The Range and Validity of the Field Current Equation

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Field currents from cold cathodes under high vacuum conditions were investigated over a range from 10^{-17} to 10^{-5} ampere. Electrometer measurements of currents below 10^{-10} ampere showed no deviation from the empirical equation of Millikan and Lauritsen. Precise measurements in the higher range exhibited a consistent deviation from the Millikan and Lauritsen equation, proportional to the fourth power of the field. This deviation is shown to be not necessarily in disagreement with the basis of the Fowler-Nordheim theoretical field current equation.

ONE of the early successes of wave mechanics was the development of an equation for cold cathode emission under high electrical fields, giving the first theoretical justification of the empirical equation

$$I = Ce^{-b/F} \tag{1}$$

obtained by Millikan and Lauritsen[1] in which I is the total emission current, b and c are constants, and F is the electric field intensity. In all cases where good vacuum conditions prevailed, the equation gave good agreement with experiment.[2] This investigation was undertaken to determine the range of validity of Eq. (1) and to search for possible deviations at high fields as predicted by wave-mechanical theory. The latter phase of the subject was briefly discussed by Stern, Gossling, and Fowler.[3]

The wave-mechanical solution of the problem by Oppenheimer, Fowler, and Nordheim[4] was based on the computation of the probability of penetration of the surface potential barrier by an electron incident upon the surface from within the metal. The simple form of surface barrier assumed by them leads to a transmission coefficient

$$D = \exp\left[-Bw^{\frac{3}{2}}/F\right], \tag{2}$$

where D is the probability of an electron of work function w, penetrating the surface barrier under the influence of an electric field F; and B is a numerical constant. It is apparent that theoretically any particular group of electrons of work function w would obey the empirical equation, (1). The total current is obtained from (2) by integration over the entire range of energies of the electrons and all values of the field over the emitting area.

Fowler and Nordheim assumed a Fermi distribution of energy and computed the emission per unit area under uniform field. A final assumption of an image force contribution to the surface potential barrier led to the theoretical field current equation

$$J = 1.55\cdot 10^{-6}(F^2/w)10^{-2.98\cdot 10^7 w^{\frac{3}{2}}\phi(y)/F}, \tag{3}$$

in which J = emission current in amperes per square centimeter; F = electric field in volts per centimeter; w = work function in electron volts; $y = 3.78\cdot 10^{-4}(F/w)^{\frac{1}{2}}$ is the reduction in height of an image force potential barrier due to an impressed field F; ϕ is an elliptic function of y, the values of which have been computed by Nordheim, and yield the curve of Fig. 1.

In a strict sense, Eq. (3) applies only at 0° absolute. The temperature dependence deduced is expressed approximately by

$$J(T)/J(0) = 1 + (4\cdot 10^8 T^2 w)/F^2, \tag{3a}$$

in which $J(T)$ and $J(0)$ are the specific emission currents at T°C and 0°C. With $F = 10^8$ volts per centimeter at room temperature,

$$J(T)/J(0) = 1.01.$$

At lower fields and higher temperatures the dependence increases slightly, but for logarithmic

[1] R. A. Millikan and C. C. Lauritsen, Proc. Nat. Acad. Sci. **14**, 45 (1928).
[2] R. A. Millikan and C. F. Eyring, Phys. Rev. **27**, 51 (1926). R. A. Millikan and S. S. Mackeowan, Phys. Rev. **31**, 900 (1928). General Electric Company of London Staff, Phil. Mag. **7** (1), 609 (1926).
[3] T. E. Stern, B. S. Gossling and R. H. Fowler, Proc. Roy. Soc. **A124**, 699 (1929).
[4] H. A. Bethe and A. Sommerfeld, *Handbuch der Physik* Vol. 24, Part 2, Sec. 3, Art. 19.

scales of J, as used experimentally, the temperature effect on field currents can ordinarily be disregarded. The almost complete temperature independence up to 1000° Kelvin is one of the outstanding characteristics of field emission.[5]

The usual check of the experimental field currents is to plot log I against the reciprocal of the applied voltage $1/V$. If Eq. (1) were of the correct form, a straight line would result since the field intensity F is proportional to the applied voltage.

