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Spin-Orbit Coupling in the Alpha-Model of Light Nuclei

D. R. INGLIS
Rowland Physical Laboratory, The Johns Hopkins University, Baltimore, Maryland
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If light nuclei are approximated by the alpha-model, with the particles localized in clusters of four (or less), the particle with uncompensated spin is accelerated by the relatively weak attraction of alphas, or by the difference of the attractions of two clusters. Then the Thomas relativistic term is not in general larger than the Larmor magnetic term in the spin-orbit coupling in contrast to the result of the central model, wherein the acceleration may be attributed to unsaturated forces toward one center. The relative magnitudes of these terms are estimated for the alpha-model of Li^7 and C^{13}, two of the few nuclei to which it is here considered that the model might apply. The magnetic moment consistent with the Larmor-Thomas coupling in the alpha-model of B^{11} is found to be considerably larger than the experimental value. The quadrupole moment of the deuteron implies a spin-orbit coupling arising directly from angle-dependent nuclear forces. The appropriate "spin-orbit-spin" angle dependence suggested by the meson theory causes no first-order coupling in the alpha-model of Li^7, however, and a rough estimate of the second-order doublet splitting shows that it may be considerably smaller than that due to the Larmor and Thomas terms.

FOR nuclei which may be approximated by a central-field model, it has been pointed out[1] that the Thomas relativistic term may be expected to be considerably larger than the Larmor magnetic term, and to determine the sign and order of magnitude of the spin-orbit coupling due to spherically symmetric forces between the particles in light nuclei. This may not be said in general for nuclei which may be approximated by the alpha-model, in which the protons and neutrons are treated[2, 3] as being clustered into alphas and possibly an extra particle or a triton or He^3. The difference arises largely from the fact that the Thomas term contains the acceleration of the particle with uncompensated spin toward the center of mass of the system, and in the alpha-model this acceleration is due to inter-alpha-forces which are supposedly considerably weaker than intra-alpha-forces, in keeping with the saturation requirement.

The validity of explaining nuclear doublet splittings by means of the Larmor-Thomas precession depends on whether or not its effect is masked by the effect of the angle-dependent nuclear interaction responsible for the deuteron quadrupole moment. Such an interaction, of "spin-orbit-spin" angle dependence $(\mathbf{s}_1\cdot\mathbf{r})(\mathbf{s}_2\cdot\mathbf{r})$, is suggested by the meson theory, but specification of its radial dependence requires an arbitrary cut-off. The contribution of this interaction to the doublet splitting in the alpha-model, of Li^7, for example, is a second-order effect and is expected to be quite sensitive to details of the model and of the radial dependence,

[1] D. R. Inglis, Phys. Rev. **50**, 783 (1936).
[2] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).
[3] H. Bethe, Phys. Rev. **53**, 842 (1938).

so it does not at present seem wise to try to compute the splitting. Since a very rough estimate below indicates that this contribution and the Larmor-Thomas splitting have about the same order of magnitude, an estimate of the Larmor and Thomas terms remains physically interesting, especially in view of such minor successes as its explanation of the sign of the spin-orbit coupling in the relatively simple nuclei Li^7 and O^{15}.

Among the $(4n-1)$-particle stable nuclei, Li^7 is probably the only promising application of the alpha-model.[4] This is indicated first by the fact that the magnetic moment of Li^7 calculated by the alpha-model[3] disagrees[5] with experiment considerably less severely than that calculated by a perturbation theory starting from the central model;[6] second, by the relatively high ratio of "internal" to "external" binding of the triton in Li^7; and third, by the failure of the alpha-model to furnish a correlation between the electrostatic energies of the various $(4n-1)$-particle nuclei.[7] For the $(4n+1)$-particle nuclei, such as C^{13}, which do not contain an easily distorted triton, the use of the alpha-model seems more plausible.

