The Scattering of Protons by Protons. III

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Our work of 1936 on the anomalous scattering of protons by protons in the energy region 600 to 900 kv was repeated during 1938 with an entirely new scattering apparatus designed for better angular measurements. Particular care was taken in order to know the energy (velocity) of the protons at the scattering volume with accuracy. The number of protons in the primary beam during each observation was determined by counting the protons scattered backward from a gold foil placed in the path of the proton-beam beyond the hydrogen-scattering volume. The scattering of protons by spectroscopically pure argon for several voltages and various angles was found to obey the Rutherford-Darwin formula. These measurements with argon provided a calibration of the voltage-scale in absolute units which was found to agree with our standard voltage-scale within about one percent. On the latter scale the midpoint of the gamma-ray resonance for protons on lithium is at 440 kv, the molecular ions thus giving a calibration point at 880 kv. A strong resonance for protons on fluorine occurs at 867 kv. The scattering anomaly arises from the failure of the Coulomb law of repulsion for very close distances of approach, because of the existence of a strong attraction of very short range which is the nuclear force between two protons. The newer measurements are in good agreement with our results in 1936, which were shown by Breit and his colleagues to correspond in the wave mechanics to a simple s wave scattering. They also agree well with the data of Herb and his colleagues where the measurements overlap at about 860 kv. No clear-cut evidence for higher order scattering is found in our newer data, although the Mott ratios for angles from 20° to 30° are higher than would be predicted from the values for angles from 40° to 45° on the basis of s wave scattering only. The departures are from five to ten percent, whereas from the internal consistency of the data one would hardly expect statistical errors of this magnitude. The discussion indicates that systematic errors affecting the scattering at 20° to 30°, although unlikely, cannot definitely be excluded. These observed systematic deviations are of the type which would be contributed by a small amount of p wave scattering superposed on the spherically symmetrical s wave scattering.

I. INTRODUCTION AND DISCUSSION

DURING 1936 we made a series of measurements on the scattering of protons by protons in the energy range 600 to 900 kv,[1] which demonstrated the existence of a scattering anomaly caused by the nuclear force acting between two protons at close distances. These measurements were shown by Breit, Condon and Present[2] to correspond to a simple s wave scattering arising from a potential well representing an attraction between two protons very nearly equal in magnitude to the attraction between a proton and a neutron. Their analysis of our measurements showed that the range of these proton-proton forces could not be much above 4×10^{-13} cm, although a lower limit on the range could not be set by the measurements over this restricted region of energies. During 1937 we carried out measurements on proton-proton scattering in the region 200 to 600 kv, using the more sensitive but less quantitative Geiger point-counter instead of the usual linear-amplifier arrangement.[3] These measurements gave added and unequivocal proof that the anomaly is due to an attraction superposed on the Coulomb repulsion and not an added repulsion, since the scattering at 45° falls to less than five percent of the Coulomb value at a voltage of 400 kv. This is because of the opposing effects of the electrical forces of repulsion and the nuclear forces of attraction.

These measurements were subject to several sources of error which made a repetition seem desirable. The major features of the results could not be called into question, but it appeared that if the measurements could be made more reliable in detail, important quantitative improvements in the theoretical deductions might become possible. By an oversight the original apparatus was disassembled before the alignment was checked, and it appeared that an error in the angular measurements of as much as 1° conceivably might have been present. A second source of error was the fact that the number of protons in

[1] M. A. Tuve, L. R. Hafstad and N. P. Heydenburg, Phys. Rev. **50**, 806–825 (1936).
[2] G. Breit, E. U. Condon and R. D. Present, Phys. Rev. **50**, 825-845 (1936).
[3] L. R. Hafstad, N. P. Heydenburg and M. A. Tuve, Phys. Rev. **53**, 239–246 (1938).

the primary beam, measured by using the scattering chamber as a Faraday cage, could not be checked continuously during the observations, but was measured before and after each observation and was assumed to be constant during the intervening period. A third source of error was an uncertainty regarding our voltage-scale, which had been determined by reference to ordinary standards at lower voltages. Finally, the scattering observations for individual values of voltage and angle were not sufficiently extensive to reduce the statistical fluctuations as far as seemed desirable.

For the measurements of 1938 reported in this paper, these sources of error were considerably reduced. An entirely new scattering chamber was constructed, with careful mechanical checks to insure that the axes of the beam-diaphragm and the detector-slit system, and also the axis of rotation of the detector, intersected in one point. Careful measurements were made of the defining slits and diaphragms, and an accurately centered circle of good quality was used for the angular measurements. Provision was made for continuous measurement of the proton beam-current by counting protons recoiling at 135° from a gold foil larger than the whole beam, placed beyond the gas-scattering volume. These gold-foil counts as a function of voltage were reduced to microcoulombs of protons by Faraday-cage measurements made before and after the scattering observations. The 10,000-megohm voltmeter-resistor was dismantled and rechecked thoroughly, and then recalibrated at 440 and 880 kv against the proton-lithium resonance. The accuracy of our standard voltage-scale, defined by this resonance, was checked in absolute units by measuring the scattering of protons by spectroscopically pure argon gas as a function of voltage and of angle. The argon scattering agreed within one percent with the expected Coulomb scattering, indicating that our measurements of voltage, and corrections for stopping power of window and gas in the path of the proton-beam, were satisfactory. The proton-scattering measurements in this series were restricted to three voltages (specified at the scattering volume), namely, 867, 776 and 680 kv. The first value, 867 kv, coincides with one of the strong gamma-ray resonances for protons on fluorine, and was so selected and specified in order to provide an accurately repeatable voltage-point for the intercomparison of results with other laboratories. Comparisons made at this resonance-voltage are obviously independent of voltmeter-scales. The statistical fluctuations of our observations in this series are considerably better than in the earlier measurements. They are not as satisfactory as we might have desired, because the high humidity conditions of Washington summer set in before we had quite finished the observations, putting an end to accurate work. The observations with argon, in particular, are less extensive than they would have been except for this difficulty. A third series of measurements with hydrogen was similarly limited to a few points.

The results of this series of proton-scattering measurements do not differ in any very important way from the measurements of 1936, and the conclusions then drawn are not significantly altered. Our experimental data are also in close agreement, even on an absolute basis, with the values obtained by Herb and his colleagues[4] in the voltage-region (near 860 kv) common to both investigations. As shown by the analysis made by Breit, Thaxton and Eisenbud,[5] the major features of the observations are accounted for as an s wave scattering by a potential well which is very nearly the same as that for the proton-neutron attraction, provided the Coulomb force between two protons is not "cut off" at the edge of the well but is assumed to exist also for shorter distances, superposed on the nuclear attraction. One feature of the measurements which is not entirely in accord with this picture is the fact that the scattering observed for angles from 20° to 30° is higher than would be computed from our observations for angles from 40° to 45° on the basis of simple s wave scattering, by an amount considerably in excess of the apparent accuracy of the measurements as judged by their internal consistency (about two percent). The deviations in the scatterings at 20° to 30° are as large as six to ten percent for some voltages and angles. Although they are not entirely consistent, they might be due to a small amount of p wave scatter-

[4] R. G. Herb, D. W. Kerst, D. B. Parkinson and G. J. Plain, Phys. Rev. **55**, 998–1017 (1939).