Equation (3) in logarithmic form becomes

$$\log J = \log \frac{(1.55 \cdot 10^{-6})}{w} + \log F^2 - 2.98 \cdot 10^7 \frac{w^{\frac{3}{2}}}{F} \phi(y). \quad \text{(3b)}$$

For a constant emitting area under a uniform field the ratio of the observed total emission I to the emission per unit area J should be constant. If in Eq. (3b) the log F^2 could be disregarded and $\phi(y)$ remained unity, then log J would be linearly related to $1/F$ or, in measurable quantities, log I would be linearly related to $1/V$. However, the log F^2 and the factor $\phi(y)$ contribute toward a deviation from linearity in a log J, $1/F$ curve. In Fig. 2, log J, as computed from Eq. (3b), is

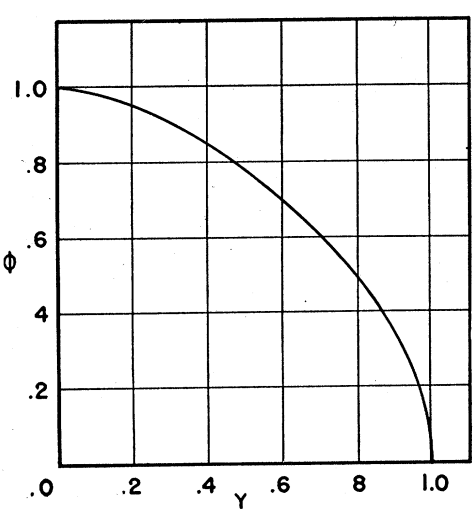

FIG. 1. Correction factor $\phi(y)$ introduced into the exponent of the Fowler-Nordheim equation, (3), to account for a reduction y in the height of an image force potential barrier due to an applied field F.

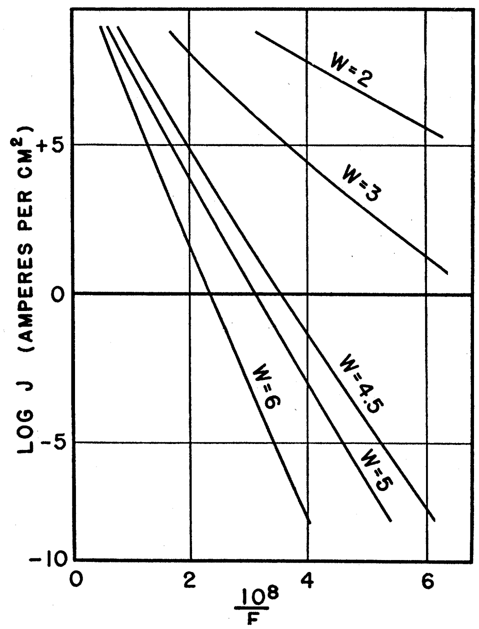

FIG. 2. The theoretical curves relating the emission to the field intensity for several values of the work function w.

plotted against $10^8/F$ for several values of the work function w. The deviation[6] of the $w=4.5$ curve from linearity as F increases from $1.5 \cdot 10^7$ to 10^8 is shown in Fig. 3. It is apparent that the deviation becomes increasingly pronounced as F approaches 10^8 volts per centimeter. The relative importance of the variable terms in Eq. (3b) is shown by Fig. 4. The main effect of the factor $\phi(y)$ is a vertical displacement of approximately two units of the straight line representing $-2.98 \cdot 10^7 (w^{\frac{3}{2}}/F)$ for $w=4.5$ and designated by $-b/F$. The two unit increase in log J would be equivalent to a hundred-fold increase in the emission J. The term, log F^2, from the figure deviates appreciably from a straight line function of the abscissa $10^8/F$.

Since experimentally the emitting area and the actual field intensity are difficult to even estimate, there appears no way to experimentally measure the effect of the ϕ factor. Measurements sufficiently precise should show evidence for or against the nonlinear log F^2 term, since F can be varied experimentally over a broad range.

[5] A. J. Ahearn, Phys. Rev. **44**, 277 (1933). Millikan and Eyring, reference 2.

[6] Deviations are the computed ordinate differences between the log J curve and a chord which closely approximates the log J curve in the vicinity of $10^8/F=3$.

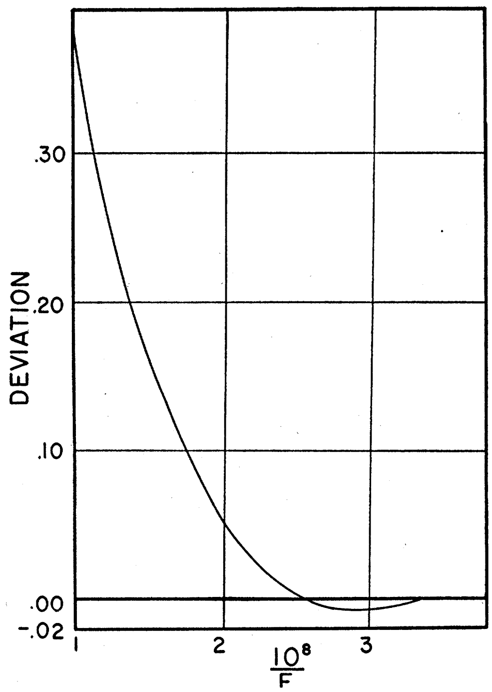

FIG. 3. Deviation of the $w=4.5$ curve of Fig. 2 from a straight line.

