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Dependence of Ferromagnetic Anisotropy on the Field Strength

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In evaluating the anisotropy constants of ferromagnetic materials a common method is to measure the torque exerted on a specimen of circular section by a uniform magnetic field, to determine the maximum torque by extrapolation to infinite field strength, and to multiply this by an appropriate number. The law of approach of the torque, T_H, to its limiting value, T_∞, is now found to be

$$T_H = T_\infty(1 - H_0/H_a)$$

for a number of disks and ellipsoids of single crystals of iron-silicon alloys containing up to 7.4 percent silicon, and for polycrystalline disks of iron and of silicon steel. Here H_a is the strength of the applied field uncorrected for demagnetizing field and H_0 is a constant which depends mostly upon the dimension-ratio of the disk. This law is discussed in relation to a similar law proposed by Schlechtweg, and to the familiar law of approach of magnetization to saturation. It is concluded that for disks the approach to saturation of the torque peaks is connected with the lack of magnetic saturation at their edges.

It is a well-established fact that ferromagnetic single crystals generally show considerable anisotropy in their magnetic properties. The simple theory of ferromagnetic anisotropy and the several methods of measuring it have recently been reviewed by Bozorth.[1] The simplest of these methods involves the use of a magnetic torquemeter to measure the torque exerted by a strong field on a suitably held single crystal disk, the torque arising from a lack of parallelism between the direction of the magnetization intensity in the disk and the direction of the applied field. This deviation between I and H occurs because of the anisotropy of the material. If the orientation of the disk with respect to the crystal axes is known, the anisotropy constants can easily be calculated from the torque data.[2–4] It is usually preferable to have some simple crystallographic plane lying in the plane of the disk because the resulting torque curve must then be symmetrical, unless the crystal is imperfect; in this case, the degree of symmetry throws some light on the trustworthiness of the calculated anisotropy constants.

In order to calculate the anisotropy constants from the torque data, it is necessary to use an applied field strong enough to make the torque independent of it. It has generally been thought that a field of several thousand oersteds is sufficient for a disk with not too large a ratio of thickness to diameter and some evidence to this effect was found[2] when a moderately accurate torquemeter was used. On the other hand, some later work by Schlechtweg[3] indicated that the torque did not reach a saturation value in fields of the same order of magnitude. Unfortunately, his highest field strength was only 2400 oersteds, and under the conditions of the experiment this was too low to prove conclusively that the torque would not have reached a limiting value at a somewhat higher field. By using a torquemeter of very high sensitivity, the author has been able to show in the present paper that a saturation value of the torque cannot be obtained experimentally, but can be calculated by extrapolating to $1/H=0$.

Apparatus

The torquemeter used in this work was of the familiar torsion type, but built on a larger scale than usual. The electromagnet was about the same size as the one illustrated elsewhere,[4] mounted so as to rotate around a vertical axis. With the flat faces of the pole pieces $1\frac{3}{4}$ inches apart, a field of 4000 oersteds could be obtained, but measurements ordinarily could not be made above 3500 oersteds because of overheating. The field was calibrated in terms of the magnetizing current in the usual manner with the aid of a search coil and a ballistic galvanometer. The fields could be measured with an accuracy of one-tenth of a percent relative to each other, but the uncertainty involved in reading the ammeter

[1] R. M. Bozorth, J. App. Phys. **8**, 575 (1937).
[2] R. M. Bozorth, Phys. Rev. **50**, 1076 (1936).
[3] H. Schlechtweg, Ann. d. Physik [5] **27**, 573 (1936).
[4] L. P. Tarasov and F. Bitter, Phys. Rev. **52**, 353 (1937).

during the actual use of the torquemeter brought the possible error in H to about one-half percent. The field was uniform to at least this extent over the area of the single crystal disks used in this investigation, which were mostly half an inch in diameter; however, the field probably varied somewhat more over the area of the one-inch polycrystalline disks which were also used. The uncertainty in the field was in all cases too small to affect the results appreciably. The highest fields used were too low to require any correction for the image effect described by Weiss and Forrer.[5]

