We can now define the absolute thermodynamic temperature by the equation

$$\gamma = 1/kT$$

and verify without difficulty that it has the usual properties of thermodynamic temperature. This justifies the statement following Eq. (3).

As a final point in the argument one can derive the second law of thermodynamics from the statistical theory in the following manner. Let us assume that we have a self-acting cyclic mechanism which can pump heat out of a cold reservoir and supply it to a hot reservoir. We can then suppose that we have an ensemble of cold reservoirs in thermodynamic equilibrium at temperature T_2 and a similar ensemble of hot reservoirs in thermodynamic equilibrium at temperature T_1. Let the mechanism be used to pump heat from each cold reservoir to the corresponding hot reservoir, the combination being isolated. Then entropy decrease of the cold reservoirs would exceed the entropy increase of the hot reservoirs. As the mechanism and working fluid return cyclically to the same condition they contribute no entropy change. Thus the sum of the entropies of the parts of the combination system increases in time contrary to the principle of increase of entropy for such cases as stated at the end of the preceding section. Hence the supposed mechanism is contrary to our statistical theory and we have derived the second law of thermodynamics.

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A Precision Determination of the Viscosity of Air

J. A. Bearden
Department of Physics, Johns Hopkins University, Baltimore, Maryland
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The rotating cylinder method has been used for making a precise determination of the viscosity of air. The inner cylinder was rotated, which made possible a higher degree of accuracy in the construction of the apparatus and in its adjustment than was possible if the outer cylinder had been rotated. Two rotating cylinders were used, and the error introduced into the viscosity by inaccuracies in machining and measurements was for the best cylinders 2.01×10^{-3} percent. Torsion wires were studied in detail and a method of treatment used which removed the defects of zero drift and variability of torsion constant for static and dynamic conditions. A precision telescope and calibrated comparator were used in measuring the angular deflection of the deflecting cylinder. The timing and driving mechanism consisted of a tuning fork controlled d.c. motor whose speed was constant within one part in 10^6, as checked by N.A.A. time signals. The viscosity of air at 20.00°C is $(1819.20\pm0.06)\ 10^{-7}$ c.g.s. unit.

Introduction

Redeterminations[1–5] of the viscosity of air during the past two years have shown that the major part of the discrepancy between Millikan's oil-drop value of the electronic charge and that obtained by x-rays[6] was primarily due to an error in the viscosity measurements. Not only are the oil-drop and x-ray methods intrinsically the most accurate available from the standpoint of experimental difficulties, but neither, fortunately, requires a knowledge of Planck's constant h. Since the electronic charge is one of the most important constants in physics, it is essential that its evaluation be made by both methods with the highest precision attainable with modern experimental technique.

The experimental value of the viscosity of the gas surrounding the oil drop enters the final calculation of e to the three-halves power. This is the most important and most difficult measure-

[1] W. N. Bond, Proc. Phys. Soc. **49**, 205 (1937).
[2] G. Kellström, Phil. Mag. **23**, 313 (1937).
[3] W. V. Houston, Phys. Rev. **52**, 751 (1937).
[4] G. B. Banerjea and B. Plattanaik, Zeits. f. Physik **110**, 676 (1938).
[5] P. J. Rigden, Phil. Mag. **25**, 962 (1938).
[6] J. A. Bearden, Phys. Rev. **47**, 883 (1935). Jesse W. M. DuMond, Phys. Rev. **56**, 153 (1939), reference 17.

ment that has to be made in the evaluation of e by this method. This measurement has been undertaken first, and the present report is entirely on this phase of the problem.

Experimental Method

The only methods available for making accurate measurements of the viscosity of a gas are: the rotating cylinder method and the flow through a capillary tube. The latter method is experimentally much simpler than the rotating cylinder method, but the numerous corrections which must be made make the results unreliable for precision measurements. There are also theoretical questions concerning the velocity gradients near the ends of the capillary which have not been completely solved. The rotating cylinder method possesses the distinct advantages of precision in the mechanical construction and freedom from theoretical objections. This method has therefore been chosen for the present experiments.