It is not difficult to produce a field which computed from the geometry approaches 10^7 volts per centimeter. Schottky[7] has shown that due to minute surface irregularities field intensity computed from the microscopic geometry may well be low by a factor of 10. Thus an experimental investigation of the deviation from linearity for geometrical fields of 10^6 and 10^7 volts per centimeter might yield information which would serve as a test of the theoretical Eq. (3).

The investigation to be described was divided into two parts. 1. By electrometer measurements of small emission currents and galvanometer measurements of larger currents from the same experimental tube, one curve of very broad range was obtained. 2. By the use of type K potentiometers a series of precise curves was obtained for the larger currents and higher field range.

Experimental Procedure

Part I. Extended range field current measurements

The tube, constructed from a one-liter Pyrex flask employed a 0.001-centimeter tungsten filament and a one-centimeter diameter copper cylinder as anode. The entire internal assembly filament leads and their supporting inseal were surrounded by a grounded copper shield. The external filament circuit was completely shielded and evacuated along with the electrometer case for small current measurements.

The high vacuum system included a two-stage Kurth type mercury diffusion pump and a liquid-air trap immersed after thorough bake-out process. The metal parts were subsequently bombarded until, after cooling, consistent ionization gauge pressure indications were below $4 \cdot 10^{-8}$ mm of mercury.

Emission currents were measured by a FP54 pliotron electrometer circuit with floating grid. The modified Barth[8] type of circuit employed is shown by Fig. 5. For currents immediately below the galvanometer range a 0.001-microfarad radio type variable condenser with pressed amber insulation was introduced into the control grid circuit to reduce the rate of drift.

The calibration was checked at the junction with directly read galvanometer currents. Currents from 10^{-10} to 10^{-5} ampere were measured by a Leeds and Northrup Type HS galvanometer and Ayrton shunt.

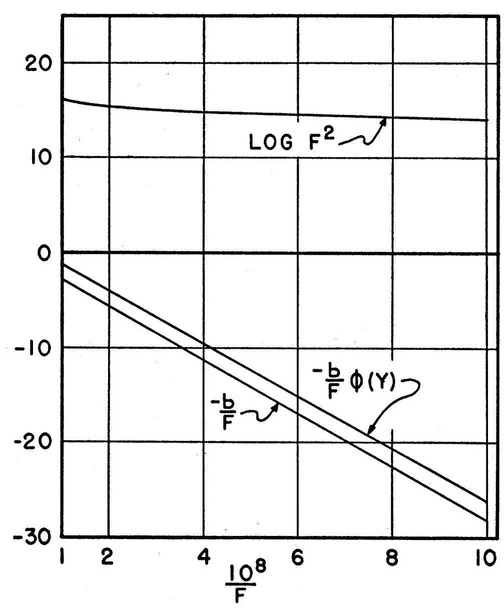

FIG. 4. Curves of the component terms in log J showing the relative magnitudes of log F^2, $-b/F$, and $-b\phi(y)/F$.

[7] W. Schottky, Zeits. f. Physik **14**, 63 (1923).

[8] O. B. Pennick, Rev. Sci. Inst. **6**, 115 (1935).

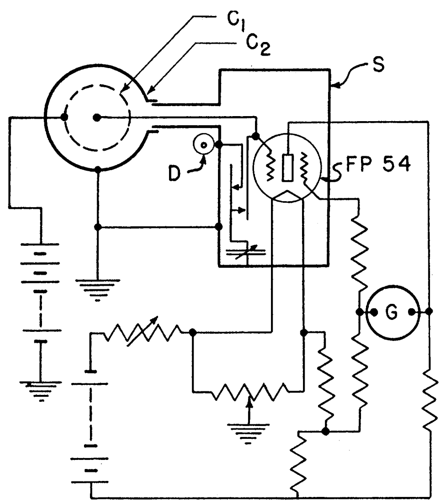

FIG. 5. Modified Barth type of FP54 Pliotron electrometer circuit used for small field current measurements. C_1—copper cylinder high voltage anode; C_2 and S—grounded electrostatic shielding system; D—control grid grounding device; G—Type H.S. galvanometer.

