The Absorption of High Energy Electrons. IV

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The scattering of electrons in lead and in aluminum has been studied, mainly under the conditions used previously for the study of energy loss. The primary object of the investigation was to determine the extent to which the existing energy loss data (up to 13.5 Mev) are affected by multiple scattering. The conclusion reached is that the measurements so far made in lead with electrons below 9 Mev are so much affected by multiple scattering within the absorbing material that they are of very little use as a direct check on the theory for energy loss. In the measurements from 9 to 13.5 Mev the scattering is found to be small enough so that a comparison of the energy losses with the theoretical values is possible. The observed losses are roughly 40 percent in excess of the theoretical. Some measurements on the multiple scattering of 0.9-Mev electrons in thin sheets of aluminum were made, and it was found that the most probable angle of scattering and the average angle of scattering were in good agreement with theoretical predictions.

INTRODUCTION

A CONSIDERABLE amount of data is now available on the loss of energy suffered by electrons passing through solid materials such as lead, carbon and aluminum. During the past two years the work carried on in this laboratory has covered the energy range from 0.5 Mev to 17 Mev.[1–4] Results from the University of California[5] and the California Institute of Technology[6] are in essential agreement with those obtained here. It may be said with some certainty that, for the experimental method used at present, the results on energy loss are well established. The main uncertainties lie in the interpretation of the results, and one difficulty which overshadows all others is the estimation of the actual distance traveled by an electron inside the absorber.

On the assumption that the measured thickness of a given absorber corresponds to the actual path traversed by an electron, the experimental results have consistently shown losses greater than those predicted by theory. The discrepancy is not the same over the whole range but varies from a factor of more than two at the low energies to about one and a half at the high end of the range. The question is: How much of this discrepancy is due to the fact that the path taken by an electron does not correspond to the actual thickness of the absorber? The number we wish to find is called the path length-thickness ratio, or the ratio of the actual path length to the thickness of the absorber. This ratio is greater than unity for all absorbers, because the electron suffers an extremely large number of deflections, taking a zigzag path. In cases where there is much scattering the direction of emergence of a particular electron from the material tells little or nothing about its path length, since its final direction is a result of very many deflections in random directions. It therefore seems to be of no advantage to restrict the data to those electrons which emerge in a direction normal to the surface, or to try to make individual allowance for the effective thickness traversed by each electron, depending upon its angle of emergence. The best procedure seems to be to include all tracks in the data and to try to make a correction which applies to the average path length.

The complexity of the theoretical treatment of the path length-thickness ratio may be indicated by listing some of the important facts to be taken into account.

(1) The probability of a deflection varies with the size of the deflection.

(2) The probability of a deflection of a given

[1] J. J. Turin and H. R. Crane, Phys. Rev. **52**, 63 (1937).
[2] J. J. Turin and H. R. Crane, Phys. Rev. **52**, 610 (1937).
[3] A. J. Ruhlig and H. R. Crane, Phys. Rev. **53**, 618 (1938).
[4] B. R. Curtis, Phys. Rev. **53**, 986 (1938).
[5] L. J. Laslett and D. G. Hurst, Phys. Rev. **52**, 1035 (1937).
[6] W. A. Fowler and J. Oppenheimer, Phys. Rev. **54**, 320 (1938).

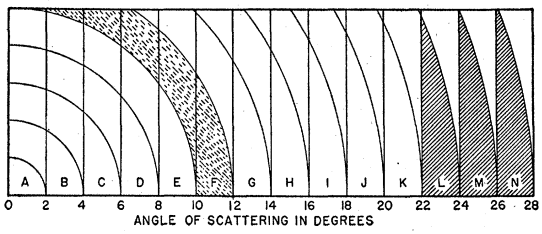

FIG. 1. Graphical device which is used in obtaining the true (three-dimensional) angular distribution of scattered electrons, from the cloud-chamber data.

size is a function of the energy of the electron, while the energy of the electron changes at an appreciable rate along its path due to two processes: ionization and radiation.

(3) The electrons finally must be separated into two classes: those which lose their entire energy in the absorber, and those which succeed in passing through the absorber. It is the average path length for the latter group alone that must be found, if the result is to be compared with the experimental data that are available.

(4) The situation has been made even more difficult to handle because the validity of the single scattering formulae has been placed in some doubt recently,[7–10] both in regard to large angles of scattering and in regard to the small angles which are important in multiple scattering. This is of direct concern because the theoretical estimates of path length are derived necessarily from the laws of single scattering.