[5] G. Breit, H. M. Thaxton and L. Eisenbud, Phys. Rev. **55**, 1018–1064 (1939).

ing superposed on the spherically symmetrical s wave contribution; a repulsive p wave term would give rise to added scattering at these angles without affecting the values at 45°. Since the scattering values obtained by Herb and his colleagues[4] in the range from 880 to 2400 kv are in substantial agreement with simple s wave scattering, it does not appear reasonable to accept these deviations as sufficient evidence for p wave scattering, and it seems most interesting to examine this possibility by a search for p wave scattering in the range from 650 down to 400 kv. Our survey of 1937, in which a Geiger point-counter was used in this region, was not sufficiently quantitative to test for a small amount of p wave contribution to the scattering.

Instead of being due to a p wave contribution, the possibility cannot be excluded that the excess scattering at 20° to 30° is caused by systematic errors which affect the values at these angles more strongly than at 45°. Several types of error have this characteristic, especially because Coulomb scattering is predominantly into the forward angles. Multiple scattering in the gas and slit-edge scattering would tend to give high values at low angles, and so would the finite angular width of the slit-systems, since the excess of particles scattered through one-half degree less than the average angle will more than compensate for the diminished scattering of particles through one-half degree more than the average angle. An error in determining the zero of the angular scale for the detector of the scattered protons (when the axis through the detector slit-system is in line with the incident beam) also is more important at small angles. All of these sources of error were recognized and experimentally examined, and we satisfied ourselves that they did not contribute large effects. However, one source of difficulty in connection with the zero of the angular scale was not obvious until the experiments were completed, and this may have contributed part of the excess scattering observed at 20° to 30°. The scattering chamber was not directly beneath the high voltage tube since magnetic analysis was used, the magnet being adjusted to deflect the beam (through about 17°) until it was centered on the aluminum-foil window just preceding the diaphragm-system of the scattering chamber. A shift of the beam by five or eight mm due to focusing readjustment would be immediately obvious if the motion were at right-angles to the plane of the magnetic deflection, and would be compensated for by adjusting the position of the ion-source. However, a similar motion in the plane of the magnetic deflection would escape notice, being compensated for by a change in the magnet current. This would result, however, in a slight change in the initial direction of the beam entering the diaphragm-system, with a correspondingly different correct position for the zero-setting of the angular scale. The latter was determined by scattering observations to each side of zero during the initial calibration of the apparatus, after which a slit-guard was installed and observations were made to one side of zero only, as discussed below. It is our impression that an error from this cause should fluctuate from day to day, and it appears unlikely that the excess observed at 20° to 30° is to be explained entirely on this basis.

II. Computations

In order to be completely independent of our earlier work, the theoretical Coulomb-law values of the scattering were recomputed from Mott's formula[6] for each voltage and angle used in this series of observations. The same values were obtained independently by two computers and are given in Table I. We used the older set of Birge's constants (1929), as follows: Electronic charge: $e=4.770\times10^{10}$ e.s.u.; Energy of 1000-kv proton: $(\frac{1}{2})\ mv^2=1.5911\times10^{-6}$ erg; Avogadro's number: $N=6.06436\times10^{23}$ atoms per mole; Perfect gas volume: $R=22{,}415$ cc per mole at 0°C. With these constants we have $N_0=3.7710\times10^{14}$ protons in our beam per microampere-minute (6.285×10^{12} per microcoulomb), and 6.5888×10^{16} hydrogen atoms per cc in our scattering volume at 22°C under a pressure of 1 mm of mercury (15.7 mm of Apiezon oil).

If N_0 is the number of protons per second in the incident beam having kinetic energy T, then the numbers of primary particles N_s scattered per second into a solid angle ω at an angle θ with respect to the initial beam by a target having N_t hydrogen nuclei are as follows according

[6] N. F. Mott, Proc. Roy. Soc. **A126**, 259–267 (1930).

to Mott:

$$N_s = N_0 N_t \omega (e^4/T^2) \cos\theta \{\csc^4\theta + \sec^4\theta - \csc^2\theta \sec^2\theta \cos[n \log((1+\cos 2\theta)/(1-\cos 2\theta))]\},$$

where $n=(1/137)(c/v)=0.161$ for $T=1000$ kv, $N_0N_t\omega(e^4/T^2)=2.04498$ for $N_0N_t\omega=10^{26}$ and $T=1000$ kv.

The values of N_0, N_t, and ω, calculated from the dimensions of the slit-systems of our scattering chamber (see next section) are: $N_0=3.7710\times10^{14}$ protons per microampere minutes; $N_t=1.37047\times10^{16}\times(1/\sin\theta)$ hydrogen nuclei, at one mm pressure of mercury and 22°C (the $1/(\sin\theta)$ factor is included to take into account the variation with angle of the target-thickness as discussed in reference 1); $\omega=0.869202\times10^{-4}$ steradian. With these values, and with a hydrogen pressure at the target of 12 mm of mercury at 22°C

$$N_0N_t\omega(e^4/T^2)=110.2334\times(1/\sin\theta) \quad \text{for} \quad T=1000 \text{ kv}.$$

For the convenience of experimentalists we have listed in Table I the numerical factors required for quick computation of the Mott values for various angles at different voltages. Our values for the "Mott function" agree, where angles coincide, with those computed by Breit and his colleagues for this series,[5] but our numerical values for the theoretical number of scattered protons from our scattering volume are 0.43 percent lower than theirs, due to the slightly different values adopted for the fundamental constants.

In Table I, columns 2, 3 and 4 give the functions of angles occurring in Mott's formula. For most uses one needs cot θ instead of cos θ because of the factor $(1/\sin\theta)$ occurring in N_t to take account of the change in our scattering volume with angle. Column 5 gives the angular function for $n=0$, that is, for an "infinite velocity" of the incident protons. The absolute value of $N_s(\theta)$ is given in column 6 for $n=0$, $T=1000$ kv, and $N_0N_t\omega=10^{26}$. Columns 7 and 8 show the effect of taking into account the factor n, which depends on the velocity, at a proton-energy 685 kv. At this energy the correction is about one percent and at higher energies it will be correspondingly smaller. The values of $N_s(\theta)$ in these two columns were those used to compare directly with the experimental data from our apparatus, for which the coefficient of the Mott function is

$$N_0N_t\omega(e^4/T^2)=110.2334\times(1/\sin\theta)(1000 \text{ kv}/685 \text{ kv})^2.$$

III. Apparatus and Technique

The requirements for an apparatus to measure accurately the scattering of protons by gases are evident from the Mott formula given in Section II. It is necessary to know the number and energy

Table I. *Tabulation of angle functions used in calculating scattering values from Mott's formula, also $N_s(\theta)$ at 1000 kv and at 685 kv for $n=0$ and $n=0.191$.*