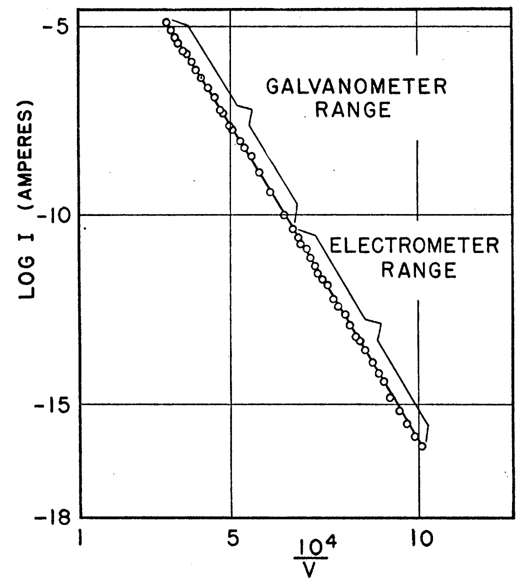

FIG. 6. Combined galvanometer and electrometer field current curve.

Throughout the electrometer range sufficiently steady high potentials were obtained from radio type B batteries. A small 3000-volt d.c. generator was used for larger currents and voltages of the galvanometer range.

Figure 6 shows the combined electrometer and galvanometer field current curve taken. Neither section exhibits curvature. No significance is attached to the slightly different slopes of the two sections. It was probably due to a partial breakdown of the emitting point as voltage was raised to the highest value at the beginning of the galvanometer run which was taken some time after the electrometer run.

The curve, extending over a current range of 10^{11} demonstrates the broad validity of the empirical Eq. (1). We conclude either that at the fields employed deviations do not exist, or else they are too minute to be observed with the instruments used.

Theory predicts increasing deviation with increased field, and this is the direction in which increased accuracy seemed relatively attainable by use of Type K potentiometer measurements. The foregoing work indicated that under best vacuum conditions the effect could be made steady enough to justify their use.

Part II. Precise field current measurements

A new tube was constructed, as shown in Fig. 7, to give higher fields at lower voltages. A 300-cc Pyrex flask was used for the tube itself. The anode was a carbon block and the cathode, a sharp tungsten point, was brought extremely close to the anode surface. A filament was provided for bombarding the electrodes. The evacuation process was essentially the same as that previously described. After bombarding, and while both electrodes were still considerably warmer than the glass walls, the cathode inseal was heated to the yield point and the tungsten point allowed to move into electrical contact with the carbon anode. Subsequent further cooling of the anode and cathode supports resulted in moving the point back a very small distance from the carbon. With this geometrical arrangement, galvanometer field currents were obtainable with less than 1000 volts applied. One objection to this method of producing high fields with low potentials is that the heating effect of currents approaching a milliampere produces changes in the anode cathode spacing due to expansion of the parts. This was in most cases avoided by opening the potential circuit for several minute intervals between the larger current readings.

Potentiometer readings of the drop across 20 ohms of an 885,000-ohm voltmeter resistor gave an accurate measure of the applied voltage.

Standard resistors of 1000 ohms to 1,000,000 ohms introduced in the cathode circuit were used in connection with a second Type K potentiometer to measure the field emission current.

Results and Discussion

The log I curve of Fig. 8 is typical of the results of six runs taken by this method with different separations of the electrodes. The actual form of the point was altered by honing three times during the course of the work. The double point near the top shows the reproducibility attainable for measurements taken at the beginning and end of a run. The curvature, although apparently slight, is evident when a straight edge is laid along the curve and is well beyond the probable error of measurement.

A comparison of the slopes of the theoretical curves of Fig. 2 with the experimental curve of Fig. 8 permits an estimate of the form factor θ relating applied voltage to the effective field ($F=\theta V$) for any assumed value of the work function w. From these form factors curves of log (I/F^2) as a function of $10^4/V$ were plotted. By theory these should yield straight lines if emission occurs from constant areas under uniform fields. This was done for assumed values of the work function from 3 to 9 electron volts. The

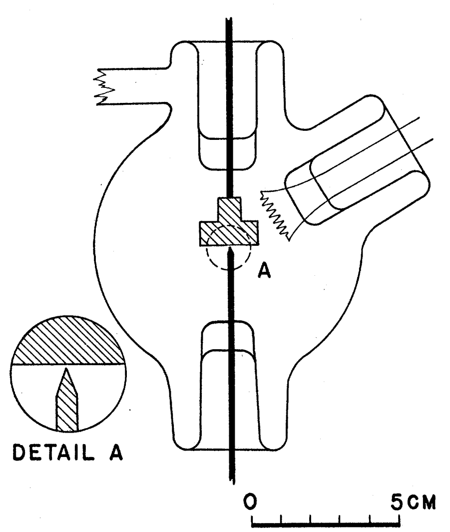

Fig. 7. Experimental tube for high field intensity at moderate applied potentials.