The specimen was held by the pressure of a thumbscrew in an aluminum holder which was suspended between the pole pieces. Phosphor bronze wires 0.032 inch in diameter were used for the suspension. The upper wire, which was fastened securely to the frame of the torquemeter, was 10 inches long; the lower wire, held at the other end in a wire chuck attached to an accurately machined gear, was 24 inches long. This gear was meshed with a worm gear, a complete revolution of which corresponded to a 12-degree twist of the lower end of the suspension. On the same shaft with the worm gear was a large dial graduated into 100 divisions. The mechanical torque required to balance the magnetic torque exerted by the field on the specimen could be reproducibly measured to a tenth of a division, and since the torque peaks of most of the specimens examined amounted to one or two complete revolutions of the dial, an accuracy of one part in a thousand or better could be reached. A galvanometer mirror was mounted on the upper suspension wire just above the aluminum disk-holder, and the image of a cross-hair thrown on a scale clamped to the torquemeter frame showed accurately when the specimen was in its zero position, i.e., the position in which the magnetic and mechanical torques were properly balanced. To convert the arbitrary dial readings to absolute units it was found best to calibrate the suspension by means of a specimen whose torque peaks had been measured on a balance-type torquemeter, which gave the results in absolute units directly.[6] It was thus found that one dial division corresponded to 65.4 dyne-cm. In making the measurements, it was necessary to introduce a slight correction for the torque exerted by the aluminum disk-holder which probably arose from ferromagnetic impurities. Because the precision was so high it was also necessary to correct for any temperature change during a set of readings. This correction was found experimentally to be one part in 700 for 1°C, a value very close to the known temperature variation of the anisotropy constant of iron at 25°C. The temperature was read on a thermometer suspended from the frame of the torquemeter in such a way that the bulb was only a millimeter or two above the specimen and slightly to one side of it.

Measurements on Single Crystal Disks

The first set of measurements was made on a group of fourteen well-annealed single crystal disks of iron containing from 2.4 to 7.4 percent of silicon by weight,[7] as found from subsequent chemical analyses on the disks. Eleven of these had the (100) plane in the plane of the disk, the others had (110). Back-reflection Laue patterns of the disks proved that the deviations from these ideal orientations were a few degrees at most, not enough to affect the validity of the results to be described. After the measurements on these disks were complete, some of them were

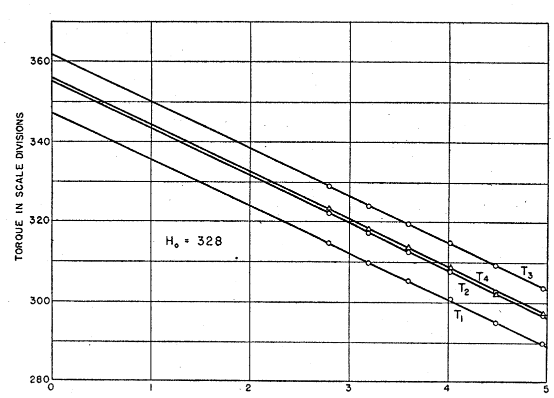

FIG. 1. Variation with $1/H_a$ of the magnetic torque peaks of a (100) disk. The successive peaks are numbered in order. H_a is the applied field.

[5] P. Weiss and R. Forrer, Ann. de physique [10] **12**, 297 (1929).

[6] This was done through the kindness of Dr. A. R. Kaufmann at the Massachusetts Institute of Technology, who used the torquemeter described in reference 4.

[7] These disks, to be described in greater detail in a subsequent paper on the anisotropy of iron-silicon alloys, were prepared several years ago by Dr. K. J. Sixtus, at that time of the General Electric Research Laboratory.

polished and etched deeply to reduce the thickness and were remeasured. There were thus available data on disks ranging from 0.37 to 0.75 inch in diameter, and from 0.008 to 0.035 inch in thickness. The ratio t/D of thickness to diameter varied from 0.015 to 0.090.