θ	$A+B$ ($A=\csc^4\theta$, $B=\sec^4\theta$)	D ($\csc^2\theta \sec^2\theta$)	cot θ	$(A+B-D)\cot\theta$ $n=0$	$N_s(\theta)$ $n=0$ $N_0N_t\omega=10^{26}$ 1000 kv	$N_s(\theta)$ $n=0$ 685 kv	$N_s(\theta)$ $n=0.191$ 685 kv
15°	224.000	16.0000	3.73205	776.266	410.863	182,367	184,980
20	74.3616	9.68110	2.74747	177.708	124.293	41,748	42,210
22.5	48.0000	8.00000	2.41421	96.5684	75.5726	22,687	22,940
25	32.8300	6.81636	2.14451	55.7864	48.2131	13,106	13,251
27.5	23.6131	5.96116	1.92098	33.9089	32.0191	7966.2	8049.6
30	17.7777	5.33333	1.73205	21.5543	21.9273	5063.7	5111.3
32.5	13.9751	4.86977	1.56969	14.1513	15.5489	3324.6	3384.4
35	11.4602	4.52990	1.42815	9.89748	11.6092	2325.2	2339.3
37.5	9.80560	4.28719	1.32704	7.19176	8.95302	1689.6	1696.3
40	8.76166	4.12437	1.19175	5.52649	7.26453	1298.33	1300.9
42.5	8.18464	4.03062	1.09131	4.53332	6.26308	1065.0	1065.6
45	8.00000	4.00000	1.00000	4.00000	5.78407	939.72	939.72
50	8.76166	4.12437	0.83910	3.89115	6.09566	914.14	916.40
55	11.4602	4.52990	0.70021	4.85265	8.12888	1140.0	1146.9
60	17.7777	5.33333	0.57735	7.18477	12.7243	1687.9	1703.8
65	32.8300	6.81636	0.46631	12.1304	22.4821	2849.8	2881.2
70	74.3616	9.68110	0.36397	23.5418	45.2390	5530.7	5591.5

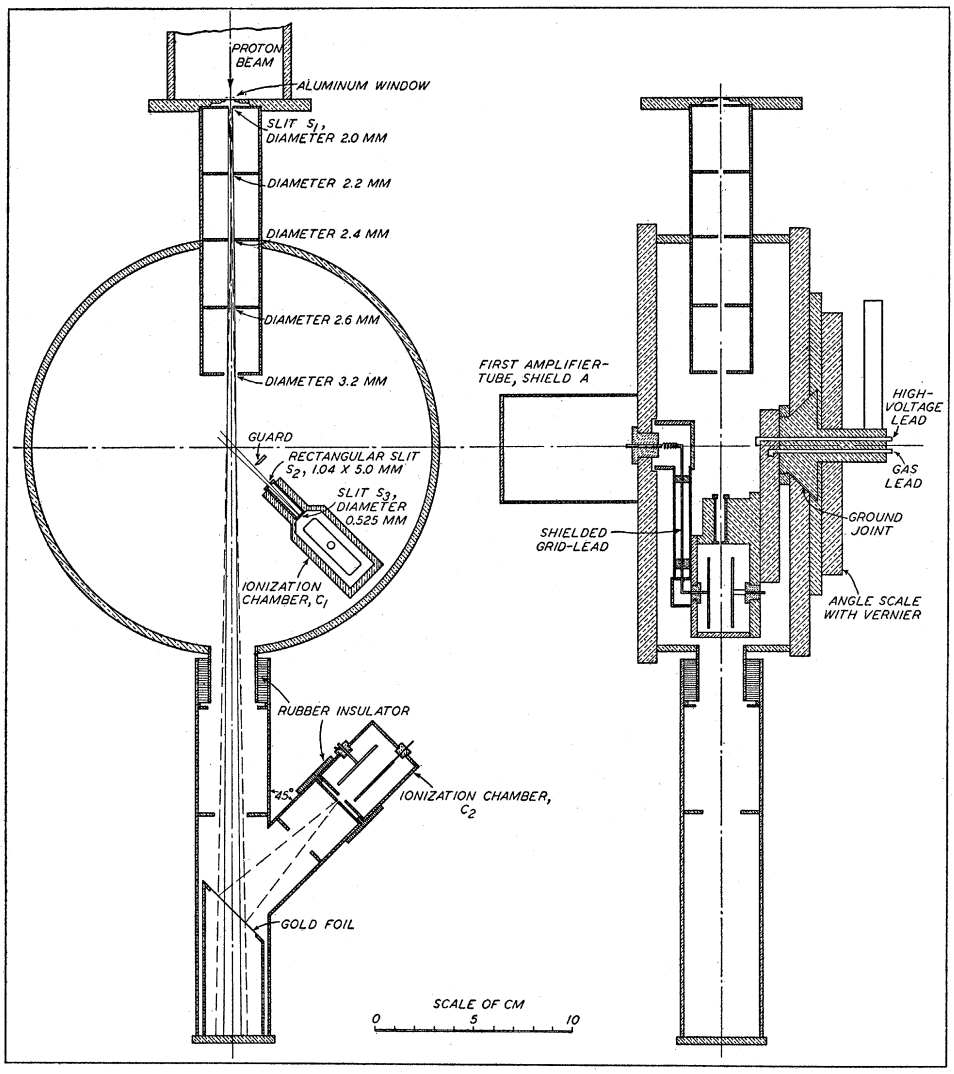

FIG. 1. Proton-scattering apparatus.

of the protons entering the scattering volume, the pressure of the gas scatterer, and the solid angles defining the scattering volume. With these quantities known, the number of protons scattered is measured at different voltages as a function of the scattering angle. Angular dispersion must be kept as small as possible and precautions must be taken to prevent spurious counts. The scattering apparatus used in this work was entirely distinct from our first scattering apparatus and was designed to overcome the difficulty of measuring the number of incident protons, which was experienced in our first work, and to give greater accuracy throughout. (This new chamber with its carefully aligned slit and rotational axes was also used in our second series of observations, made with Geiger counters.)

A diagram of the scattering apparatus is given in Fig. 1. The magnetically analyzed homogeneous proton beam from the 1200-kv electrostatic generator entered the scattering chamber through the thin aluminum window and was defined by the first diaphragm S_1, diameter 2 mm. The succeeding diaphragms were each 0.2 mm larger in diameter than the preceding one. The detector of the protons from the scattering volume of gas defined by this diaphragm-system and that of the movable detector was the ionization-chamber C_1,

the plates of which were parallel to the path of the entering protons. This chamber and the defining system attached to it could be rotated, allowing the scattering angle to be varied at will. The connection from the collector plate of the chamber passed through an insulator and a shielding tube to the grid of the first amplifier-tube located in the shielding box A outside the gas chamber. This tube was an "acorn type" (954), which was found to be very satisfactory because of its low microphonic characteristics and compactness. However, the input capacity on the grid was still too large to measure protons scattered at 45° below 650 kv.