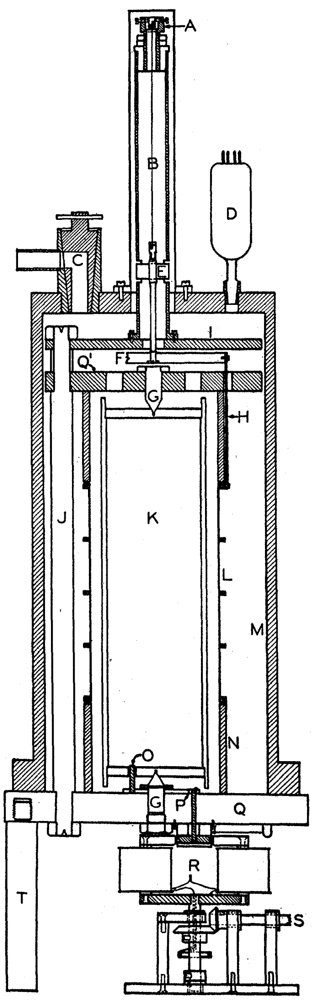

FIG. 1. Cross section of apparatus. A, adjustable torsion wire support; B, torsion wire; K, inner cylinder rotating on centers G; L, suspended cylinder; N, guard cylinders; M, bell jar; R, magnetic drive.

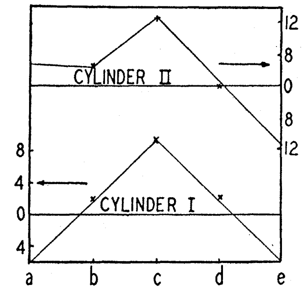

FIG. 2. Diameters of cylinders plotted as a function of their length. The ordinates are in cm$\times 10^6$.

The design finally adopted for the apparatus differs mainly in one respect from that used by previous experimenters. In this work the constantly rotating cylinder was made the inner cylinder and the deflecting cylinder was on the outside. This method has four important advantages: first, it is possible to machine the external diameter of a long cylinder with much greater precision than can be accomplished on the internal diameter of a cylinder of the same length. The deflection cylinder is short, permitting its internal diameter to be accurately machined without difficulty. Second, the measurements of the diameters and lengths can be made with high precision. Third, the assembly and adjustments can be made easily and accurately. Fourth, the rotating cylinder can be rotated on centers.

It is obvious that the theoretical expression for the viscosity is exactly the same for the present

TABLE I. *Diameters of rotating cylinders.*

Position	Cylinder I Diameter	Cylinder II Diameter
a	12.44218	13.57041
b	12.44226	13.57040
c	12.44233	13.57048
d	12.44226	13.57035
e	12.44218	13.57011
Average	12.44224	13.57035

TABLE II. *Measured diameter of suspended cylinder.*

Position A	14.95296 cm
Position B	14.95302
Position C	14.95295
Average	14.95298±0.00050 cm

arrangement as for the previous one. This expression may be written

$$\eta=\frac{F}{8\pi^2}\frac{b^2-a^2}{La^2b^2}\varphi t,$$

$$\eta=KF\varphi t/8\pi^2 \quad \text{where} \quad K=(b^2-a^2)/La^2b^2, \quad (1)$$

where b is the radius of the outer suspended cylinder, a is the radius of the inner cylinder, L is the effective length of the suspended cylinder, ϕ is the angular deflection of the suspended cylinder, F is the torsion constant of the suspension, t is the time of revolution of the inner cylinder. K may be considered a constant of the apparatus unless the spacings between the guard rings and suspended cylinder are altered. The effective L for the present apparatus is equal to the length of the suspended cylinder plus one-half the sum of the spacings between the guard rings and cylinder.

Construction and Dimensions of Viscosity Apparatus

A low density, strong, machinable, nonmagnetic metal was essential for the rotating and suspended cylinders and for the guard rings. It was also necessary that the material could be easily freed of strains so it retained its shape after machining and grinding. It was found that a magnesium alloy, Dow metal J, fulfilled the requirements better than any other available material.

Previous interferometer experience with high precision spectrometer bearings and spectrometer shafts mounted on centers showed that the method of centers for the mounting of the constantly rotating cylinder is much superior to the usual precision cone bearing. The cylindrical surface could be given its final finishing while rotating on the centers later used in the measurements. Failure of the cylindrical surface to rotate about its true center is doubtless an appreciable source of error in former work. The design of the apparatus used in the present work is shown schematically in Fig. 1.