For the present the discussion will be restricted to the (100) disks, whose four torque peaks are of equal magnitude. In Fig. 1 the torque peaks found for one of these disks are plotted individually as a function of $1/H_a$, where H_a is the applied field. This was varied between 2000 and 3500 oersteds. It is clear that at least in this range the magnitude of the torque peak changes linearly with $1/H_a$ to a high degree of accuracy. The results can be expressed by the equation

$$T_H = T_\infty(1 - H_0/H_a), \tag{1}$$

where T_H and T_∞ are the magnitudes of the torque peaks for an applied field H_a and for an infinite one, respectively, and H_0 is a constant. This linear relationship was found to hold only above 2000 oersteds or thereabouts, the curve breaking suddenly downward for lower fields. Above 2000 oersteds the law held all the way to $1/H_a = 0$, as was demonstrated by measurements on an oblate ellipsoid of revolution to be described later. For disks or ellipsoids cut parallel to the (100) plane, the anisotropy constant is given by $K_1 = 2T_\infty$.

At this point, it is desirable to mention the effect of rotational hysteresis upon the results. It is known that the two torque curves obtained by rotating the field in opposite directions are separated vertically by an amount proportional to the rotational hysteresis. The data plotted in Fig. 1 were obtained by always rotating the field in the same direction through all four peaks. Hence, two of the peaks were too small by an amount equal to the torque arising from rotational hysteresis, while the other two were large by the same amount. Since rotational hysteresis tends toward zero as magnetic saturation is approached, the error becomes less in higher fields with the result that the slope of two of the lines, such as are drawn in Fig. 1, may differ slightly from the slope of the other two lines. In some cases, as in Fig. 1, the error arising from rotational hysteresis was so small that all four lines were parallel within experimental

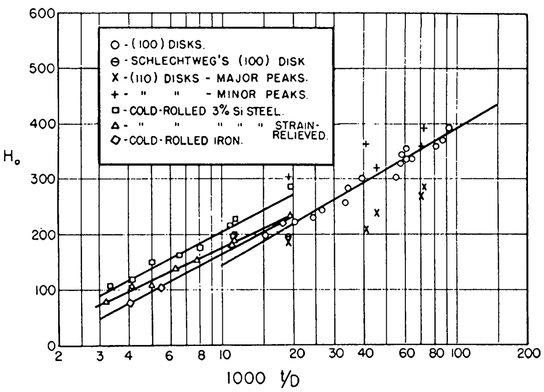

FIG. 2. H_0 as function of t/D, the ratio of the thickness of a disk to its diameter. The straight line on the right was calculated by least squares using only the circles. The three straight lines at the left, drawn in directly, connect the three sets of data for cold-rolled and strain-relieved materials.

error; in other cases, especially for polycrystalline disks, the error was large enough to give a perceptible difference in slopes. Were it not for imperfections in the specimen, such as a slight deviation between the (100) plane and the plane of the disk, the four lines of Fig. 1 would be coincident. Even where rotational hysteresis was comparatively large, it was far too small to account for the observed differences in the sizes of the supposedly equal torque peaks. To eliminate the effect of rotational hysteresis, the results were always plotted to give a single line showing the variation of the sum of the four torque peaks with $1/H_a$, for in this case the errors automatically canceled.

It was noticed that H_0 increased with the ratio of the thickness of a disk to its diameter. Several ways of plotting results were tried, but the only one that gave a reasonably linear variation was the one in which H_0 was plotted against log (t/D), where t and D are the thickness and diameter. The results for all the (100) disks are indicated by circles in Fig. 2. A dashed circle shows the result calculated from similar data obtained by Schlechtweg[3] on a (100) disk. This point is in reasonably good agreement with the others.