Angular measurements

The incident proton-beam defined by the diaphragm-system and the solid angle defined by the detector slit-system intersect in space to form a "scattering volume" from which the protons may be scattered to the detector. This volume varies with the scattering angle θ and is proportional to the csc θ (neglecting very small second-order corrections). The dimensions and spacings of the detector slit-system are given in Fig. 1. The rectangular slit S_2 was accurately machined and was found to be 1.04×5.0 mm by direct comparison on a measuring engine with a standard meter bar. The hole S_3 into the detector was 0.525 mm in diameter, and the distance from the center of the scattering volume to the slit S_3 was 50,116 mm and from S_2 to S_3 was 25.024 mm. The resolution of the slit-system was approximately 2°. The thickness of the scattering volume is determined by the width of the slit S_2, and is given by (aR/S_2S_3) when the detector slit-axis is perpendicular to the incident beam, where a is the width of slit S_2 and S_2S_3 is the distance between slits S_2S_3 and R is the distance from S_3 to the center of the beam. The thickness for any angle is then given by $(aR/S_2S_3)\csc\theta=0.208\ \csc\theta$ for the above dimensions. This value neglects the small second-order corrections due to the slight divergence of the incident beam and the finite size of the hole S_3.

The detector and its slit-system could be rotated on a ground joint about an axis passing accurately through the center of the scattering volume. The accurate intersection of these axes at right-angles also was insured by the method of construction, using a good milling machine. A vernier scale which permitted readings of 0.1° was attached to the arm controlling the setting of the scattering angle. The zero-angle of the detector was first set by eye-observations with a beam of light passing through the diaphragm-systems of the incident beam and of the detector. It was then checked by observing the scattering from air at a scattering angle of 25° on either side of zero. It was found necessary to shift the zero-setting by $\frac{3}{8}$° to make the scattering from the two sides equal.

Later, when much of the data on the scattering in hydrogen had been taken, a more careful check of the zero-position was made and a residual error of $\frac{1}{6}$° was found. Since it was desirable to use the same angle-settings throughout the measurements, it seemed preferable to correct the results for the true zero-position. This was done by calculating the percentage change in scattering for each angle in the case where the results were in good agreement with the classical theory, and by finding the percentage change for $\frac{1}{6}$° graphically for each angle by plotting the scattering curve for all angles and taking increments on the tangent at each angle, when there was an observed anomaly. These corrections are given in percentages in Table II. The second column gives the percentage correction for shift of $\frac{1}{6}$° in zero, calculated from the Rutherford scattering law. The next three columns apply to the proton-proton scattering angles, and the percentages were obtained graphically from the slopes of the experimental scattering curves. The observed counts were in all cases reduced by these percentages since with an angle-setting of 20°, for example, the true scattering angle was actually $19\frac{5}{6}$°.

TABLE II. *Percentage excess scattering due to zero-correction (the last three columns are corrections for the experimental proton-proton scattering curve).*

θ	CLASSICAL $R-D$ SCATTERING	670 KV	776 KV	867 KV
20°	4.19	4.00	4.00	4.00
25	3.31	2.97	2.90	1.84
27.5	2.97	2.76	2.00	1.82
30	2.73	2.16	1.60	1.05
32.5	2.47	1.67	1.18	0.88
35	2.29	1.11	1.11	0.83
37.5	2.10	0.88	1.02	0.77
40	1.98	0.81	0.62	0.61
42.5	1.82	0.75	0.52	0.47
45	1.71	0.70	0.42	0.40

Gas pressure

The mercury manometer used to measure the gas pressure in the scattering chamber in the earlier work was replaced by an Apiezon-oil manometer. This manometer has a sensitivity 15.7 times greater than that of a mercury manometer and has the added advantage of being easier to read because of the transparency of the oil. The vacuum-arm of the manometer was continuously pumped.

It was important to be sure that no foreign gas entered the scattering chamber during the measurements. The rate of leakage was determined by pumping the chamber to the best possible vacuum, isolating it and noting the time which elapsed before appreciable scattering was observed. A check was also made on each set of measurements by repeating an observation of the scattering at 20° at the conclusion of the run. Before using a continuous-flow procedure the gas was always pumped out and renewed after 50 or 60 minutes of observation—during this time the scattering would not change by more than one or two percent. In the later series of measurements the apparatus was arranged so that a hydrogen flow was maintained through the chamber continuously during the observations. This was done by pumping on the chamber through a needle-valve which was adjusted to give the same rate of flow as the hydrogen leak into the chamber through the palladium tube. It was found in practice that a given pressure could be maintained to within a tenth of a mm of mercury, or better than one percent, with an adjustment of the needle-valve about once every five minutes. The rate of flow was adjusted to give a complete change of hydrogen every ten minutes, so that the necessity of stopping the observations to renew the gas was avoided. Tank hydrogen, purified by passing through palladium (palladium tube not contaminated with deuterium), room air, spectroscopically pure argon from Linde, and purified Linde nitrogen were used as scattering gases in this work.

Current measurement

The second ionization chamber (C_2) served to detect protons scattered at about 135° to the beam from the gold foil located in the chamber beyond the scattering volume. The number of protons scattered from the gold foil was of course proportional to the number of particles passing through the scattering volume. It was therefore possible to know the relative number of protons in the incident beam during a given measurement of the gas scattering by simultaneously counting the protons scattered from the gold foil throughout the same period. A calibration of the scattering from the gold foil when a known current impinged upon it with a vacuum in the scattering chamber permitted the calculation of absolute numbers of protons in the incident beam during the scattering experiments. This arrangement circumvented the objection to our previous work that it was impossible to measure the current in the beam simultaneously with the gas-scattering observations, because of the production of secondary electrons in the Faraday cage.

An important item which had to be taken into account for each measurement was the change in the energy of the protons between the scattering volume and the gold leaf due to the stopping power of the intervening gas. The number of protons scattered from the gold per microampere-minute with a vacuum in the scattering chamber was determined over the range of voltage to be encountered, using the insulated lower part of the scattering chamber as a Faraday cage. (Tests with a magnetic field showed that the current-readings were correct to one-half percent or less.) In any later observation with gas in the chamber the energy of the protons at the gold foil was then known from the voltage and stopping powers of windows and gas, and the proton-current passing through the scattering volume was taken directly from the curve of the vacuum Faraday-cage observations. No correction was made for the negligible proton-current lost from the beam by scattering in the gas in the chamber or the negligible gas-scattering into the gold counter. Inhomogeneity in the gold leaf could give rise to an error due to the spreading of the beam over a larger area of the gold when gas was present than when the current calibrations were made, with a vacuum in the chamber. This was checked by measuring the number of protons scattered from the gold foil (under steady conditions of beam current and voltage) first with a vacuum and then again with 1.5 mm of air in the scattering chamber. The decrease in the energy of the protons of