Two inner rotating cylinders K were constructed of different diameters. In each case the inside of a Dow metal tube was machined to a uniform diameter except at the ends where it was enlarged to receive a metal disk. The wall thickness was about 1.1 cm. The finished disks contained the centers, two holes each for air to pass freely in and out, and a hole 0, Fig. 1, for the driving pin. These disks were pressed in and secured by Dow metal pins. The outside surface was rough machined (within 0.001 inch) and then the entire cylinder was annealed to remove all strains. Afterwards the outside surface was finished on a cylindrical grinding machine in which the cylinder rotated on dead centers and water was flowed over the cylinder to maintain it at constant temperature.

The diameter of each cylinder was measured at five positions along its length and around its circumference by the National Bureau of Standards. Measurements were made with different pressures on the contact surfaces and the final result obtained by extrapolating to zero pressure. All measurements were made at 20.00°C and this same temperature has been maintained in all the experimental work, thus avoiding any correction for expansion or contraction. The results of the measurements on the two cylinders are given in Table I. Positions a and e are at points corresponding to the ends of the suspended cylinder L Fig. 1; b, c, and d are equally spaced between a and e. The result for each point is an average of the diameters along different directions. These diameters differed by only one part in 10^6.

In Fig. 2 the diameters of the cylinders are plotted as a function of their lengths. The diameters are greatest at the center of the cylinders and taper to a smaller diameter at each end.

TABLE III. *Diameter of suspended cylinder by volumetric method.*

Trial 1	4573.831 cm³
2	0.807
3	.843
4	.783
5	.779
6	.807
Average	4573.808±0.006
Diameter = 14.95289±0.00005 cm	

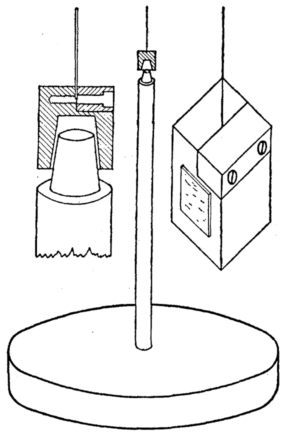

FIG. 3. Lower clamp for the suspension wire and the ground joint for attaching it to the rod which holds the inertia disk. Inertia disk and supporting rod.

Cylinder I is better than II and for this reason has been used in most of the measurements. The value of a which should be used in Eq. (1) is $(n^{-1}\Sigma a^2)^{\frac{1}{2}}$, but the variations are so small this is exactly the same as the average value which is given in Table I. The probable error can be estimated from the irregularities of the curves in Fig. 2 plus the errors in the gauges used in measuring the diameters. For cylinder I this amounts to ± 0.00002 cm, and for cylinder II ± 0.00004 cm.

The suspended cylinder L was designed with five sustaining ribs, as shown in Fig. 1. This gave the cylinder sufficient strength to retain its accurate dimensions and was also light enough to be supported by a small torsional wire. The end ribs were made larger than the remaining three in order that they could be used for supporting the cylinder on a precision face plate for final finishing.

The cylinder was machined from a well-annealed Dow metal tube. It was then annealed again and its diameter changed in different places by as much as 0.007 cm. The cylinder was remachined, removing only sufficient material to make it true. Again the annealing distorted the cylinder. The process of machining and annealing was repeated with a higher annealing temperature. Successive attempts finally located the best annealing temperature and thence the cylinder retained its dimensions accurately. The final machining removed less than 0.002 cm of material from both outside and inside after the last annealing.

The length and diameter of this cylinder were measured by the National Bureau of Standards. The length is

$$L = 26.04583 \pm 0.00008 \text{ cm}.$$

The only method available for measuring the inside diameter must make use of spherical contact points. The contact pressure for precise measurements is such that with the present metal slight indentations were undoubtedly made. The pressure was prevented from warping the cylinder by applying an equal pressure on the outside directly in line with the internal pressure. The average results for the three positions are given in Table II. These results are very consistent, but inside diameter measurements are difficult, and correction for the contact indentation cannot be made accurately. The $\pm$ error given in Table II is a limiting error of the measurement and not a probable error.