It is obvious that there is considerable scatter in H_0, when only the points for the (100) disks are considered. An examination of the original data showed that this scatter could not be accounted for on the basis of variations in the silicon content of the disks which, as has already been mentioned, contained from 2.4 to 7.4 percent, nor because of slight deviations from the ideal orientation. The only explanation of the scatter is that it arises from certain peculiarities of the specimens whose nature is unknown at present, for the amount of scatter is larger than can be ascribed to errors in the measurements. The straight line drawn through these points was calculated by least squares, and its equation is

$$H_0 = 637 + 246 \log_{10} (t/D), \tag{2}$$

or in a slightly different form,

$$H_0 = 246 \log_{10} (385t/D). \tag{2a}$$

It is obvious that this equation is valid only between certain limits of t/D; moreover, it applies only to the torque peaks of (100) single crystal disks. The line corresponding to this equation in Fig. 2 is drawn somewhat beyond the experimental points to indicate the probable range of validity.

The discrepancy between the experimental values of H_0 and those corresponding to the line was generally less than 5 percent. If it is assumed that H_0 can be calculated for a (100) single-crystal disk from its dimensions with an error not exceeding 5 percent, then the torque peaks can be extrapolated to $1/H_a = 0$ according to Eq. (1) with an error of less than 0.5 percent, using the reasonable values of $H_a = 3000$ and $H_0 = 300$. Thus this equation should be useful when experimental circumstances make impossible the accurate determination of H_0.

We now turn to the (110) disks, which have two major and two minor peaks instead of four of the same size. Similar measurements on the (110) disks showed that H_0 for the minor peaks is considerably larger than for the major ones. The two sets of points are plotted in Fig. 1 as crosses. The scatter is considerable because few disks were available and also because they were smaller than the (100) disks so that the variation of the peaks with $1/H_a$ was harder to determine accurately. There can be no doubt, however, that H_0 for the (100) disks is smaller than for the minor peaks of the (110) disks and larger than for their major peaks. At the same time, it appears that H_0 for both sets of (110) peaks does not increase as rapidly with t/D as does H_0 for the (100) peaks. More data on (110) disks are needed before it will be possible to calculate H_0 from the dimensions with any reasonable accuracy.

Measurements on Polycrystalline Disks

Some 3 percent silicon steel was cold rolled from 100 mils down to various thicknesses ranging from 20 to 3 mils. Disks one inch in diameter were punched from this material, which is described in detail elsewhere,[8] and were measured both in the cold-rolled condition and after being annealed just below the recrystallization temperature to relieve the strains as much as possible without changing the texture. The predominant texture of severely cold-rolled silicon steel is one having the (100) planes in the rolling plane or deviating by various amounts from this ideal orientation. The torque curve of this material is similar to that of a (100) single crystal disk in that the four torque peaks are of approximately the same size; furthermore, the torque curve remains practically the same after the disks have been annealed for strain relief. Some work was also done on three disks of fairly pure iron, all of which were cold rolled to a 95 percent reduction in thickness.

The three sets of results are shown, distinctively marked in Fig. 2 on the left. Although the points are very scattered, it is possible to draw three straight lines, one through each set. The same disks were used before and after the strain relief so that the decrease in H_0 following the anneal is known to be a real effect. The values of H_0 for the three iron disks are even smaller, but not enough specimens were available to make sure that this was not an accidental variation. Although the lines drawn through the three sets of points have roughly the same slope as the line for the (100) single-crystal disks, the lines themselves are all somewhat higher. One reason for this is the difference between the strain conditions in the two cases. It may also be that instead of drawing a straight line for the single-crystal disks and another one for the strain-relieved disks, there should be drawn a slightly curved line passing through both sets of points.

[8] L. P. Tarasov, Trans. A. I. M. E. **135**, 353 (1939). See Table I.