It is here shown that the magnetic term is at least of the same order of magnitude as the Thomas term in the alpha-model of the ground state of Li^7 and of C^{13}, and in Li^7 probably even larger. Only in the odd-neutron case C^{13} (and Be^7) can a preponderant magnetic term change the sign of the spin-orbit coupling, since the negative gyromagnetic ratio of the neutron here makes the magnetic and Thomas terms have opposite signs. It is further shown that the Larmor-Thomas splitting in B^{11} is too small to lead to agreement of the result of the alpha-model with the observed magnetic moment.

The Larmor-Thomas Precession in Li^7

For the sake of numerical simplicity in the dynamical problem we approximate the alpha-model of Li^7 by two clusters of $3\frac{1}{2}$ particles each, having their center of mass at the geometrical center. Each cluster has a displacement $\mathbf{r}$ and a velocity $\mathbf{v}$, relative to the center of mass, and a mass $(7/2)M$. The magnetic field at the triton due to the motion of the alpha relative to the triton is then

$$\mathbf{H}=(2e/c)(2\mathbf{r})\times(2\mathbf{v})/(2r)^3=(e\mathbf{L}/7Mcr^3),$$

where $\mathbf{L}=7M\mathbf{r}\times\mathbf{v}$ is the orbital angular momentum.

The Larmor energy is

$$E_L=-(ge/2Mc)\mathbf{S}\cdot\mathbf{H},$$

where $\mathbf{S}$ is the spin angular momentum, $(ge/2Mc)\mathbf{S}$ the spin magnetic moment, and g the spin gyromagnetic ratio (5.6 for the proton in Li^7 and about -4 for the neutron in Be^7). $\mathbf{S}\cdot\mathbf{L}$ contributes a factor $3\hbar^2/2$ to the doublet splitting, $\Delta E=E(^2P_{1/2})-E(^2P_{3/2})$, so we have the Larmor splitting of the Li^7 doublet

$$\begin{aligned}\Delta E_L&=3ge^2\hbar^2/(28M^2c^2r^3)\\&=(3g/28)(137/1830)^2(e^2/mc^2r)^3mc^2\\&=0.003_5(e^2/mc^2r)^3mc^2.\end{aligned}$$

The Thomas relativistic term is

$$E_T=\mathbf{S}\cdot\mathbf{a}\times\mathbf{v}/2c^2$$

the mean value of which is due mainly to the radial component of the acceleration $\mathbf{a}$ and the tangential component of the velocity $\mathbf{v}$. We may consider the radial acceleration in two parts. First there is a centripetal acceleration a_e accompanying the exchange of particles between the two clusters, and, second, the familiar centripetal acceleration $a_r=v^2/r$ of a particle at rest in a rotating coordinate system.

In treating the exchange, each particle may be described by a wave function having two

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[4] Compare, however, B. O. Grönblom and R. E. Marshak, Phys. Rev. **55**, 229 (1939).

[5] D. R. Inglis, Phys. Rev. **55**, 329 (1939).

[6] D. R. Inglis, Phys. Rev. **53**, 882 (1938).

[7] H. Brown and D. R. Inglis, Phys. Rev. **55**, 1182 (1939). In that paper it appears that the electrostatic energy of a light nucleus is as much as about half as great as the binding energy *between alphas*. This fact furnishes an easy rough indication of the preponderance of the magnetic term in the alpha-model if the spin stays with a triton; it favors the Thomas term by only a factor 2 in a comparison of nuclei with the well-known situation in atoms (wherein the electric field is the accelerating field, and the magnetic term is twice as large as the Thomas term). The magnetic term in nuclei is equally favored by the anomalously large gyromagnetic ratios of the proton and neutron, leaving the magnetic term preponderant as in atoms. Values of nuclear masses newer than used in that paper (compare W. H. Barkas, Phys. Rev. **55**, 691 (1939)) alter the comparison with experiment but do not decrease the discrepancy. The following may be substituted in Table I:

	H^3-He^3	Li^7-Be^7	$B^{11}-C^{11}$	$N^{15}-O^{15}$
α-model	0.86*	2.00	3.14*	4.28
obs.	0.86	1.87	3.14	3.66

extremes, one at each of the clusters.[2] Each of these "molecular orbital" wave functions may be considered in the distant-cluster approximation to be a symmetric or antisymmetric combination of the s functions of the two clusters, as in the perturbation treatment of the H_2^+ ion. If we call the energy difference between the antisymmetric and symmetric states $\hbar\omega$, the "frequency of exchange" between the clusters in the single-particle problem is $\omega/2\pi$. The order of magnitude of the corresponding acceleration we take from simple harmonic motion, $a_e \approx -\omega^2 r$. Using this and $\mathbf{L}=7M\mathbf{r}\times\mathbf{v}$, we have

$$\Delta E_T \approx (3/28M)(\hbar\omega/c)^2.$$

The magnitude of $\hbar\omega$ is given in Fig. 4 of reference 2 as $\hbar\omega = Q \approx mc^2$, but this depends upon the application of the alpha-model to B^{11} and N^{15} where it is probably not valid. A more generous estimate of $\hbar\omega$ may be made as follows: The binding energy of the alphas in Be^8 would be about $5mc^2$ if there were no Coulomb energy,[7] and the binding energy of the triton to the alpha in Li^7 would be about $7mc^2$. Of this, roughly $(3/4)5mc^2 \approx 4mc^2$ might be due to the same sort of interaction as binds Be^8, there being $\frac{3}{4}$ as many intercluster heavy-particle interactions in Li^7, so we might attribute the remaining $3mc^2$ to "exchange"—that is, to the fact that Li^7 has one more particle in the low energy symmetric state than in the antisymmetric state, whilst in Be^8 both states are filled. Using $\hbar\omega = 3mc^2$, we obtain

$$\Delta E_T \approx (m/M)mc^2 \approx 5\times 10^{-4}\ \mathrm{mc}^2.$$

The "rotational" centripetal acceleration is

$$\mathbf{a}_r = -(v/r)^2\mathbf{r} \doteq -(L/7Mr^2)^2\mathbf{r}$$

and the consequent doublet splitting is

$$\Delta E_T' \approx (3/4)\hbar^2L^2/(7^3M^3c^2r^4) = (3/2)\hbar^4/(7^3M^3c^2r^4) \approx 3\times 10^{-4}(e^2/mc^2r)^4mc^2.$$

For the value of r suggested below, this is considerably larger than the previous term ΔE_T.

The contribution of the tangential component of $\mathbf{a}$ and the radial component of $\mathbf{v}$ to $|\mathbf{a}\times\mathbf{v}|_{\mathrm{Av}}$ is $\langle(\dot{r}/r)(d/dt)r^2\dot{\varphi}\rangle_{\mathrm{Av}}$, which does not in general vanish in the alpha-model as it does in the central model. In the case of a simple harmonic motion along the figure axis (if one considers $\dot{\varphi}$ constant and so retains only the Coriolis acceleration), this becomes $\omega^2r^2\dot{\varphi}$, which is just $|a_e r\dot{\varphi}|$. The effect of the Coriolis term is thus just equal to the rather small effect ΔE_T of the radial exchange acceleration, in this approximation. In the alpha-model of more complex nuclei, in which the particle is not so nearly constrained to move on a rotating line, the tangential acceleration would be expected to be even less important than here.