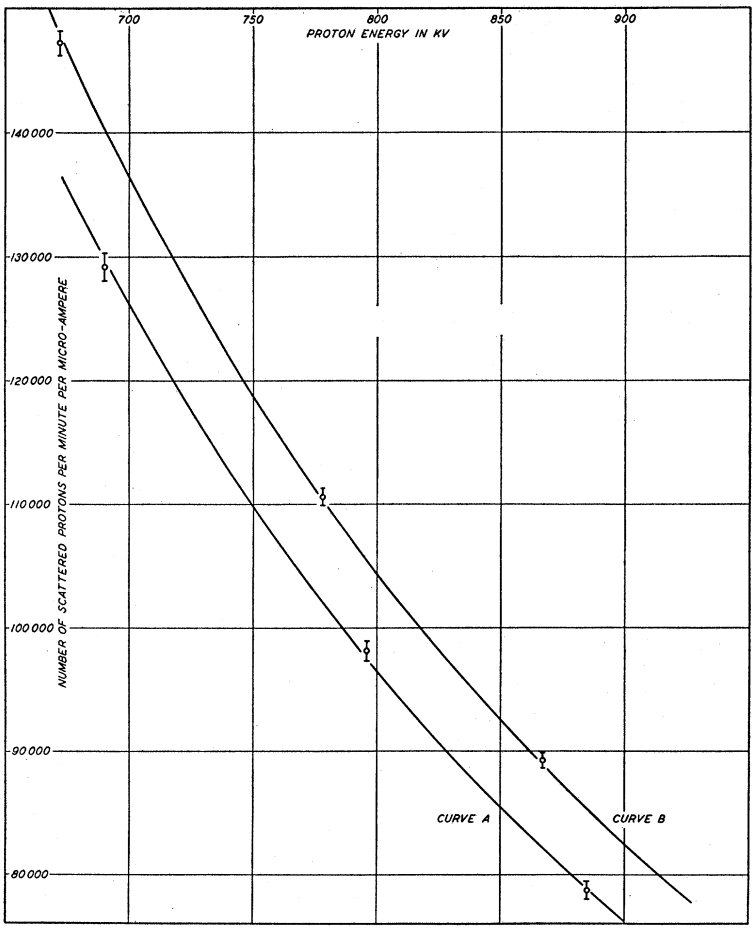

FIG. 2. Current-calibration curves, gold-foil scattering.

23 kv due to the stopping power of the added air should increase the scattering by 6.25 percent according to the classical theory of scattering. It was found that the number of counts actually did increase by 6.2 percent when the gas was added, in agreement with the theory, and indicating that the spread of the beam due to the pressure of the gas did not affect the counting rate from the gold leaf.

The gold leaf, which had a stopping power for protons of less than one mm, was mounted on a brass ring. The ionization chamber C_2 was placed to observe protons scattered at approximately 135°. The opening into the chamber was 0.5 mm in diameter, covered with a thin collodion window. The size of this hole was chosen to give a counting rate of about 2000 protons per minute with the current used in the scattering observations. As any single observation lasted for five minutes or longer, the statistical error introduced by counting only the small fraction of the total beam scattered by the gold was one percent or less. An independent amplifier and counting unit was used for the gold-foil scattering, and counts were recorded simultaneously with the counts from the gaseous scatterer.

The calibration-curves for the gold-foil scattering are shown in Fig. 2. The number of scattered protons per minute per microampere beam current is plotted as the ordinate and the energy of the proton as the abscissa. The length of the lines through the points indicates the probable error, each point representing about 25,000 counts. According to the Rutherford scattering law, which should be obeyed by gold for protons of these energies, the number of scattered par-

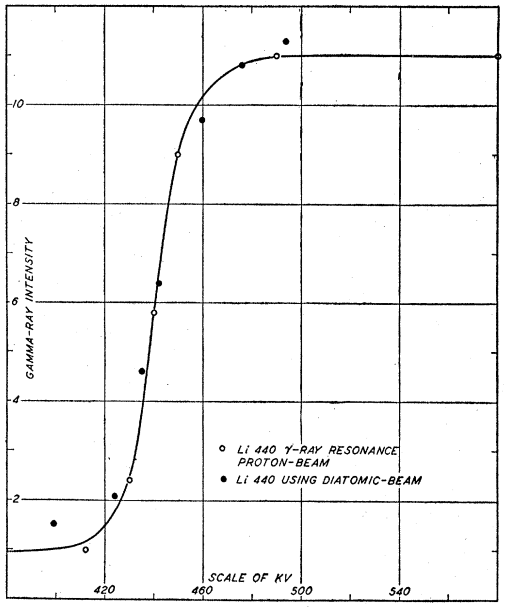

FIG. 3. Lithium gamma-ray resonance at 440 kv, using proton monatomic and diatomic beams.

ticles from the gold should vary as $1/V^2$, where V is the voltage of the incident protons. The curves in Fig. 2 are proportional to $1/V^2$ and represent the best adjustment to the points. These curves were calculated by taking the average value of the product of the square of the voltage by the observed counts at that voltage.

Curve A is the calibration curve used during the second series of scattering measurements. After this series was completed it was noticed that the counter, recording protons scattered from the gold leaf, missed an appreciable, but constant, fraction (eight percent) of the counts, due to excessive bias. This condition was corrected by adjusting the amplifier to give much larger pulses which were assuredly counted by the Thyratron-unit. Curve B was taken after this correction had been made and was used for the third series of the scattering observations. No error is introduced in the second series of measurements provided that the fraction of the counts missed remained the same during the calibration and during the measurements. Fortunately, all adjustments of the amplifier and of the Thyratron-unit remained unchanged during the calibration of curve A and the second series of scattering measurements, and evidence obtained from the series itself shows that any error introduced was very small.

At all times the counting rates of both Thyratron-units were sufficiently low that the number of counts missed through failure to resolve simultaneous counts was negligible (less than one percent). To insure that this was the case a test was made by counting with much larger currents and observing the resultant decrease in the number of counts per minute per microampere. With very large currents the number missed was proportional to the square of the counting rate so that it was possible to extrapolate back to determine safe counting rates.

Voltage measurements

In any experiment dealing with scattering of particles it is important that the energy of the incident particle be known as accurately as possible. When dealing with Coulomb scattering, an error of one percent in the voltage measurement is sufficient to introduce an error of two percent in the final result. The situation is even more serious when observing the anomalous scattering of protons by protons around 45°, where an error of one percent in voltage can cause a final error of four or five percent, due to the rapid change of the scattering anomaly with voltage. The greatest feasible precautions have therefore been taken to insure a minimum contribution from this source of error. In these experiments it is necessary to know the energy (velocity) of the proton at the scattering volume and at the gold foil. This necessitates measuring the voltage applied to the accelerating tube, the stopping power and uniformity of the aluminum window of the scattering chamber, and the stopping power of the gas traversed in the chamber before reaching the small scattering volume defined by the diaphragm-systems.