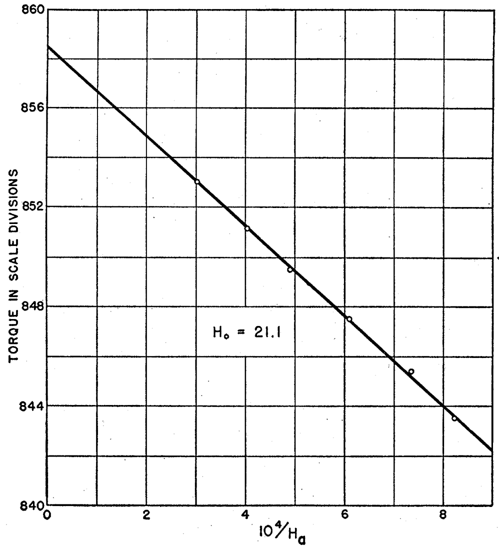

FIG. 3. Sum of four torque peaks of the (100) ellipsoid as function of $1/H_a$. This ellipsoid was filed from the (100) disk referred to in Fig. 1. The straight line is drawn to have a width of 0.1 division, the limit of accuracy in the measurements.

The considerable scatter that occurs in H_0 for polycrystalline material shows that it is best not to depend too much upon the values shown in Fig. 2.

Measurements on Single Crystal Ellipsoids

After the measurements were finished on the (100) single-crystal disk for which the results are shown in Fig. 1, this disk was filed into a highly accurate oblate ellipsoid of revolution.[9] The torque peaks of this ellipsoid, which contained 2.4 percent silicon, were measured with the highest possible precision after it had been etched and annealed to remove the distorted surface.

The results, shown in Fig. 3, indicate that an ellipsoid follows the same type of law as do the disks. However, H_0 is only 21.1 as compared with 328 for the same single crystal when it was still a disk. The accuracy with which the $1/H_a$ law is followed can be seen from the thickness of the line which corresponds to 0.1 division, the limit of accuracy of a single reading. The linear relationship extends to considerably lower fields than for disks, entirely on account of the relative ease of "saturating" an ellipsoid and a disk. The deviations from linearity are so negligible that the relatively short extrapolation to $1/H_a=0$ needs no further justification.

The work on the ellipsoid made it possible to check the validity of the linear extrapolation to $1/H_a=0$ in the case of the (100) disks. The effective anisotropy constants were calculated ($K_1=2T_H$) for various fields for both the disk and the ellipsoid into which it was made, and the values were plotted as in Fig. 4. The anisotropy constant extrapolated to $1/H_a=0$ was the same within experimental error in the two cases. This establishes the validity of Eq. (1) for the (100) single crystal disks, and it is reasonable to assume that the same is true of polycrystalline disks.

A (110) disk containing 5.9 percent silicon was similarly filed into an ellipsoid and it was found that H_0 for the major peaks was very close to 20. The variation of the minor peaks with $1/H_a$, which was hard to determine with any certainty, gave H_0 equal to about 15. The validity of the linear extrapolation to $1/H_a=0$ for (110) disks was also checked and Eq. (1) was again found to be satisfactory in the extrapolated range.

Torque Behavior in Lower Fields

It has already been mentioned that the linear relation given by Eq. (1) was followed only for

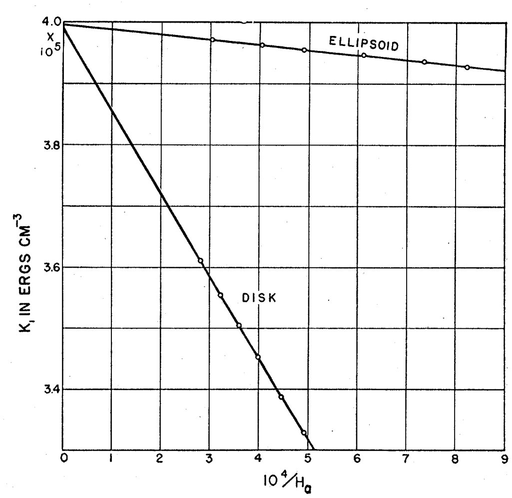

FIG. 4. Apparent anisotropy constants calculated for various fields for a (100) single crystal disk and for the ellipsoid into which it was filed. The true anisotropy constant is the value of K_1 for $1/H_a=0$.