The value of r to be used in the comparison of the various terms should be indicated by the mass difference of Li^7 and Be^7, which is attributed to Coulomb energy according to the usual assumption that the Hamiltonian is otherwise symmetrical between protons and neutrons. The atomic mass difference Be^7-Li^7 is roughly[8] 1.0 Mev$=2.0mc^2$, and that of He^3-H^3 is about zero. The Coulomb energy difference of Be^7-Li^7 minus that of He^3-H^3 is then about $2mc^2$ and the corresponding radius $r=\frac{1}{2}r_{t\alpha}=\frac{1}{2}\cdot 2e^2/(2mc^2) = \frac{1}{2}e^2/mc^2$. Using this value we obtain $\Delta E_L \approx 7\Delta E_T' \approx 50\Delta E_T$ and a total splitting $\Delta E \approx 0.03mc^2 = 15$ kev. Although these estimates are rough, they indicate that the magnetic term is almost certainly larger than the Thomas term in the alpha-model of Li^7. The same is true of the alpha-model of Be^7, the sign of its Larmor-Thomas coupling being thus altered and its ground state $^2P_{1/2}$.

The fact that the "rotational" centripetal acceleration appears to be greater than the acceleration associated with exchange is an indication of the manner in which the alpha-model should be treated in rotational problems. Since the triton and alpha change places by rotation more frequently than they exchange identity by migration of a particle, it is more consistent to consider a triton and an alpha rotating about a center of gravity somewhat nearer the alpha than it is to picture two clusters of $3\frac{1}{2}$ particles each rotating about the geometrical center. This makes a slight difference[3,9] in the expected magnetic moment of Li^7, favoring the

[8] L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **54**, 657 (1939), especially p. 680.

[9] R. G. Sachs, Phys. Rev. **55**, 825 (1939). I wish to thank Dr. Sachs for discussion of his results, especially on C^{13}, in connection with the present paper, and for pointing out that his reason for neglecting the Larmor term was a desire to make a formal comparison between the results of the two models, introducing the same assumptions, as nearly as possible, in both.

value 0.405 $\mu_N+\mu_\pi$ of reference 3 over the value 0.433 $\mu_N+\mu_\pi$ of reference 9.

The Larmor-Thomas Precession in C^{13}

C^{13} has an extra neutron which, in the alpha-model, occupies free space on both sides of the plane containing the three alphas. It is attracted by the alphas and may be pictured as being accelerated continually toward the plane in which they lie. In the ground state, the nucleus rotates about an axis in the plane of the alphas with one unit of angular momentum.[2] About this axis the moment of inertia is almost the same as that of the Li^7 nucleus, if the alpha-alpha distance $r_{\alpha\alpha}$ is nearly equal to the triton-alpha distance $r_{t\alpha}$. The angular velocities are therefore about the same. The C^{13} calculation differs from that for Li^7 principally in the fact that the extra particle has a greater centripetal acceleration because it is not bound into a cluster with other particles.

We can get a rough idea of the average distance of the neutron from the plane of the alphas in C^{13} by assuming that it is the same as (or slightly less than) that of the proton in N^{13}, and by using the observed Coulomb energy of the latter, 3.21 mMU$=6mc^2$. This is $6e^2/r_{\pi\alpha}$, where $r_{\pi\alpha}$ is the average proton-alpha distance in N^{13}. We thus have for the average neutron-alpha distance in C^{13}, $r_{\nu\alpha}\approx e^2/mc^2$. Taking $r_{\alpha\alpha}\approx e^2/mc^2$, as suggested by the Li^7-Be^7 Coulomb energy, we have the average position of the neutron on each side of the plane forming a regular tetrahedron with the alphas. The average radial component of the electric field at the neutron is only slightly reduced by the vectorial addition of the fields of the three alphas. This electric field is thus

$$F\approx 6e/r_{\pi\alpha}{}^2\approx 6m^2c^4/e^3$$

or somewhat less. With the magnetic field $\mathbf{H}=\mathbf{v}\times\mathbf{F}/c$ and with $\mathbf{v}\times\mathbf{r}=\mathbf{L}/2M$, the Larmor splitting is roughly