The calculations can be simplified if the scattering is so small that at all times the deviation from the initial direction is not great ($\sin\theta \leqq \theta$). The treatment simplifies also if the scattering is so great that the mathematics of diffusion become applicable. It is desired, in the measurement of energy loss, to approach as nearly as possible the condition in which the scattering is small. Unfortunately the choice of an absorber which is thin enough to insure the electron path is nearly straight, and yet thick enough to cause a loss of energy large enough to be accurately measurable seems to be difficult in the range of energy with which we have been concerned, at least when a material of high atomic number is used. However, with the aid of the experimental results which we shall present, we have been able to arrive at some general statements which simplify the problem somewhat, at least in regard to the interpretation of the energy loss measurements already existing.

[7] A. Barber and F. C. Champion, Proc. Roy. Soc. **A168**, 159 (1938).
[8] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).
[9] N. L. Oleson, K. T. Chao, J. Halpern and H. R. Crane, Phys. Rev. **56**, 482 (1939); Phys. Rev. **56**, 1171 (1939).
[10] W. A. Fowler, Phys. Rev. **54**, 773 (1938).

PROCEDURE

Electrons were allowed to enter the cloud chamber through a thin window in the side wall and to fall normally upon a piece of absorbing material placed across the center of the chamber, as described in previous papers.[1–4]. The energy and the angle of deflection in the horizontal projection were measured for each track passing through the absorber. Although stereoscopic pairs of photographs were taken in all cases, it was found more satisfactory to measure only the component of deflection in the horizontal plane and to convert the distribution thus obtained into a three-dimensional one by a graphical method. This conversion can be made uniquely, because of the cylindrical symmetry about the initial direction of motion of the electrons. The reason for adopting this procedure rather than the direct three-dimensional measurement from the stereoscopic pictures is the following: Because of the limited depth of the chamber many tracks deflected through large angles away from the plane of the chamber are invisible or at least unmeasurable. Therefore an important correction would have to be made in this case, and the amount of the correction would be difficult to determine. By restricting the data to those tracks which lie within definite, small angular limits in the vertical direction (the plane of the chamber is horizontal), the correction to be applied becomes more definite and calculable. The data pertaining to the geometry are as follows.

TABLE I. *Area ratios for angles larger than those given in Fig. 1.*

ANGLE	PERCENT	ANGLE	PERCENT	ANGLE	PERCENT
26–28	42	36–38	28	46–48	22
28–30	37	38–40	26	48–50	21
30–32	35	40–42	24	50–52	21
32–34	32	42–44	23	52–54	20
34–36	30	44–46	23	54–56	20

(1) The depth of the visible region of the chamber is well defined by the light beam and is taken to be 3 cm.

(2) The incident electrons are passed through a slit system outside the chamber, so that their paths lie within 5 degrees of the horizontal plane and so that they strike the center of the absorber.

(3) The scattering is cylindrically symmetric about the direction of the incident electron.

(4) Only those tracks were included in the data which, after passing through the absorber, did not pass out of the light beam. This means that only those tracks were included whose vertical component of scattering was smaller than 10 degrees. All angles in the horizontal plane (up to 90 degrees) were included.

For the actual operation of making the conversion to the three-dimensional case the diagram in Fig. 1 is used. This shows the fraction of each scattering cone which falls within the horizontal and vertical angles defined above, and together with the supplementary data in Table I gives the numerical values necessary for conversion of the data to the three-dimensional case. The actual measurements yield the number of tracks in a particular vertical strip. This number then has to be distributed among those parts of the circular zones which lie within the vertical strip. It is necessary to start with the largest angle and work toward the center. An example of the procedure is as follows: Suppose we measure the number of tracks which fall in the rectangular column between 26 and 28 degrees. Since the angle is large, we can assume that the density of population in this rectangle is uniform, and therefore we can determine the number of tracks to assign to the shaded area N. Let us call this number N. The density of population in the *unshaded* part of the 24–26-degree rectangle is the same as that of the area N, because of the cylindrical symmetry about the origin. The number of tracks in the unshaded part of the 24–26-degree rectangle is therefore $0.47N$, because this is the ratio of the areas. The number of tracks which belong to the shaded area M is now determined by counting the number in the 24–26-degree rectangle and subtracting $0.47N$.