Calibration of resistance voltmeter

The accelerating voltage was measured by observing the current which passed through the corona-free high resistance unit (10,000 megohms) previously described.[7] The 440-kv gamma-ray resonance in lithium was used as the nominal standard of voltage. The measured voltage of 440

[7] L. R. Hafstad, N. P. Heydenburg and M. A. Tuve, Phys. Rev. **50**, 504–514 (1936).

kv originally assigned by us was considered to be accurate to within two percent as it was measured immediately after the original calibration of the resistance unit of the voltmeter in 200-kv sections at the National Bureau of Standards. It was necessary for the present work to recalibrate the high resistance unit and to recheck its performance in respect to the absence of corona, because the rubber tubing in which the individual unit resistors were originally enclosed had been replaced by varnished cambric tubing since the first calibration. This was accomplished by again assigning the value 440 kv to the voltage at which the lithium resonance occurred; the voltage at which the same resonance was observed when bombarding with molecular hydrogen ions was correspondingly assigned the value 880 kv. These points do not lie quite on a straight line through the origin, possibly because of heating effects in the voltmeter resistor, so a parabola drawn through these points and zero was taken as the correct calibration curve of the voltmeter. Fig. 3 shows the measurement of the lithium resonance used in the recalibration of the voltmeter. The equation of the calibration curve is

$$V=9.85\ I-0.00766\ I^2,$$

where V is in kv and I is in microamperes. The correction term in I^2 is presumably due to the heating of the resistors. To obtain strictly consistent results it was necessary that the voltage be applied to the voltmeter for about 45 minutes to insure thermal equilibrium before any measurements were made. This effect was observed directly by measuring the 867-kv fluorine gamma-ray resonance at five-minute intervals after first applying the voltage. The equilibrium reading of the current through the voltmeter resistor at the resonance voltage corresponded to a voltage approximately 20 kv lower than the initial reading. The same effect was also observed in bench measurements on individual resistor units. In all calibrations and scattering measurements the voltage was turned on at least 45 minutes previous to any observations to preclude any errors of this sort. Because of a peculiar construction of our apparatus the total accelerating voltage is measured in two parts, the voltage applied to the first two focusing gaps and the voltage applied to the rest of the tube. The agreement of the calibration of the two voltmeters was checked by observing the extremely sharp 334-kv fluorine resonance first with 100 kv applied to the first two gaps and later with 20 kv. The corresponding change in the voltage applied to the rest of the tube showed that the calibrations of the two sections were in good agreement.

The fluorine gamma-ray resonance at 867 kv is a more useful voltage reference point for the higher voltage range than the 880-kv point of lithium because of the greater intensity of the gamma-radiation. The voltage at which this resonance occurred was therefore measured in comparison with the 880-kv point of the lithium resonance and thereafter was used as a reference point. Some difficulty was encountered in this measurement due to the charging up of the CaF_2 crystal used as a target. This effect was called to our attention by Drs. Herb and Breit. It was found that the displacement of the resonance was proportional to the bombarding current and was due to the IR potential drop across the target. For one microampere of impinging protons the displacement was approximately 15 kv. The effect was avoided by covering the crystal target with a fine gauze, otherwise by using a low resistance target composed of a pasty mixture of calcium fluoride powder, water, and phosphoric

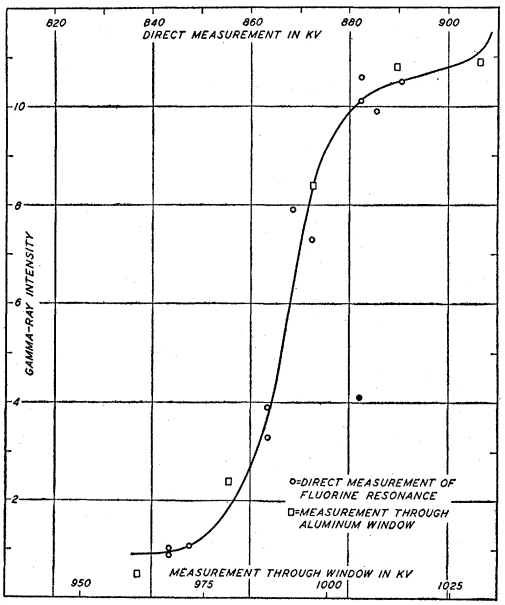

FIG. 4. Fluorine 867-kv gamma-ray resonance before and after passing through aluminum window.

acid. Our previously published[7] voltage for this resonance is accordingly too high because of this error. Using a thin target Herb and colleagues[8] assigned the voltage 862 kv to this resonance, but the best value from our thick target observations seemed to be 867 kv. Fig. 4 shows a typical determination of this resonance voltage. Throughout the scattering measurements frequent observations were made of this resonance to insure that there was no change in the calibration of the

[8] E. J. Bernet, R. G. Herb and D. B. Parkinson, Phys. Rev. **54**, 398–408 (1938).

TABLE III. *Observed scattering of protons by protons, second and third series, Washington, D. C., spring of 1938, voltage-scale based on lithium gamma-ray resonance at 440 kv.*

θ	Ratio $\times 10^3$, hydrogen to gold	Probable error, from fluctuations among data	Observed N_s from hydrogen per μa minute	N_s Corrected for zero-angle and strays	Total counts	Calculated N_s per μa minute	Ratio N_s observed to N_s Mott	Probable error of ratio
Second series: Five runs; scattering at 867 kv; 843 kv at Au foil; total tube voltage 1011 kv								
20°	235.48	±1.33	20,439	19,434	11,511	26,285	0.74	±0.004
25	83.489	2.18	7246.8	7028.8	2852	8251.5	0.85	0.02
27.5	63.252	2.18	5490.3	5379.5	2415	5013.2	1.07	0.035
30	50.569	0.85	4389.4	4340.5	2942	3184.0	1.36	0.024
32.5	43.136	0.16	3744.2	3711.8	2051	2108.8	1.76	0.03
35	40.230	0.60	3492.0	3462.6	2820	1459.0	2.37	0.04
37.5	34.969	0.30	3035.3	3013.0	2093	1058.0	2.85	0.04
40	31.583	0.11	2741.4	2725.8	2735	811.81	3.36	0.05
42.5	29.324	0.42	2545.3	2533.8	2125	665.00	3.81	0.06
45	27.232	0.41	2363.7	2354.7	2801	586.59	4.01	0.06
Second series: Four runs; scattering at 776 kv; 752 kv at Au foil; total tube voltage 924 kv								
20°	225.95	±1.46	24,674.	23,276	11,669	32,841	0.71	±0.005
25	72.412	0.45	7907.3	7574.5	3352	10,310	0.73	0.008
27.5	51.828	1.59	5659.6	5508.8	2092	6263.7	0.87	0.03
30	40.230	0.60	4393.1	4324.1	1935	3977.9	1.09	0.016
32.5	32.847	0.31	3586.9	3546.2	1874	2634.3	1.35	0.02
35	26.636	0.65	2908.7	2877.0	2074	1821.4	1.58	0.04
37.5	23.330	0.30	2547.6	2521.7	1931	1341.3	1.88	0.03
40	21.639	0.22	2362.9	2349.3	1936	1013.5	2.31	0.03
42.5	19.527	0.56	2132.3	2122.7	1878	830.25	2.55	0.07
45	18.474	0.23	2017.4	2009.0	1985	732.24	2.75	0.04
Second series: Three runs; scattering at 670 kv; 640 kv at Au foil; total tube voltage 827 kv								
20°	205.13	±3.40	30,934	29,437	8134	44,119	0.67	±0.011
25	58.068	2.47	8756.6	8493.8	2376	13,850	0.61	0.008
30	27.428	0.16	4136.1	4032.6	1016	5342.5	0.75	0.014
32.5	20.866	0.67	3146.6	3099.4	967	3537.5	0.87	0.02
35	16.356	0.20	2466.5	2440.4	1439	2445.1	0.99	0.02
37.5	13.807	0.28	2082.1	2064.9	1451	1773.0	1.17	0.02
40	12.095	0.05	1823.9	1809.4	1474	1359.7	1.33	0.02
42.5	11.347	0.06	1711.1	1699.3	1445	1113.8	1.52	0.03
45	9.844	0.19	1484.5	1474.9	1330	982.22	1.50	0.03
Third series: One run; scattering at 867 kv; 843 kv at Au foil; total tube voltage 1011 kv								
20°	215.1	....	....	19,276	17,822	....	0.73	....
25	78.95	....	....	7220	4048	....	0.87	....
30	48.26	....	....	4498	2760	....	1.41	....
35	36.29	....	....	3388	1806	....	2.32	....
40	28.75	....	....	2694	1673	....	3.32	....
45	24.73	....	....	2321	1503	....	3.96	....
Third series: One run, scattering at 776 kv; 752 kv at Au foil; total tube voltage 924 kv								
20°	204.7	....	....	22,991	15,944	....	0.70	....
45	16.81	....	....	1982	503	....	2.71	....
Third series: One run; scattering at 670 kv; 640 kv at Au foil; total tube voltage 827 kv								
20°	193.3	....	....	30,956	6288	....	0.68	....
45	8.81	....	....	1468.6	110	....	1.46	....