$$\Delta E_L\approx(9/4)g(137/1830)^2mc^2=-0.05mc^2.$$

The binding energy of the last neutron in C^{13} is about $10mc^2$. Although it moves in a non-Coulomb field so that we cannot apply the virial theorem exactly, we may assume as a rough approximation that the average kinetic energy has the same magnitude. Of this, part corresponds to motion in the two dimensions parallel to the plane of the alphas, but probably as much or more corresponds to motion along the z axis. To calculate the partition would require quite detailed treatment of the forces, so we merely assume that about $5mc^2$ corresponds to z motion. Approximating the z motion by a simple harmonic motion of frequency $\omega/2\pi$ and amplitude r, and considering E_T to be mainly due to this motion, we have $\ddot{z}\approx-\omega^2z\approx-4(5mc^2/Mr^2)z$ and, due to the rotation, $v_{\text{tangential}}=(L/2Mr_{\alpha\alpha}{}^2)z$. These lead to a Thomas splitting

$$\Delta E_T\approx(15/4)(137/1830)^2mc^2=0.02mc^2$$

(or somewhat more with the Coriolis term). This is thus weaker than the magnetic effect, but so slightly that it remains for a more accurate estimate to decide definitely whether the spin-orbit coupling arising from the Larmor-Thomas precession is positive or negative in the alpha-model of C^{13}. This rough estimate serves at least to show that the Larmor term may not be merely neglected, and suggests that the coupling is more apt to be negative, in agreement with the central model[10] and with experimental indications.[11]

Larmor-Thomas Coupling in B^{11} and the Consequent Magnetic Moment

A study of Coulomb energies does not exclude the possibility that the alpha-model applies to B^{11}, but makes this seem unlikely by showing that the model does not apply both to B^{11} and to N^{15}. Recent papers[9, 12] suggest that the alpha-model of B^{11}, with the Larmor-Thomas coupling, stands in agreement with the observed magnetic moment, but this is due to apparently excessive caution in avoiding specification of parameters in the model: Even a relatively safe order-of-magnitude estimate of the spin-orbit coupling limits the plausible μ to a narrow range which does not include the experimental value,[12] 2.68 μ_N. The treatment of Li^7 suffices to estimate the coupling due to body rotation and exchange acceleration (corresponding to β of reference 9)

[10] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937).
[11] C. H. Townes, Phys. Rev. **56**, 850 (1939).
[12] S. Millman, P. Kusch and I. I. Rabi, Phys. Rev. **56**, 165 (1939).

as $10^{-2}mc^2$. An upper limit of the coupling due to rotation of the "hole" around the triangle (the parameter α of reference 9) is had by considering the entire axis-projection $\Lambda=\hbar$ of the angular momentum to be due to rotation of one particle about the circumscribed circle. This gives an upper limit because a hole has less mobility (greater effective mass in the theory of metals) than a particle. Taking a radius $3^{-\frac{1}{2}}e^2/mc^2$, one finds the Thomas term $\Delta E_T \approx \frac{1}{4}mc^2$ (or $\alpha \approx mc^2/6$) and the magnetic splitting much smaller. The ground state might differ from ${}^2P_{3/2}$ by admixture of ${}^2D_{3/2}$. The rotational kinetic energy difference of the states is $\epsilon \approx 3\hbar^2/2I \approx 7mc^2$, so the admixture of D wave function is of the order of magnitude $\frac{1}{4}mc^2/7mc^2=0.03$. The operator $g_L L_z + g_S S_z$ being diagonal, the magnetic moment μ of the ground state would thus be expected to differ from $\mu({}^2P_{3/2})=\mu_\pi+5/11=3.23\mu_N$ by only about one-tenth of one percent, $(0.03)^2$. (Specializing the formulas of reference 9 gives $b_2=3^{\frac{1}{2}}\alpha/16\epsilon$ and $\mu \approx (1-10^{-5})\mu({}^2P_{3/2})$. The lower limit 1.5 μ_N suggested in reference 9 and quoted in reference 12 corresponds to about equal mixture of P and D states, which is quite implausible.)