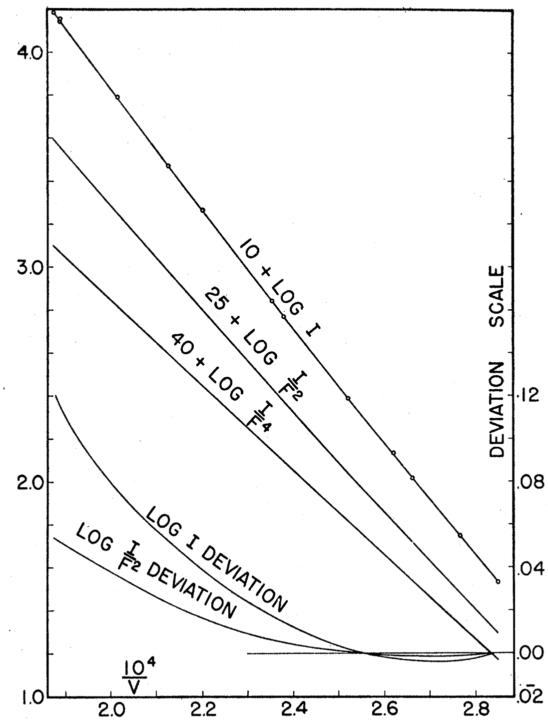

Fig. 8. Typical precise field current curves for the ordinates given adjacent to the curves. The double point at the top represents the reproducibility of the data; one point was taken at the beginning and the other at the end of the run. Deviations from linearity may be observed with a straight edge and are plotted at the bottom of the figure. The deviation of the $40+\log(I/F^4)$ curve is less than the probable error of measurement and is not plotted.

log (I/F^2) curve for $w=4.5$ is shown in Fig. 8. A consistent deviation appeared to remain regardless of the assumed value of the work function. Further investigation indicated that experimental values of log (I/F^3) or log (I/F^4) would more closely approach linearity as a function of $10^4/V$. The log (I/F^4) is shown on Fig. 8. The deviation (computed as in Fig. 3) of the log I and log (I/F^2) curve are shown at the bottom of Fig. 8. The deviations of the log (I/F^4) curves are less than the probable error of measurement, and hence are not shown. We therefore conclude from this investigation that the functional dependence of the total field current emission upon the field is best represented by an equation of the form

$$I=CF^4e^{-b/F}, \qquad (4)$$

where C and b are constants.

Although the form of this equation is different from the Fowler-Nordheim equation, (3), it does not necessarily contradict the basis upon which this equation is founded since this equation refers to the specific emission under constant area and uniform field. Consideration of this equation shows that any reasonable assumption as to the variation in field as a function of the distance from the point of maximum intensity leads to a higher power of F than occurs in the expression for J, the emission per unit area under constant field.

For example, in the case of an axially symmetrical point for which the field intensity at distance r from the axis is expressible in series form, the field in the vicinity of the axis is given by

$$F=F_0+r(\partial F/\partial r)_{r=0}+r^2(\partial^2 F/\partial r^2)_{r=0}\cdots.$$

Since the axis is a maximum point $(\partial F/\partial r)_{r=0}=0$ and thus

$$F=F_0+r^2(\partial^2 F/\partial r^2)_{r=0}$$

and to first approximation putting

$$(\partial^2 F/\partial r^2)_{r=0}=-\alpha$$

we have

$$F=F_0-\alpha r^2.$$

The total emission is given by

$$I=\int_0^\infty 2\pi r J(r)dr.$$

Introducing the above expression for F into the Fowler-Nordheim expression for J, Eq. (3), and integrating, we obtain approximately

$$I=\alpha F^3 e^{-\beta/F}, \qquad (5)$$

in which all factors such as the work function w and the function ϕ have been included in constants α and β.

Only in the case of uniform field over the entire emitting area such as might result from a patch of low work function material covering a small area on the cathode would the Fowler-Nordheim expression, (3), describe the total emission. Under the more probable physical condition of an approximately symmetrical pointed electrode and constant work function over the area, Eq. (5) should more closely represent the total emission current. The experimental equation, (4), requires a still higher power of F. The correct power of F in the coefficient in any physical case depends on the exact microscopic shape of the point.

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