The length of the cylinder was accurately known, so a determination of the volume of the cylinder was used as a more precise method of determining the above diameter. The cylinder was clamped to a flat (ground) Dow metal plate. A fine ground flat glass plate was used as a top for the cylinder. The cylinder with ends was weighed on a large balance in a constant temperature room at 20.00°C. The cylinder was then filled with distilled water at the same temperature and reweighed. Buoyancy corrections for air were made and the weights were calibrated by the National Bureau of Standards. The density of water at 20.00°C was taken as

$$\rho = 0.998232,$$

which is given in the *International Critical Tables*. Landolt and Börnstein give 0.998230 which differs by only two parts in 10^6 from the above value. The resulting values for seven independent determinations are given in Table III. These measurements give a diameter in good agreement with that of Table II. A weighted average of the two results has been used for calculating b^2 and the probable error has been taken as that of Table III.

$$2b = 14.95290 \pm 0.00005 \text{ cm}.$$

The guard cylinders N were made with an inside diameter the same as the finished diameter of the suspended cylinder. The outside diameters were 0.0005 cm larger than the outside diameter of the ribs on the suspended cylinder. The inside and outside cylindrical surfaces were concentric with the centers which supported the rotating cylinder. The inside edges of the guard cylinders and suspended cylinder were cut with sharp right angle edges.

In constructing the guard cylinders, the steel base Q and the secondary base Q' were first rough machined and all holes drilled and tapped, including those (not shown) for attaching the guard cylinders. The top and bottom surfaces were then finished plane and parallel. The two plates were then bolted together and the holes for the centers G and three supporting rods J were carefully bored perpendicular to the surfaces with a boring mill. Only a small amount of metal was removed in this finishing process in order to reduce warping to a minimum. The lower guard cylinder whose end had previously been finished was fastened permanently to the base Q with eight machine screws. The base was then bolted to the lathe face plate which had just previously had a finishing cut taken over its surface making it accurately perpendicular to the axis of the lathe spindle. A tight fitting ground rod was inserted in the center hole G and was used to accurately center the base Q on the lathe axis. The center G was inserted and its cone, together with the inside and outside diameters of the guard cylinders, were finished without disturbing the position of the work on the face plate. The final 0.0025-cm cuts were taken without letting the lathe spindle stop, in order to avoid changes in oil films on the bearings. The top guard cylinder was machined in exactly the same manner as the bottom one.

The secondary steel base Q' was supported by three posts J. These were made of steel and were 28 mm in diameter except at the ends, where they were 20 mm. The rods were carefully machined on centers, and particular care was taken to make the small diameters so the rods could just be pushed into the holes in the bases. A special device was arranged to make the length between the shoulders of all three rods exactly the same. This distance determines the separation and parallelism of the guard cylinders.

A shelf I was necessary for supporting the torsion head assembly. A steel disk with parallel sides was supported above Q' by three cylinders of equal length slipped over the rods J.

The spider F which supported the suspended cylinder was milled from a finished Dow metal disk containing a central hole bored perpendicular to the surfaces. This hole was used as a guide in milling away the material to leave three arms 120° apart and the center of gravity exactly in the center of the hole. Similar Dow metal strips H and screws were used for connecting the spider to the suspended cylinder L.

The diameter of the rod where the mirror E was mounted was enlarged and one-half of the rod milled away, so that the reflecting surface of the mirror was on the axis of rotation. The lower end fitted snugly into the spider and the top end was tapered to fit the torsion wire clamp as shown in Fig. 3. The size of rod and mirror were adjusted to keep the center of gravity as near the axis of the rod as possible.

The torsion wire clamps were found to affect the characteristics of the torsion wire B considerably. If the wire was not clamped correctly, its damping constant was increased, the zero position was not stable, and the period of oscillation was not constant. These effects introduced errors of the order of one part in 10^4, which were entirely too great for the present work.

