Ridges in a Liquid Surface Due to the Temperature Dependence of Surface Tension

The occurrence of tears in strong wine was explained many years ago by James Thomson.[1] It arises from evaporation, which depletes the alcohol in the wine at the rim of a glass. The result is a greater surface tension at the rim than at the center, and the wine is continuously drawn up onto the side of the glass, whence it returns from time to time in the form of tears. As recently as 1931 the process in various other liquids was studied by Loewenthal,[2] who concluded that it can occur only in mixtures. An analogous phenomenon can occur in pure liquids, however, if temperature gradients are allowed to exist.

It is a matter of common observation that a layer of liquid often has a ridge at each border when it is allowed to drain from a flat vertical surface. That this phenomenon is an illustration of the effect of temperature upon surface tension has been shown in this laboratory by experiments with a layer of liquid on a microscope slide. The effect was found to occur in a volatile liquid such as water or acetone, but not in octoil-*s*, a substance with a very low vapor pressure. It did not occur even in a volatile liquid, if the surrounding atmosphere was saturated with vapor, but a ridge appeared immediately on exposure to the air of the laboratory. Reinsertion into the vapor caused the ridge at each border to slide over the surface toward the center until the layer was divided into three strips of equal width. Thus the greater evaporation or condensation at the border set up a local temperature gradient, and the corresponding inequality of surface tension created a motion in the liquid.

As a direct test of the effect of temperature a slide with a layer of liquid was held in a horizontal position, and the back was touched with a glass rod which had been cooled in liquid air. The liquid was drawn up into a mound over the point of contact in all of the substances investigated regardless of volatility. In the case of water, circulation in the mound was made visible by a dilute suspension of bentonite clay. At the free surface the water flowed toward the center of the mound, while underneath it flowed away. The slide was turned into a vertical position and was supported at the bottom by a pair of greasy forceps. A well-defined monolayer spread up over the surface. The mound collapsed and disappeared when the monolayer covered it.

The contour of the free surface of a layer on a horizontal slide under conditions of steady flow may be found by an analysis of the motion of the liquid. The layer may be assumed to be thin, and the thickness h may be assumed to vary slowly with the surface coordinates. Inequality in the surface tension γ is then accompanied by a tangential stress in which the force per unit area is equal to the gradient of γ. If the surface is clean, the liquid flows in the direction of increasing γ with a local velocity which is a linear function of the distance z above the fixed plane. The liquid experiences also a force which arises from the gravitational field and from inequality in the pressure. The force is directed parallel to the surface, and the liquid flows with a velocity which is a quadratic function of z.

The resultant velocity $\mathbf{v}$ is expressed by the equation

$$\eta\mathbf{v} = z\nabla\gamma + (\tfrac{1}{2}z^2 - hz)\rho g\nabla h,$$

in which η is the viscosity and ρ the density. $\nabla\gamma$ and ∇h are the vector gradients of γ and h. The current flux in the layer is expressed by $\int_0^h vdz$. At the steady state the flux vanishes, and h is related to γ by the differential equation

$$\tfrac{1}{2}h^2\nabla\gamma - \tfrac{1}{3}h^3\rho g\nabla h = 0.$$

The integral of the differential equation,

$$3\gamma - \rho g h^2 = c,$$

is the equation of the free surface. Thus, for example, a 1.0°C temperature difference between two points in a layer of water at 20° creates a 0.15 dyne per cm difference in surface tension, and if the thickness were 0.2 mm the difference in elevation would be 0.09 mm.

If the layer is held in a vertical position the liquid is drawn into the cooler regions and then continues downward under a ridge. When an insoluble monolayer spreads over the surface it takes up the tangential stress, and the liquid is allowed to flatten out.

The writer takes pleasure in thanking Dr. Irving Langmuir for calling this phenomenon to his attention, and for making several constructive suggestions during the progress of its investigation.

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[1] Cf. the article "Capillary Action" in the ninth edition of the *Encyclopedia Britannica*.
[2] M. Loewenthal, Phil. Mag. **12**, 462 (1931).

Klein's Fifth Dimension as Spin Angle

Although originally "classical," Klein's five-dimensional theory has been translated into quantum matrix form by Flint,[1] and its relations with Dirac theory well clarified. One attractive feature of the five-dimensional point of view appears to have been overlooked: by choosing the arbitrary scale factor so that the wave function is 4π periodic in the fifth dimension it follows that (a) this fifth dimension can be interpreted as spin angle with the electron spinning round once between each de Broglie node; and (b) the fifth component of the charge and current vector turns out to be exactly one Bohr magneton associated with the spinning electron.

This point of view removes the objection that the fifth dimension is a purely *ad hoc* hypothesis without physical meaning. It further illuminates the frequently ignored fact that in order to follow classical laws under classical conditions the electron must forget that it has spin when it gets free from nuclear fields. By treating the spin as a fifth dimension the Hamiltonian for continuous energy states is found to be independent of the spin, which contributes at most a constant mechanical energy (mass) independent of magnetic fields.[2]

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[1] H. T. Flint, Proc. Roy. Soc. **A131**, 172 (1931); **A159**, 45 (1937).
[2] W. Band, Communicated Phil. Mag. paper giving details.