2)}$	−3.0	−2.7	−2.4	−2.1	−1.9				
G_S	−2.0	−1.3	−1.0						
$H_{00}+\cdots+G_S$	−18.5	−18.7	−18.7						

$$-G_S<(1/6T\sigma)\{(V^2)_{00}-V_{00}{}^2-\Sigma_D V_{0i}{}^2-\Sigma_Q V_{0i}{}^2\}.$$

Values of G_S and of the sum $H_{00}+E_C+E_D{}^{(2)}+E_Q{}^{(2)}+G_S$ are given in Table II for the P state.

VI. Further Variational Calculations with Two Parameters

In an effort to improve the first-order energy (6) for the P state, the function

$$\psi={}_p\psi_0+\mu V_p\psi_0 \tag{8}$$

has been used. Here ${}_p\psi_0$ is the P state function (2) in two parameters, μ is another parameter, and $V=\sum_{i>j}J_{ij}$. The function (8) iterates the ordinary potential. From test work on H^3, it appears that such iteration may be almost as effective as iteration of the total Hamiltonian.

With (8), the variational energy is

$$E=\frac{H_{00}+2\mu(HV)_{00}+\mu^2(HV^2)_{00}}{1+2\mu V_{00}+\mu^2(V^2)_{00}}. \tag{9}$$

The parameter μ is to be eliminated by setting $\partial E/\partial\mu=0$ and solving for μ.[8] The calculation of the denominator of (9) is simple. In connection with the numerator, it is to be noted that the

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[8] Since ${}_p\psi_0$ is complex, in general μ will also be complex. In the present work, however, μ is real.

$\nabla_i \cdot \nabla_j$ terms cannot be evaluated by the scheme[2] used for one-parameter work. The fundamental relation between coordinate and momentum matrices, upon which the scheme depends, is not valid for a set of functions in which some functions have one parameter and the rest have a different parameter. Hence, the $\nabla_i \cdot \nabla_j$ integrals must here be computed in a straightforward fashion. On this account, the numerator of (9), especially $(HV^2)_{00}$, involves considerable integration.

For $\sigma = 2.6$, and with μ given its minimizing value, (9) yields energies almost uniformly lower than (6). The lowering is slight: between 0.3 and 0.4 Mev for $0 < \kappa < 2$.

It is worthy of mention that for $\kappa = 0$ (alpha-particle referred to c.m. of He^5), ${}_pH_{00}$ is lowered 0.37 Mev. A four-particle function similar to (8), where ${}_p\psi_0$ is replaced by ϕ_0, lowers $\mathcal{H}_{00}$ exactly the same amount. Thus, the difference in the kinetic energy terms due to change of center of mass has no effect upon the more refined calculation of this section; the lowering given by (9) at $\kappa = 0$ is a true lowering of the alpha-particle energy. It is interesting that iteration of the potential is not nearly so effective as a linear variation function scheme.[9]

VII. Discussion

The order and the spacing of the S and P states will be discussed first. The Hartree model gives about 5 Mev. for the difference ${}^2S - {}^2P$. Table II shows that the second-order calculations diminish the interval so radically that, with the inclusion of the bounds G_Q, the S state apparently may lie below the P state. Examination of Table II shows, however, that as second-order energies from functions of higher excitation are gradually included, the minimizing value of σ becomes progressively smaller. At the smaller values of σ, convergence is slower, and the bounds G are larger. This is true for both states. In addition, the convergence of the second-order energies is slower for the S than for the P state; the bounds G_Q are poorer for the former than for the latter. In view of these facts, it may be expected that further second-order approximation would locate the S state slightly above the P state.

For the S state, some matrix elements H_{0i} involve only s functions. Such integrals are large, because, for oscillator functions, the strongest combination occurs between functions of the same symmetry. On the other hand, for the P state, most H_{0i} contain 1s and various p functions, and these integrals are small because of the difference in the orbital (angular momentum) part. For He^5, the Hartree model appears to give correctly the *order* of the low-lying levels but it does not give the *spacing* between levels accurately, *because second-order effects depend critically upon the orbital symmetry of the level concerned.* There are other instances of this symmetry effect. For Li^6, Margenau and Carroll[2] found that the interval ${}^3D - {}^3S$, which is 2 Mev according to the Hartree model, becomes 6 Mev upon higher approximation. Margenau[10] observes no appreciable change in the interval ${}^1S$ (He^6) $-$ ${}^3S$ (Li^6) upon the inclusion of second-order terms, because both levels possess the same orbital symmetry.