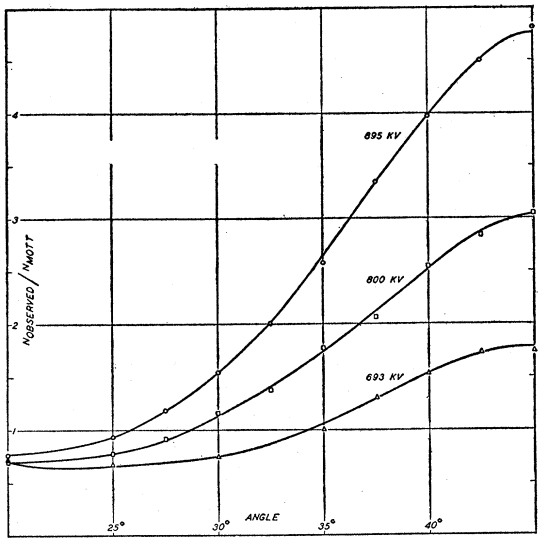

FIG. 5. Proton-proton scattering, first series of observations.

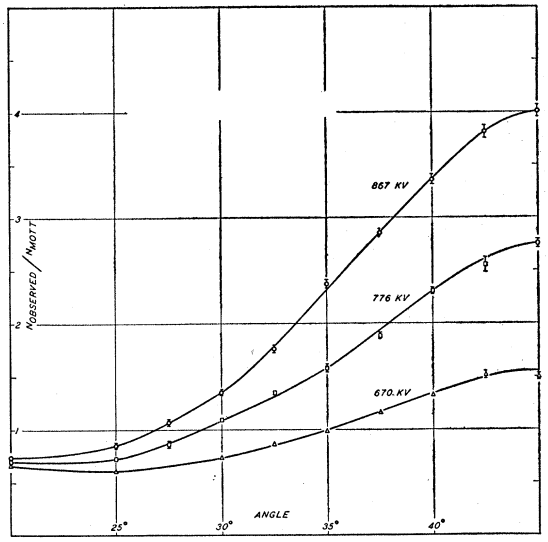

FIG. 6. Proton-proton scattering, second series of observations.

voltmeter unit. All values of voltage in this paper refer to the nominal voltage scale based on assigning the value 440 kv to the lithium resonance but, as will be noted below, the absolute calibration of this voltage scale in terms of the Coulomb scattering of protons by spectroscopically pure argon gas shows that our nominal scale is correct to within about one percent.

Stopping power of window and gas

The stopping power of the aluminum foil through which the protons entered the scattering chamber was measured by further observations of the fluorine resonance. For this purpose a calcium-fluoride crystal was placed inside the scattering chamber and bombarded by protons which passed through the aluminum window. The difference in the observed resonance voltages as measured inside and outside the scattering chamber, with due precautions to avoid charging up of the crystal, is due to the stopping power of the window. The range-curve of Parkinson and associates[9] for protons in aluminum was then used to determine the thickness of the aluminum window. When using protons of any other energy the same range curve was used to give the stopping power of the window at the other voltage. This measurement had to be repeated from time to time to measure the change in the stopping power of the window due to accumulation of carbon. As this deposit of carbon never became greater in thickness than 15-kv stopping power and the aluminum windows were approximately 100-kv stopping power, it was not necessary to calculate the stopping powers separately. The current reaching the CaF_2 target at the bottom of the scattering chamber was always so small that the charging effect of the crystal previously mentioned was not observable. Fig. 4 shows fluorine resonance curves as observed inside and outside of the scattering chamber.

The stopping power of the gas in the scattering chamber was calculated from the known pressure and length of path. Herb's range curve for protons in air and Bethe's[10] data for the relative stopping powers of the various gases were used.

For the stopping power of hydrogen we used 0.24 times the stopping power of air, for nitrogen and oxygen the same as air, and for argon 1.92 times that of air. In obtaining the voltages at the scattering volume and at the gold foil some inaccuracy is introduced because of the difficulty in reading the voltages from the range curve. This could introduce an over-all error in the final scattering results of about one percent.

[9] D. B. Parkinson, R. G. Herb, J. C. Bellamy and C. M. Hudson, Phys. Rev. **52**, 75–79 (1937).

[10] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 272 (1937).

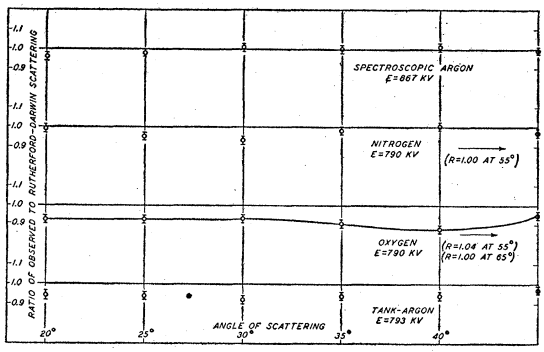

FIG. 7. Proton scattering in argon, nitrogen and oxygen.

Method of reduction of experimental data

The experimental data consist of a certain number N_H of scattered protons from the gas target for a given number N_G of scattered protons from the gold foil, the latter being a measure of the incident beam current. The ratio (N_H/N_G), if multiplied by the conversion factor from gold-scattering counts per minute to microamperes (Fig. 2), will then give the number which can be compared to the Mott value N_s as given in Table III. This conversion factor is read directly from the gold foil calibration curves in Fig. 2. The voltage to be used at the gold foil is lower than that at the scattering volume, of course, by the loss of voltage due to the intervening gas.

