On Explosion Showers

Let us consider a high energy collision between two particles of initial impulses $|p_1| \cong |p_2| \gg p_0 = hc/l$ (where p_1, p_2 are referred to a barycentric frame: $p_1 + p_2 = 0$, and l is a universal length $\sim 10^{-13}$ cm).

Heisenberg,[1] starting from a half-classical theory of explosion showers, showed that the number of particles created by collision must be sensibly proportional to the initial energy $E = 2|p_1|c \sim nhc/l$.

Our purpose is to examine whether it is permissible to apply the second quantization method and to assume the existence of a suitable relativistic interaction-operator in order to obtain some additional information about this kind of shower. We arrive at the same description of showers obtained by Heisenberg, and this seems to support the consistency of our assumption. Moreover our method leads to the result that the cross section for explosion showers decreases for $n > 1$ and tends to 0 when $E \rightarrow \infty$.

In analogy to Heisenberg's Lagrangian we shall consider here, as a plausible and typical example, an interaction-operator $G(\xi)$ which is a function of

$$\xi = l^2 \Box.$$

We suppose also that this operator corresponds to some kind of short range forces, e.g. we consider $G(\xi)$ of the type $\exp[-|\xi|]$ or $(\xi+1)^{-1}$ (for Yukawa forces), or other types used for "cutting off" purposes.

The general term of interaction, which in the formalism of second quantization, corresponds to an explosion shower, is:

$$b_{k_1}{}^* b_{k_2}{}^* \cdots b_{k_n}{}^* a_{p_1} a_{p_2} \int_\Omega \varphi_{k_1}{}^*(x,t) \cdots \varphi_{k_n}{}^* G(\xi) \psi_{p_1} \psi_{p_2}(x,t) dx,$$

where $\varphi_{k_s} \psi_{p_s}$ represent plane waves with impulses $k_s p_s$ and $b_{k_s}{}^* a_{p_s}$ are the usual operators representing the creation and the annihilation of particles. $\varphi_{k_s} \psi_{p_s}$ are normalized in a volume Ω, and thus assume the form:

$$\Omega^{-\frac{1}{2}} u_p \exp (i/\hbar)(px - Et),$$

where u_p are only spin dependent factors. We have:[2]

$$G(\xi)\psi_{p_1}\psi_{p_2} = G\{-p_0^{-2}[(p_1+p_2)^2 - c^{-2}(E_1+E_2)^2]\} \times u_{p_1} u_{p_2} \exp (i/\hbar)[(p_1+p_2)x - (E_1+E_2)t].$$

Applying the usual perturbation method in order to obtain the Fourier coefficients of the function χ which represents a state:

$$\chi(N_{k_1} N_{k_2} \cdots N_{p_1} \cdots t) = \Sigma c(N_{k_1} \cdots N_{p_1} \cdots t) \exp [-(i/\hbar)(N_{k_1} E_1 + \cdots)t]$$

(with obvious signification of the symbols) we obtain:

$$|c(1_{k_1} \cdots 1_{k_n} 0_{p_1} 0_{p_2} t)|^2 = t \frac{2\pi}{\hbar} \frac{1}{\Delta E} \left| \int_\Omega \varphi_{k_1}{}^* \cdots \varphi_{k_n}{}^* G \psi_{p_1} \psi_{p_2} dx \right|^2_{(t=0)},$$

where

$$\frac{1}{\Delta E} = \left(\frac{\delta \pi \Omega}{h^3}\right)^n \left(\frac{k_1^2}{v_1}\right) \cdots \left(\frac{k_n^2}{v_n}\right)$$

is the number of quantum states referred to unit energy interval. In a barycentric frame we have $p_1 + p_2 = 0 = k_1 + k_2 + \cdots + k_n$. An easy calculation shows that: $|c|^2$ results independent of Ω and proportional to:

$$k_1^2 k_2^2 \cdots k_n^2 G\left[\left(\frac{E_{p_1} + E_{p_2}}{c p_0}\right)^2\right].$$

In order to study how this probability varies with n and with the spatial distribution of vectors k_s, we make use of the conservation law:

$$E_{p_1} + E_{p_2} = E_{k_1} + E_{k_2} + \cdots + E_{k_n}$$

and find that this probability is the maximum in the case:[3] $|k_1| \sim |k_2| \sim \cdots \sim |k_n| \sim p_0$.

Thus n is proportional to the initial energy and the frequency of explosion showers results reduced by the factor G for incident energies greater than cp_0.

GLEB WATAGHIN

Department of Physics,
Sao Paulo University,
Brazil,
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[1] W. Heisenberg, Zeits. f. Physik **110**, 251 (1938); **113**, 61 (1939).
[2] Following a remark of Mario Schönberg, the operators of the type $\exp (\Delta/K_0^2)$ applied to a function give in the corresponding Fourier-development a cut-off factor.
[3] G. Wataghin, Nature **142**, 393 (1938); Comptes rendus, August (1938).

Ferromagnetic Anisotropy of Low Nickel Alloys of Iron

Although the ferromagnetic anisotropy constant K_1 has been determined accurately for iron-nickel alloys containing 50 percent or more of nickel, nothing at all is known about the low nickel alloys. The marked discontinuity in magnetic properties of 30 percent nickel makes it impossible to interpolate between iron and the high nickel alloys. Until the time when suitable low nickel single crystals of iron are grown, the only way of obtaining any idea as to the behavior of the anisotropy constant K_1 in the low nickel range is to make use of cold-rolled material. This is possible because the texture of cold-rolled iron-nickel alloys has been carefully investigated by McLachlan and Davey,[1] using standard x-ray diffraction technique. They found that for a given total percentage of cold reduction in thickness, the texture was independent of the composition up to at least 20 percent nickel.

The magnetic torque curve of cold-rolled iron is similar to that of an iron single crystal disk having (100) in its plane.[2] The exact shape and the amplitude of the polycrystalline torque curve depend upon the nature of the deviations from the ideal (100) orientation and upon the fraction of material having a random texture. It is convenient to speak of a texture factor, which is defined as the ratio of the amplitude of the predominant 4θ term in the torque curve of the cold-rolled material to the corresponding amplitude for a single crystal of the same composition and volume For all practical purposes, the texture factor can be taken as the ratio of the average of the torque peaks of the polycrystalline material to that of the single crystal. In view of the work of McLachlan and Davey, it can be assumed that if iron and iron-nickel alloys are cold-rolled in the same manner, their texture factor will be the same, at least as a first approximation. The average values of the torque peaks will then be directly proportional to the corresponding values of K_1.