The two-parameter variational calculations of Sections III and VI constitute a partial check upon the reliability of the perturbational work with one parameter. A 2p or 2s function with a parameter κ is equivalent to an expansion of p or s functions, respectively, with the parameter σ. For $\sigma = 2.6$ and $0 < \kappa < 2$, calculation shows that several coefficients in each expansion are large. The curves of Fig. 1 show, therefore, the variational energy corresponding to several important functions. Further variational refinement may change the shape of the curves, especially the curve for the P state, but marked lowering is not to be expected.

It is to be emphasized that the general potential (1) has been used throughout. In regard to more extended variational calculations, one may anticipate that, since there is a considerable correlation[2, 10] between variational and second-order perturbational results, many one-parameter functions would be needed for a close approximation to the convergence limit. Likewise, in an iteration scheme, many powers of V or H would be required.[11]

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[9] H. Margenau and W. A. Tyrrell, Jr., Phys. Rev. **54**, 422 (1938).

[10] To be published soon.

[11] G. Horvay, Phys. Rev. **55**, 70 (1939).

As to the results of experimental investigations involving He^5, Williams, Shepherd and Haxby,[12] from a study of the reaction Li^7 (d, α), concluded that He^5 is unstable by 0.84 Mev against dissociation into an alpha-particle and a neutron. The work of Rumbaugh, Roberts and Hafstad[13] seems in agreement with this. Staub and Stephens,[14] from experiments on the scatterin of neutrons by alpha-particles, support this energy value and conclude that this unstable state of He^5 has a ^2P symmetry. In marked contrast to these harmonious results, Joliot and Zlotowski[15] claim that He^5 has a stable state, located 2.2 Mev below dissociation. Bethe[16] has indicated reasons for rejection of the Joliot result. It appears, then, that the lowest energy of He^5 is $-27.6+0.8=-26.8$ Mev.

In the scattering of neutrons by He^4, only one resonance was observed.[14] The present calculations suggest that both s and p waves of the incident neutron participate in this resonance. Staub and Stephens have concluded that the scattering of p waves must predominate in order to account for the height of the resonance peak. The present suggestion in no way vitiates their analysis, for, from the figures they give, a considerable amount of s scattering would give only a small contribution to the total resonance.

VIII. CONCLUSIONS*

From a study of Table II and Fig. 1, it is concluded that the ^2P and ^2S levels of He^5 lie close together, with the ^2P probably lower. With a symmetric Hamiltonian, the value -20 Mev may be set as a probable lower bound to the position of these states. From the work of Carroll[17] on Li^7, it is inferred that the set of parameters recently proposed by Breit[3] will give levels for He^5 which are even higher. The estimate given above falls some 7 Mev short of the experimental value. The discrepancy between theory and experiment has the same direction and about the same magnitude as for Li^6 and Li^7. The general conclusions which may be drawn from the present results have already been given.[18]

The writer is indebted to Professor Henry Margenau for suggesting this problem and for advice and supervision throughout the course of the work.

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[12] J. H. Williams, W. G. Shepherd and R. O. Haxby, Phys. Rev. **51**, 888 (1937); **52**, 390 (1937). Cf. also M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 318–319 (1937).

[13] L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **54**, 657 (1938).

[14] H. Staub and W. E. Stephens, Phys. Rev. **55**, 131 (1939).

[15] F. Joliot and I. Zlotowski, Comptes rendus **206**, 1256 (1938).

[16] H. A. Bethe, Phys. Rev. **55**, 434 (1939).

* *Note added in proof:*—Since this manuscript was submitted for publication, there has appeared an article by S. Watenabe, Zeits. f. Physik **112**, 159 (1939), in which the nuclear five-body problem is also treated. Watenabe's analysis differs in some detail from the present work; his general results, however, are in substantial agreement with the author's conclusions.

[17] K. G. Carroll, Phys. Rev. **55**, 1128(A) (1939).

[18] W. A. Tyrrell, Jr., K. G. Carroll and H. Margenau, Phys. Rev. **55**, 790 (1939).