Angle-Dependent Nuclear Forces in Li^7

The above remarks have been directed toward an investigation of the possibility that nuclear properties may be derived from rather simple spherically symmetric exchange forces between the particles. The quadrupole moment of the deuteron indicates that such forces (if retained at all) must be supplemented by a torque between spin and position vectors, so as to mix virtual states with the low S state of the deuteron. The appropriate angle-dependent interaction, or "spin-orbit-spin coupling," suggested by the meson theory, has the form

$$H' = \{\mathbf{s}_1\cdot\mathbf{s}_2 - 3(\mathbf{s}_1\cdot\mathbf{u})(\mathbf{s}_2\cdot\mathbf{u})\}f(r),$$

where $\mathbf{u}=\mathbf{r}/r$, and $\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$, the difference of the position vectors of the particles. Because of the present difficulties in the meson theory,[13] it still seems desirable to investigate the possibility of supplementing the old central exchange forces by a term H' only strong enough to account for the deuteron quadrupole moment. We shall here estimate the magnitude of the spin-orbit coupling thereby introduced in the alpha-model of the relatively simple nucleus Li^7.

Considering H' as a perturbation, the first-order spin-orbit coupling would be due to a difference of the ${}^2P_{1/2}$ and ${}^2P_{3/2}$ diagonal elements of H'. According to the sum rule, this may be calculated as the difference between the sum of the diagonal elements for the two states with $(M_L, M_S)=1, -\frac{1}{2}$, or $0, \frac{1}{2}$ and that for the state with $(M_L, M_S)=1, \frac{1}{2}$. (Here M_L and M_S are the usual orbit and spin projection quantum numbers.) The wave function of the ground state in the alpha-model, as in a molecule, is a product of a function of the internal coordinates, with M_S as a subscript, and a function of the orientation of the line between the clusters, with M_L as a subscript. The function of internal coordinates may be written as an antisymmetric sum of products of the form

$$s^{\nu+}s^{\nu-}s^{\pi+}s^{\pi-}a^{\nu+}a^{\nu-}a^{\pi\pm},$$

where s and a are the symmetric and antisymmetric "molecular orbital" wave functions, orthogonal to one another, the superscripts ν and π are the isotopic spin quantum numbers[14] referring to neutron and proton, respectively, and the superscripts + and − refer to single-particle spin projection m_s, or to the spin functions α and β, respectively. For calculating the matrix elements, the essential "spin-orbit-spin" coupling factor of H' may be written

$$\begin{aligned}(\mathbf{s}_1\cdot\mathbf{r})(\mathbf{s}_2\cdot\mathbf{r}) = \tfrac{1}{4}[&(s_x+is_y)_1(s_x-is_y)_2 \\ &+(s_x-is_y)_1(s_x+is_y)_2](x^2+y^2) \\ &+s_{z1}s_{z2}z^2+\text{terms which alter } M_S,\end{aligned}$$

and the spin operators may be used in the form

$$(s_x+is_y)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=\begin{pmatrix}0\\ \alpha\end{pmatrix},\quad (s_x-is_y)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=\begin{pmatrix}\beta\\ 0\end{pmatrix},$$
$$s_z\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=\begin{pmatrix}\alpha/2\\ -\beta/2\end{pmatrix}.$$

The diagonal element of this coupling factor consists of direct terms, between the product written above and itself, and of exchange terms, between the product as written and a similar product with two of its factors interchanged, with

[13] H. A. Bethe, Phys. Rev. **55**, 1261 (1939).