The final design of the lower clamp is shown

TABLE IV. *Separation of guard rings.*

Position	Separation
1	26.0760 cm
2	26.0763
3	26.0761
4	26.0765
5	26.0766
6	26.0764
7	26.0764
8	26.0762
9	26.0763
10	26.0760
11	26.0764
12	26.0767
13	26.0765
14	26.0762
15	26.0763
16	26.0765
Average	26.07634 ±0.00020

in Fig. 3. An ideal clamp would be a cylindrical hole of exactly the size of the wire such that the wire is held rigidly up to the point where it leaves the holder. This was approximated by cutting two 90-degree grooves in the flat clamping surfaces to such a depth that when clamped together with the wire between them, the surfaces were separated by about 0.001 cm. The grooves were accurately machined so that the wire was concentric with the mirror rod. The tapered joint made it possible to remove the wire and clamp from the mirror rod, permitting the torsional constant of the wire to be measured in a separate apparatus.

The upper torsional wire clamp A was made similar to the lower except that no tapered joint was necessary. This support was made so that it could be moved horizontally in any direction, rotated about its vertical axis or adjusted as to height. When it was necessary to remove the torsion wire for calibration, the tapered lower joint was loosened from the mirror rod and then the wire with the entire upper assembly was raised out of the supporting brass tube.

A magnetic drive R rotated the inner cylinder. This allowed the cylinder to be rotated in vacuum if desired and also made it possible to evacuate the apparatus and fill it with the desired gas without disturbing any of the driving mechanism.

A soft iron rod $1.0\times1.0\times4$ cm was attached perpendicularly to the shaft which rotated the spur gear P, Fig. 1. This drive was enclosed in a vacuum-tight brass housing U. The soft iron bar was driven by the electromagnet R. The weight of the coils was balanced so in rotating they caused no perceptible vibration of the apparatus. The 0.1-ampere exciting current was carried through a small armature on the lower part of the rotating shaft. The driving was through S and the two-to-one reduction bevel gears.

An 1800-r.p.m. d.c. motor was connected to S Fig. 1 through a reduction and ratio changing gear system. A 100-to-one worm gear reduced the motor speed to 18 r.p.m. Spur gears were used to change the 18-r.p.m. shaft speed in 12 steps from 3 r.p.m. to 108 r.p.m. An arrangement of four spur gears was used which allowed the shaft S Fig. 1 to be rotated in either direction. The inner rotating cylinder could be rotated in either direction at 12 speeds, ranging from one revolution in 6.67 seconds to one revolution in 240 seconds.

A test of the possible effect of the rotating magnetic drive on the suspended cylinder was made. A Celluloid cylinder was placed between the inner cylinder K Fig. 1 and the suspended cylinder L such that it did not touch either. The inner cylinder was rotated at various speeds with the magnetic drive, but no deflection of the suspended cylinder was observed. The current through the magnetic coils was increased to 1.0 ampere and no deflection was observed. A deflection could have been observed which would have caused an error in the final viscosity measurements of one part in 5×10^4.

The bell jar M was made of cast Dow metal and painted on the outside with Glyptol. A mercury metal diffusion pump with a speed of 10 liters per second was used for evacuating the apparatus through the large metal stopcock C Fig. 1. An ionization gauge D and a Piranni gauge (not shown) were used for measuring the low pressure, and a mercury-in-glass manometer for pressures up to one atmosphere. Apiezon Q was used for sealing all joints. Gases were admitted through a needle valve (not shown) screwed into the base Q Fig. 1. The vacuum usually attainable was about 10^{-5} mm, and when the stopcock C was closed the pressure did not increase to more than 10^{-4} mm in 12 hours.

A heavy steel subbase $2.5\times50\times61$ cm mounted about 15 cm from the floor was used as a base upon which to mount the viscosity apparatus. The base was supported on four steel rods 32 mm in diameter and 60 cm long. These rods were rigidly secured to the concrete floor of the room with nuts and washers above and below the floor. The steel base was also fastened by nuts which allowed it to be leveled.

Temperature Control

The measurements of the important dimensions were all made at a temperature of 20.00°C. In order to avoid expansion corrections, this temperature was chosen for the room temperature in which all measurements have been made.

The normal temperature of the room used is about 21 to 22°C. In the winter months the necessary cooling was obtained by admitting air from the top of the building through a small shaft that could be connected to the room. A blower type fan operated by a thermostat admitted cold air as needed. The speed of the blower and electrical heating were adjusted to give the optimum heating and cooling cycle. A recording clock was used to keep the heating and cooling cycle adjusted.