TABLE II. *Ratio of the complete ring about the origin to that of the curved figure.*

ANGLE	FACTOR	ANGLE	FACTOR	ANGLE	FACTOR
10–12	5.2	30–32	18.5	50–52	28.0
12–14	7.2	32–34	19.6	52–54	28.7
14–16	9.3	34–36	20.6	54–56	29.4
16–18	10.5	36–38	21.6	58–60	30.8
18–20	11.7	38–40	22.5	62–64	32.1
20–22	12.9	40–42	23.5	66–68	33.5
22–24	14.1	42–44	24.5	70–72	34.4
24–26	15.2	44–46	25.4	74–76	35.1
26–28	16.3	46–48	26.3	78–80	35.6
28–30	17.4	48–50	27.2	82–84	35.8
				86–88	36.0

TABLE III. *Energy distribution of electrons striking aluminum.*

THICKNESS OF ALUMINUM	NUMBER	ENERGY MEV
0.0025 cm	31	0.6 to 0.8
	87	0.8 to 1.0
	32	1.0 to 1.2
0.01 cm	45	0.6 to 0.8
	78	0.8 to 1.0
	35	1.0 to 1.2
0.025 cm	43	0.6 to 0.8
	99	0.8 to 1.0
	53	1.0 to 1.2

The number in the shaded area L is equal to the number in the 22–24-degree rectangle, minus $0.52M$. By continuing this process to zero degrees, the density of population in all parts of the diagram is found. A continuation of these area-ratios for larger angles than those which appear on the diagram is given in Table I.

To get the data finally in terms of the number of tracks in a given interval of solid angle or interval of angle, we make use of Table II. This table gives the ratio of the complete ring around the origin to that of the curved figure (for example the shaded one whose base lies between 10 and 12 degrees in the diagram).

EXPERIMENTAL RESULTS

The experimental results are represented by the curves in Figs. 2 and 3. In these figures the points represent number of tracks scattered into an angular interval between ϕ and $\phi+\Delta\phi$, in the plane of the chamber. In each case two curves are given. The full line curve is drawn smoothly through the experimental points, the dashed curve is the angular distribution which includes the entire cone of scattering, obtained by the

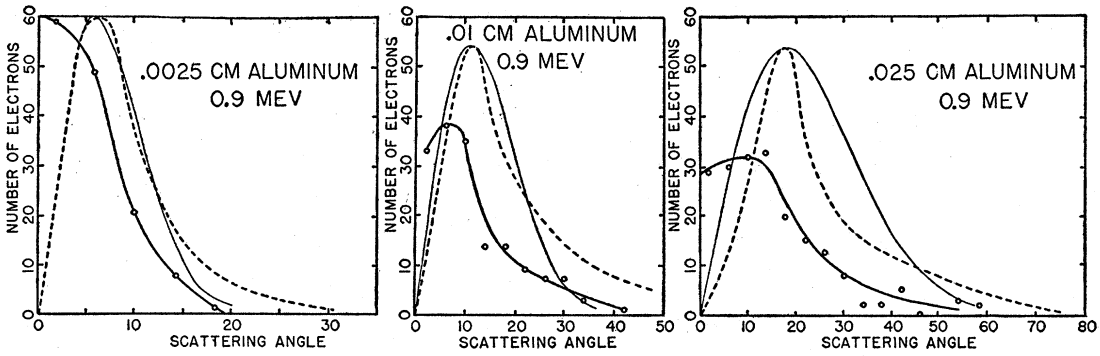

FIG. 2. Angular distribution of electrons scattered by aluminum.

use of Fig. 1. In Fig. 2 an additional curve is given to represent a theoretical distribution of the form: $y=Axe^{-Bx^2}$ with the parameters chosen arbitrarily. The reason for including the latter curve will be indicated in the discussion which follows later in the paper. The electrons which were incident upon the aluminum scatterers had an average energy of 0.9 Mev, and the actual distribution in energy is given in Table III. The electrons incident upon the 0.0038-cm and 0.0066-cm lead scatterers had an average energy of 0.9 Mev and a distribution as given in Table IV.