[9] The method is essentially the one described by L. W. McKeehan, Rev. Sci. Inst. 5, 265 (1934). This and another ellipsoid were prepared at M. I. T. through the kind cooperation of Dr. A. R. Kaufmann.

fields above a certain minimum value, which in the case of disks generally was in the neighborhood of 2000 oersteds. Disks with a high ratio of thickness to diameter required a stronger field for the $1/H_a$ law to hold than did disks with a low ratio. This behavior is clearly connected with the relative ease of approaching magnetic saturation in the disks in the two cases.

For the (100) ellipsoid, the minimum satisfactory field was considerably lower than for a disk of the same dimension ratio, as would be expected on the basis of ease of saturation. The deviations from the $1/H_a$ law were negligible down to 1100 oersteds, but just below this there was an abrupt drop in the magnitude of the torque peaks. Had the ellipsoid been thinner with respect to its diameter, the break would not have occurred until even lower fields were reached.

The major peaks of the (110) ellipsoid behaved exactly like the peaks of the (100) ellipsoid, but it was a different story for the minor peaks. The variation of a major and a minor peak of the (110) ellipsoid is shown in Fig. 5, where it can be seen that at 1000 oersteds or so the minor peak is 25 percent larger than the value it attains at much higher fields. Below 800 oersteds, the major and minor peaks are of practically the same magnitude. A satisfactory explanation of this phenomenon has not yet been given.

Discussion of the $1/H_a$ Law

The only theoretical work on the approach of torque measurements to their saturation value is that of Schlechtweg,[10] who predicted that the torque peaks of a homogeneously magnetized disk should approach saturation according to a $1/H_a$ law. The restriction to homogeneously magnetized disks is nothing more than a restriction to the use of ellipsoids instead of disks. Calculation of H_0 for the torque peaks of (100) and (110) ellipsoids according to Schlechtweg's equations gave $H_0=0$ in both cases, although for other orientations it was not zero. This indicated that the experimentally observed values of H_0 for the two ellipsoids had to be explained by a different theory than the one proposed by Schlechtweg.

It is interesting to compare the approach to saturation of the torque peaks with the similar approach to saturation found for the intensity of magnetization in the case of iron and iron-silicon alloys.[5, 11] One highly important difference between the two cases is that the *applied* field is used in connection with the torque measurements while the *effective* field is used for the magnetic intensity. The coefficient a appearing in place of H_0 in the similar equation of approach to saturation of magnetization has been measured by Fallot,[12] who found that a varied between 5 and 24 for iron-silicon alloys. The values obtained depended entirely upon some unknown property of the specimen measured and not upon the silicon content. Hence, all that can be said about H_0 and a is that they are of the same order of magnitude.

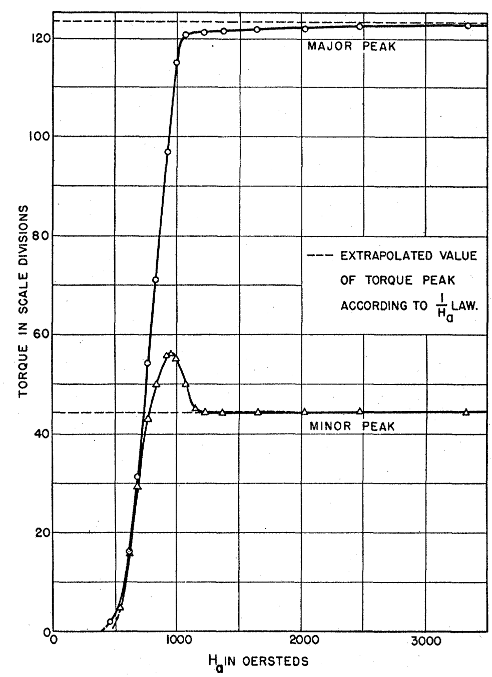

FIG. 5. Major and minor torque peaks of (110) ellipsoid plotted against magnetic field.