V. RESULTS

After the first set of observations of this group were made on proton-proton scattering, it was discovered that the microammeter used to measure the current through the voltmeter resistor between the concentric spheres of the electrostatic generator had changed its calibration some time during the period the data were being taken. When recalibrated, the meter read 12 percent low. This would amount to a correction of a little over one percent in the total voltage on the tube. This meter had not been properly protected against transients when sparking occurred on the generator. We are, therefore, uncertain as to the exact energy at which these curves were taken and hence the absolute values of the ratios of the observed counts to the Mott-theory scattering do not quite agree with the data taken later. However, the curves, as shown in Fig. 5, have the same general contour and therefore add weight to the later observations. Each curve is the average of from two to three independent runs. The probable error based on the spread of the data is given by the lines through the points on the curves.

After the first set of observations described above, the aluminum window and gold foil were accidentally damaged due to a sudden burst of air into the scattering chamber. New aluminum and gold foils were then installed and calibrated. This aluminum window had a stopping power of 113 kv. The calibration curve for the scattering of protons from this gold foil is given in Fig. 2; curve A has been used for the second series of observations and curve B for the third series as discussed in Section II. The gold foil curve for the first series is not shown.

Table III gives rather completely the data for the second and third series of observations on proton-proton scattering. In this table we have included the initial voltages of the protons, their voltage at the scattering volume and at the gold foil.

The observed ratio of hydrogen to gold scattering given in column 2 of Table III is the average of three to five independent runs; a new filling of

TABLE IV. *Proton scattering in oxygen and nitrogen.*

θ	Ratio $\times 10^3$, gas to gold	N_s corrected for zero-angle	Total counts	Calculated $N_s(R-D)$	Ratio N_s observed to $N_s(R-D)$	Probable error (statistical)
		Oxygen: 790 kv; 780 kv at Au foil				
20°	1792.4	177,700	7,144	190,518	0.93	0.014
25	594.2	59,470	14,672	63,880	0.93	0.014
30	245.0	24,680	2,565	26,406	0.93	0.014
35	113.7	11,500	1,662	12,632	0.91	0.014
40	58.59	5,943	1,115	6,735.7	0.88	0.02
45	36.41	3,705	971	3,906.9	0.95	0.02
55	16.13	1,652	278	1,581	1.04	0.03
65	7.59	777	187	777.1	1.00	0.04
		Oxygen: 684 kv; 669 kv at Au foil				
25°	601.3	80,540	18,480	85,212	0.95	0.014
		Oxygen: 882 kv; 870 kv at Au foil				
25°	629.6	50,540	21,656	51,247	0.99	0.014
		Nitrogen: 684 kv; 669 kv at Au foil				
20°	1454	144,200	6,184	145,864	0.99	0.014
25	461.9	46,220	13,900	48,908	0.95	0.014
30	185.9	18,720	3,353	20,217	0.93	0.014
35	93.31	9,435	1,980	9,672.0	0.98	0.014
40	50.98	5,169	1,442	5,157.1	1.00	0.02
45	28.45	2,894	885	2,991.1	0.97	0.02
55	11.79	1,208	343	1,211.0	1.00	0.03
		Nitrogen: 790 kv; 780 kv at Au foil				
25°	445.3	59,630	14,568	65,240	0.91	0.014
		Nitrogen: 882 kv; 870 kv at Au foil				
25°	485.5	38,960	13,816	39,236	0.99	0.014

hydrogen was used for each run. In column 5 labeled "Observed N_s, corrected for zero-angle and strays" we have applied the percentage correction for the shift in the zero position as discussed in Section III and given in Table II. The stray-count correction, due to the counting rate when the scattering chamber is evacuated, amounted to about one percent of the counting rate with the hydrogen in the chamber. The ratios of the observed N_s to that calculated from the Mott formula of Section II are given in column 8 and are plotted in Fig. 6.

After completion of the second group of observations, some improvements were made on the amplifier which counted the protons scattered from the gold foil. The oscillograph then showed that the pulses were about five times the noise level, thus removing any doubt of their being recorded on the Thyratron counter. At this stage it was decided to recalibrate the gold foil and to repeat some points on the proton-proton scattering curves. The results on this third set of observations are given in Table III. It is seen that the agreement with the second series is good. The entire curve at 876 kv was checked but only the points at 20° and 45° for the other two voltages. This agreement with the results of the second series indicates that they were reliable even though the gold-foil scattering counter was missing a few counts (eight percent). The number missed must have remained approximately constant during the calibration of the foil and the scattering measurements. This is reasonable since the amplifier gain was adjusted, by means of a gain control, before (and checked after) each run to give the same grid bias setting on the selector tube of the scale-of-eight counter for the amplifier noise level. The grid bias was then set a fixed amount above this noise level.

During the course of the second series of observations we tried the scattering of protons from nitrogen and oxygen. If the scattering from these gases proved to behave classically it could be used as a check on the absolute calibration of the scattering apparatus and voltage scale. These data are given in Table IV and the ratio of the observed scattering to that predicted from the Coulomb scattering law is plotted in Fig. 7. The

TABLE V. *Proton scattering in spectroscopically pure argon.*

θ	Ratio $\times 10^3$, argon to gold, $p=1.00$ mm	N_s corrected for zero-angle	Total counts	Calculated $N_s(R-D)$ per μa minute	Ratio N_s observed to $N_s(R-D)$	Probable error (statistical)
		867 kv, 858 kv at Au foil				
20°	4404	383,250	2,960	397,951	0.96	0.018
25	1487	130,503	77,530	133,438	0.98	0.004
30	630	55,581	10,800	55,160	1.01	0.01
35	300	26,561	6,504	26,388	1.00	0.012
40	160	14,239	4,040	14,070	1.01	0.016
45	90.5	8,075	1,544	8,161	0.99	0.026
		781 kv, 773 kv at Au foil				
25	1531	166,287	17,128	164,435	1.00	0.01
		683 kv, 673 kv at Au foil				
25	1505	215,539	17,902	215.019	1.00	0.01

deviations of these ratios are larger than can be accounted for by experimental errors. Oxygen shows an appreciable dip in the variation of the scattering with angle at 40°. The variation of both oxygen and nitrogen scattering with energy is not strictly in agreement with classical theory and at present is not explained except as a true scattering anomaly. The scattering from argon was then tried. It is of course much more to be expected that argon gives rise only to Coulomb scattering, because of the larger nuclear charge. The ratios of the observed to Rutherford-Darwin scattering for tank argon and spectroscopically pure Linde argon are given in Table V and are plotted in Fig. 7. The scattering for spectroscopic argon agrees in absolute value with the classical theory using the voltage scale discussed above in Section III, in which the lithium gamma-ray resonance was given the value 440 kv. The tank argon is seen to be about five percent lower than the spectroscopic argon; this is probably due to air contamination because this argon had been stored for some time in a laboratory tank at about atmospheric pressure.

The theoretical significance and discussion of our results has already been given by Breit, Thaxton and Eisenbud.[5]

We wish to express our indebtedness to Dr. J. A. Fleming for his continued support of this program. We also want to acknowledge the important part taken in this work by R. C. Meyer, who designed and constructed the scattering apparatus.