THEORETICAL CONSIDERATIONS

There can be little doubt that in the experimental data we have presented, we are dealing mainly with the effects of multiple scattering, even for the fastest electrons and the thinnest absorbers. In such cases, where a large number of deflections occurs inside the absorber, the distribution in direction of the emergent electrons must be found by compounding statistically effects of the large number of deflections. The result can be represented approximately by the simple error curve of the form

$$N(\theta)d\theta=A\theta e^{-B\theta^2}d\theta.$$

However, there is always a small number of large deflections due to single encounters, whose effect is to distort the distribution from the form mentioned above, especially at large angles. One must expect a noticeable deviation from the error curve at angles which are several times the most probable angle; this is the region of the so-called plural scattering. At extreme angles the distribution is expected to correspond closely to that predicted by the formula for single scattering. The angle beyond which the single scattering formula applies can be estimated by the use of Wentzel's criterion.[11] The 4ω used by Wentzel is approximately equal to the most probable angle, and it is customary to use 3 or 4 times this as the limiting angle. Referring to our curves, we may say that the scattering is essentially single at angles equal to about 4 times the angle at which the peak occurs in the dashed curve. The experimental distributions will be expected to conform to an error curve in the region of the maximum, and to have a plural or single scattering "tail" at the large angles, which will lie above the tail of the error curve.

It is essential at this point to tell what happens as the most probable angle approaches 25 or 30 degrees. In cases of thick absorbers, where the scattering is very great, the electrons will lose completely their original directions of motion, and take up random directions inside the absorber. Thus the case will resemble diffusion. Obviously any further increase in the thickness of the absorber will not alter the distribution in direction of the emerging electrons, but will only

TABLE IV. *Energy distribution of electrons striking lead.*

THICKNESS OF LEAD	NUMBER	ENERGY MEV
0.0038	81	0.6 to 0.8
	75	0.8 to 1.0
	56	1.0 to 1.2
0.0066	63	0.6 to 0.8
	65	0.8 to 1.0
	38	1.0 to 1.2

[11] G. Wentzel, Ann. d. Physik **69**, 335 (1922).