[14] B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936).

a negative sign because of the Pauli antisymmetry. One thus finds six direct terms in $-z^2/4$ from like-particle, unlike-spin pairs (two $s-s$, three $a-s$, and one $a-a$). There are also three terms in $z^2/4$ from like-particle, like-spin, $a-s$ pairs, making the direct terms from $a-s$ pairs vanish. Unlike-particle pairs contribute as many positive as negative terms. (x^2+y^2) is absent from the direct terms because of orthogonality introduced by the accompanying spin operations. Among the exchange terms, we first have three terms $-\frac{1}{4}(x^2+y^2)$ from the like-particle, $s-s$ or $a-a$ pairs (two $s-s$ and one $a-a$). We may combine the terms involving $s-s$ pairs into a space integral of $-\frac{1}{2}(x^2+y^2+z^2)f(r)$, independent of the sign of M_S, $\pm$. Due to its symmetry it is also independent of M_L, and gives no splitting. Similarly for the $a-a$ pairs. Further, we have the exchange terms due to like-particle, $a-s$ pairs which amount to $-\frac{3}{4}(x^2+y^2)$ from the unlike-spin pairs and $-\frac{3}{4}z^2$ from the like-spin pairs, all involved in the same "internal" space integral of an exchange nature, again centrally symmetric, so not contributing to the splitting. Unlike-particle pairs have no exchange terms for an H' independent of isotopic spin. There is thus no first-order doublet splitting in the alpha-model of Li^7.

The second-order splitting arises from admixture of excited P and F states to the two levels of the low ^2P. (Excited S and D states are not admixed because the space factor of H' transforms as the product of two vectors.) The magnitude of the matrix elements responsible for the admixture would depend in great detail upon the form of $f(r)$ and of the alpha-model. We shall therefore make only a very rough order-of-magnitude estimate of the splitting by comparison of this problem with that of the deuteron quadrupole moment.

By numerical integration of an $f(r)$ based on cut-off of the neutral meson theory, Bethe has shown that the 7-percent admixture of D wave function which he obtains in the ground state of the deuteron is quite adequate to explain the quadrupole moment.[13] While dependent on a special form of the interaction, his result indicates roughly the amount of D admixture required by the quadrupole moment with any interaction giving about the same "size of the deuteron," except for the possibility[15] of a considerably larger admixture which requires the small quadrupole moment to be a difference of the effects of the $S-D$ and $D-D$ quadrupole matrix elements. For interactions of which H' is a small perturbation this latter possibility seems to be ruled out by comparison of the stability of the deuteron and alpha-particle,[15] together with new measurements of the range of the proton-neutron interaction.[16] So it seems that the matrix element responsible for the D admixture is about $(S|H'|D)\approx 0.07(E_D-E_S)$ in order of magnitude, and this might be about $1mc^2$. This gives us an idea of the magnitude of the corresponding matrix elements in the alpha-model of Li^7, between the excited $^{2,4}P$ and ^4F states and the low ^2P states. The "internal" space factors of these excited states have an extra node making them orthogonal to the ground state, so that the matrix elements in question would vanish for very short or very long range of H', and for intermediate range the integrations must contain strong cancellation of positive and negative parts. This leads one to expect the matrix elements to be much smaller than in the deuteron, of the order of magnitude $10^{-1}mc^2$. The energy involved in the excitation is intermediate between that for a triton and that for an alpha, perhaps about $30mc^2$. For the depression of one of the low ^2P levels by one excited F state we may thus estimate $(F|H'|P)^2/(E_F-E_P)\approx 3\times 10^{-3}mc^2$. The excited P states depress both the levels $^2P_{1/2}$ and $^2P_{3/2}$, but the ^4F depresses only the $^2P_{3/2}$, suggesting that this type of doublet splitting would also be apt to make the $I_{Li^7}=\frac{3}{2}$, as observed. Since it depends on the detailed difference of the depressions of the two low ^2P levels due to several excited states, the splitting might be either larger or smaller than the depression ($\approx 3\times 10^{-3}mc^2$) due to one excited state. We may at least make the very rough estimate that the second-order splitting due to H' is of about the same order of magnitude as, and might be considerably smaller than, the Larmor-Thomas splitting ($\approx 3\times 10^{-2}mc^2$) in the alpha-model of Li^7.

[15] D. R. Inglis, Phys. Rev. **55**, 988 (1939).
[16] E. O. Salant, R. B. Roberts and P. Wang, Phys. Rev. **55**, 984 (1939).