Before trying to reconcile this apparent similarity of the coefficients in spite of the difference

[10] Reference 3; also H. Mussmann and H. Schlechtweg, Ann. d. Physik [5] **32**, 290 (1938).

[11] A. Kussmann and E. Schoen, Physik. Zeits. **38**, 777 (1937).

[12] M. Fallot, Ann. de physique [11] **6**, 305 (1936).

between the meanings of H, an attempt was made to replot the torque data for one of the (100) disks on the basis of the effective field rather than the applied one. In order to get a linear approach to saturation using the effective field H, it was necessary to use a law of the form

$$T_H = T_\infty(1 - H_0'/H^n), \tag{3}$$

where n was very nearly equal to $\frac{1}{6}$. This equation is useless because neither n nor T_∞ can be obtained with any certainty from the data. Thus the only simple and easily obtainable equation that fits the torque data is the one involving the applied field. An examination of Fallot's data shows that it is equally hopeless to replot them on the basis of the applied field rather than the effective one and still retain a linear law; such a change, moreover, could not be justified theoretically.

The best explanation of the apparent similarity in the two laws of approach seems to be the following: The $1/H$ law for the approach to magnetic saturation holds only for polycrystalline material, where the direction of magnetization is averaged over all orientations, because it is known that the magnetization curves of a single crystal along the directions of easy magnetization do not conform to this law; instead, saturation is reached at very low fields for the [100] direction in iron, and in somewhat higher fields for the [110] direction. With the torque peaks, on the other hand, the direction of magnetization is along the same crystallographic direction within a few degrees, so that the magnetization along this direction may well approach saturation according to some other law than $1/H$. Since the magnitude of the torque peak is, among other things, proportional to the intensity of magnetization, it follows that the approach to saturation in torque will also follow this law. It seems, then, that the difference in crystallographic conditions between the two cases is accidentally balanced by the difference in the nature of the magnetic fields, i.e., applied or effective, leaving the laws formally the same and with coefficients of the same order of magnitude.

As has been mentioned, Schlechtweg's torque equations in $1/H_a$ predict a zero value for H_0 for the torque peaks of the (100) and (110) ellipsoids, and nonzero values for other orientations. For the two special orientations, H_0 was experimentally found to be in the neighborhood of 20, which appears to be an empirical constant of the material. It is probable that the torque peaks of an ellipsoid of some other orientation than (100) or (110) would have values of H_0 somewhat higher than this, the excess being given by Schlechtweg's theory.

Thus far, the discussion has been concerned only with ellipsoids. For disks, it is clear that the very much larger values of H_0 are associated with the difficulty of saturating their edges, i.e., the material outside the inscribed ellipsoid. This is in line with the results shown in Fig. 2. Probably a very small part of H_0 for a disk is to be attributed to the approach to magnetic saturation of the specimen as a whole, as in the case of the ellipsoids, but the important part is due to edge effects.

Conclusions

There are really three distinct $1/H_a$ laws for torque peaks. The first is Schlechtweg's theoretical one, which predicts an increase with field in the peaks of single crystal ellipsoids unless they are of the (100) or (110) orientation. Next, there is the experimentally determined $1/H_a$ law for (100) and (110) ellipsoids, which is attributed to the slow increase in magnetic saturation noticeable even in very high fields. And finally, there is the $1/H_a$ law observed for disks, for which H_0 is quite high because it is connected with the approach to saturation of the edges.

The usefulness of the $1/H_a$ law for disks lies in the fact that it makes possible the calculation of a unique value of the anisotropy constant, the one corresponding to $1/H_a=0$, which does not depend upon the field used nor upon the dimensions of the specimen. The validity of the extrapolation to $1/H_a=0$ has been satisfactorily checked by the measurements on two ellipsoids.