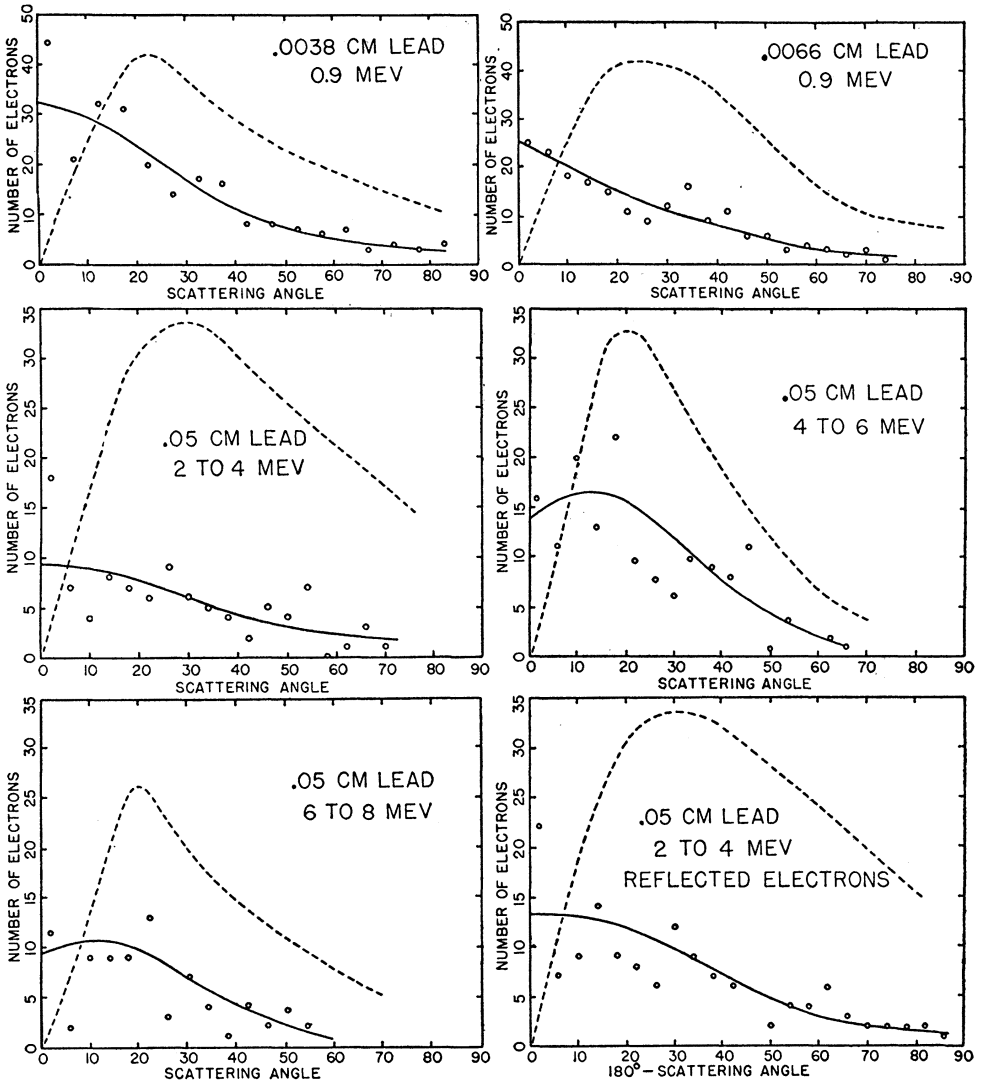

FIG. 3. Angular distribution of electrons scattered by lead.

change the number emerging. We have found experimentally that in those cases in which diffusion seems to exist, the most probable angle of emergence of the electrons is 25 to 30 degrees (measured from the normal to the surface). This means that if we start with a very thin absorber and gradually increase its thickness, we will at first get an error curve for the distribution, whose peak will move toward the right with increasing thickness. As the peak approaches 25–30 degrees the form of the distribution will change over into one which is characteristic of diffusion. No matter how much more we increase the thickness after this point has been reached, the curve will not change in any way except in total intensity. The arguments of the preceding paragraphs, which are based upon the error curve, will therefore apply only if the most probable angle is considerably less than 25 degrees.

Bethe, Rose and Smith[12] have treated the problem of multiple scattering and have given a theoretical expression for the "transport mean free path":

$$\frac{1}{\lambda}=\frac{2\pi NZ^2e^4W^2}{(W^2-m^2c^4)^2}\log\frac{2ap}{\hbar Z^{\frac{1}{3}}},$$

[12] H. A. Bethe, M. E. Rose and L. P. Smith, Am. Phil. Soc. Proc. **78**, 573 (1938).

TABLE V. *Comparison of our observed values of θ_{max} with those calculated by Bethe, Rose and Smith.*

MATERIAL	THICKNESS, CM	AVERAGE ENERGY, MEV	θ_{MAX} CALC.	θ_{MAX} OBS.	PATH LENGTH THICKNESS RATIO, CALC.	ENERGY LOSS, EXP./THEOR.
Al	0.0025	0.9	8	6	1.01	—
Al	0.01	0.9	16	11	1.04	—
Al	0.025	0.9	25	19	1.10	1.25
Pb	0.0038	0.9		22	Diffusion	3.
Pb	0.0066	0.9		25		2.7
Pb	0.05	3.0		29		1.8
Pb	0.05	5.0		21		1.8
Pb	0.05	7.0		21		1.8

where W is the energy of the electron including the rest mass, a is the Bohr radius and p is the momentum of the electron. The quantity λ is immediately connected with the most probable angle of scattering:

$$\theta_{max}=(2t/\lambda)^{\frac{1}{2}}.$$

The above formula can be applied to our data only when the most probable angle is considerably less than 25 degrees. Under this condition Bethe, Rose and Smith give as the mean path length for the electron in the absorber,

$$t=t_0(1+\tfrac{1}{2}\theta^2_{max}),$$

where t is the actual path length taken by the electron, t_0 is the measured thickness of the absorber and θ_{max} is the most probable angle of scattering.

Williams[13] has recently treated both single and multiple scattering in detail, and has developed formulae for the angular distribution and also for the average angle of scattering. He has compared our results for the average angle of scattering in 0.0025 cm aluminum at 0.9 Mev with that calculated by means of his formula and finds fairly good agreement. Our experimental value for the average angle in the plane projection is 5.5 degrees and his calculated value, after applying the appropriate correction for our geometrical conditions, is 5.9 degrees. This indicates that theory cannot be seriously wrong for the scattering in aluminum at this energy.

DISCUSSION OF RESULTS

Scattering

A comparison between our results and the values of θ_{max} calculated by the formulae of Bethe, Rose and Smith is shown in Table V. With the help of this the following can be said:

(1) The angular distributions of the electrons are those which would be expected to result mainly from multiple scattering. The positions of the maxima and the shapes of the curves are in fair agreement with the predictions of the multiple scattering formulae, in those cases in which the theory is applicable, namely $\theta_{max}\ll 25°$.

(2) The experiments on 0.0025 cm and 0.01 cm aluminum lie within the region of applicability of the Bethe, Rose and Smith formula, and we see that here the experimental and calculated values of θ_{max} are in fair agreement. The energy loss was too small to be measured in these two absorbers.

(3) The energy loss discrepancy in the 0.025 cm aluminum is somewhat larger than is accounted for by the increase in path length which is calculated, but this probably indicates that at this value of θ_{max} (19 degrees) we are already outside the region in which the formula gives the correct path length.

(4) For all the lead absorbers listed in Table V the scattering is so great that it approaches the case of diffusion. It is important to keep in mind the fact that as the amount of scattering increases θ_{max} approaches the stationary value of about 25 degrees, while the path length-thickness ratio continues to rise. Therefore in all cases in which θ_{max} is near 25 degrees, the formula gives only a lower limit to the path length. The value 25 degrees for the limit which the angle approaches was obtained by referring to the *experimental* data, and may therefore be to some extent characteristic of the particular set-up used for these measurements.

[13] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).

(5) Table VI gives a resumé of results from other papers. Some similar data exist for positrons,[3, 4, 6] but these have not been included in the table. In two of the measurements on lead (0.015 cm, 9 and 13.5 Mev) the results are reasonably free from the effects of diffusion so that the formula for the path length correction may be used. Although this statement is based upon the calculated values of θ_{max} this seems safe, since the experimental values have been lower than the calculated, in cases in which both are available.

(6) The fact that the observed values of θ_{max} are smaller than the theoretical is worth noting, in view of the fact that others[14–17] have recently reported a deficiency in scattering, both single and multiple. It is true that in the case of 0.0025 cm Al the average angle was found to be in accord with Williams' formula. This, however, is only a single case, and there is the possibility that it is in error.

(7) Single scattering could not be checked against theory for any of the cases because the thickness of the foils and the energy of the electrons were such that only those electrons scattered at very large angles would satisfy the requirements. In the case of the thinnest absorber (0.0025 cm Al, 0.9 Mev) Wentzel's criterion requires that only those electrons scattered through ~20 degrees or more be considered. Actually no electrons at all were found beyond this angle, for the 0.0025 cm Al.

Energy loss

In the light of the foregoing data, we may now make some statements about the interpretation of the measurements on energy loss.

(1) When we use an absorber of high atomic number which is thick enough to cause an easily measurable loss of energy, the effect of multiple scattering is usually very large. It appears in Tables V and VI that the only values for energy loss in lead which can be accepted (on the basis of scattering) are those for 0.015 cm thickness and 9 and 13.5 Mev. These are measurements by Fowler and Oppenheimer. The value for 0.038 cm thickness and 12.7 Mev, by Ruhlig and Crane is on the borderline (calculated $\theta_{max}=18$ degrees). In all the other measurements in lead the path in the absorber is so crooked that its average length cannot be estimated with reasonable accuracy. If we are willing to assume that positrons behave exactly as electrons, both in energy loss and in scattering, we may find some acceptable measurements on positrons in the papers already mentioned.[3, 6] These values do not change our conclusions, but only support what has been said in regard to electrons.

(2) The number of electrons which strike the lead absorber and do not emerge at all is found to be large in all those cases in which θ_{max} is large. This is understandable if we believe that in these cases the electrons penetrate the absorber by a process very much like diffusion. Those which fail to emerge are simply those which have traveled a distance equal to several times the thickness of the absorber and have thus "died" inside the material. The fraction of the electrons which fails to emerge from absorbers of various kinds has been measured and reported.[18]

(3) The fact that in some cases in which the scattering would be expected to be extremely great (for example 0.35 to 0.65 Mev in 0.0066 cm lead)[3] the specific energy loss becomes almost as low as the theoretical value, is understandable. The thickness of the absorber is not much less

TABLE VI. *Summary of results from other papers on energy loss of high speed electrons.*

Material	Thickness, cm	Average energy Mev	θ_{max} calc.	Path length Thickness ratio, calc.	Energy loss exp./thor.	Reference
Pb	0.038	9.5	22	—	1.2	3
Pb	0.038	12.7	18	1.05	1.3	3
Pb	0.014	2.5	>25	—	2.7	5
Pb	0.015	9.0	15	1.04	1.7	6
Pb	0.015	13.5	11	1.03	1.5	6
Pb	0.05	10.0	24	—	1.5	2
C	0.5	3.0	>25	—	~1	1
C	0.5	5.0	20	1.07	~1	1
C	0.058	2.5	11	1.03	0.5	5

[14] A. Barber and F. C. Champion, Proc. Roy. Soc. **A168**, 159 (1938).
[15] F. C. Champion and A. Barber, Phys. Rev. **55**, 111 (1939).
[16] W. A. Fowler, Phys. Rev. **54**, 773 (1938).
[17] N. L. Oleson, K. T. Chao, J. Halpern and H. R. Crane, Phys. Rev. **56**, 482 (1939); Phys. Rev. **56**, 1171 (1939).
[18] H. R. Crane and J. Halpern, Phys. Rev. **55**, 838 (1939).

than the maximum range of the electrons in the material. Consequently only those which happen to have a fairly straight path will emerge and will be counted. Those whose paths are more crooked will not emerge at all and will not enter into the average. Thus there is a selection in favor of the electrons which are least scattered. This probably accounts for the fact that the energy loss for carbon, 0.5 cm, 3 Mev (see Table VI) is about equal to the theoretical, in spite of the fact that diffusion clearly exists. In this case 65 percent of the electrons failed to emerge from the absorber.

After rejecting such a large part of the existing work on energy loss, it seems as though there remains little upon which to base an opinion as to whether or not theory is in accord with experiment. If, in the three most acceptable measurements on lead, we apply the indicated path length corrections we find that, as an average, the experimental values are at least 1.4 times the theoretical. Aside from the question of whether or not one is inclined to place any faith even in this result, we believe that we have been able to expose some of the pitfalls in energy loss measurement, so that future experiments can be made upon a somewhat firmer basis.

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The Spin of Carbon Thirteen

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Carbon thirteen was concentrated in a Hertz diffusion system to an abundance of 50 percent and enough 35 percent heavy carbon was obtained for a measurement of the nuclear spin of C^{13}. The spin was determined from the relative intensities of the Λ-type doublets of two lines of the O—O Swan band of $C^{13}-C^{13}$. These doublets are so close that a Lummer-Gehrke plate crossed with a 21-ft. grating was needed to obtain sufficient resolving power. An intensity analysis of the combined interference patterns of the two doublets proves that the C^{13} nucleus obeys the Fermi-Dirac statistics and strongly indicates a spin of $\frac{3}{2}$. This is in disagreement with a spin of $\frac{1}{2}$ predicted for this nucleus from the Hartree nuclear model but is the value predicted by the alpha-particle model.

THE determination of the nuclear spin of the ground state of C^{13} has become of considerable interest lately because it is one of the few magnitudes concerning which the predictions from the alpha-particle nuclear model and from the Hartree model are in complete disagreement.[1] The former predicts a spin of $\frac{3}{2}$ and the latter a spin of $\frac{1}{2}$. The most straightforward method of determining the spin is by measuring the relative intensities of alternate lines in the $C^{13}C^{13}$ molecular spectrum. In ordinary carbon, the $C^{13}C^{13}$ bands are only one ten-thousandth as intense as the $C^{12}C^{12}$ bands; so that for a measurement of the C^{13} spin, it is necessary to concentrate the C^{13} isotope considerably.

[1] R. G. Sachs, Phys. Rev. **55**, 825 (1939).

Concentration of Carbon Thirteen

The enrichment of C^{13} was carried out in a 34-member Hertz diffusion apparatus described by Wooldridge and Smythe.[2] To obtain high concentration the diffusion was in two stages. For the first diffusion, pure methane was pumped slowly and continuously through the light end at 1.4 cm pressure to maintain the normal concentration of C^{13} at this end. Six liters of methane at 8 mm pressure and about 7 percent C^{13} could be collected at the heavy end after a five-day run. Due to the large quantity of gas passed through the light end, considerable amounts of heavy impurities also collected in

[2] D E Wooldridge and W R Smythe, Phys Rev **50**, 233 (1936).