In the summer months an ammonia cooling system was used to maintain a 300-gallon tank of water at about 10°C. The tank and ammonia plant were in a room underneath the constant temperature room. The cold water was circulated through an auto radiator and a blower, controlled by the thermostat, circulated air through the radiator as needed.

A thermostat was needed which would respond very quickly to small changes in air temperature. This was attained by making a thermostat whose expanding liquid was xylol in a copper tube 6 mm in diameter and 13 meters long, coiled into a helix of 24 turns. On one end of the copper tube a small sylphon bellows was attached which permitted accurate adjustment of the thermostat. A glass capillary U-tube with electrical contacts was sealed to the other end. Mercury was used for closing the contacts. A 0.1°C change in air temperature opened or closed the contacts in about 20 seconds. A current of 6 ma at 6 volts operated a low wattage relay which in turn operated a mercury relay in the blower circuit. Four fans were used to circulate the air at a sufficient rate to maintain the same temperature in all parts of the room.

The heating and cooling cycle gave rapid changes in temperature of about 0.1°C. The temperature as read on a Beckman thermometer inside the thick-walled bell jar of the apparatus varied less than 0.01°C in 24 hours. Since practically all pieces of apparatus used were massive, their average temperature variations were less than 0.01°C.

TABLE V. *Mirror to wire distance.*

Date	Distance
November 22, 1937	2784.37 mm
March 26, 1938	2784.38
October 11, 1938	2784.37
January 3, 1939	2784.38
February 7, 1939	2784.39
Average	2784.38 mm
Window correction	2.94
Effective mirror-to-wire distance	2781.44 mm

Assembly and Adjustment of Apparatus

In order for the suspended cylinder to be accurately concentric with the guard rings and rotating cylinder, it was necessary to make the base Q accurately horizontal. This was accomplished by assembling the lower part of the apparatus and firmly fastening it to the subbase (not shown) and then leveling with a high precision level by raising or lowering the clamping nuts which held the subbase to the concrete floor. A few hours were allowed for the temperature of these parts to return to normal and the leveling was further adjusted. This process was repeated until the base Q and the top of the guard cylinder were level within three seconds of arc. The leveling has been rechecked several times, and at no time has the base been out of level by more than six seconds of arc.

The distance between the guard cylinders was measured several times at 16 equally spaced positions with a calibrated bar and inside micrometer. This was accomplished by disconnecting the spider arms H and sliding the deflecting cylinder in against the inner cylinder at the point of measurement. The results obtained for each position are recorded in Table IV, from which the parallelism can be judged. The probable error takes into account the uncertainty of the calibrated gauges. The error calculated from the table (consistency) is about one-seventh that used. The difference between this separation and the length of the suspended cylinder is 0.03051 cm, which gave a gap of 0.0153 cm at the top and bottom when the suspended cylinder was adjusted. This gap distance was also measured with the suspended cylinder in place but resting centrally on the lower guard ring. This measurement gave 0.031 cm, which is in good agreement with the above measurements.

The horizontal position of the upper guard cylinder was checked by resting the suspended cylinder on the lower guard cylinder. The outside diameter of the guard cylinders was made 0.001 cm larger than the outside diameter of the suspended cylinder ribs. A sharp angle straightedge held against the outside diameter of the lower guard cylinder with a light opposite from the observer made it very easy to locate the suspended cylinder centrally with respect to the lower guard cylinder. An examination of the top guard cylinder and suspended cylinder with the light and straight edge showed them to be concentric within 0.0005 cm.

A selected torsion wire of the correct length was inserted in the wire clamps and placed in the torsion head assembly. A small twisting force was sufficient to attach the taper joints of the mirror rod and wire clamp. The vertical and horizontal head motions were then adjusted until the suspended cylinder was centrally located between the guard cylinders and concentric with them. The vertical adjustment is relatively unimportant, but the concentric adjustment is very important. The latter adjustment was checked with the straight edge and light, as used above on both guard cylinders. It was possible to make the suspended cylinder concentric within 0.0005 cm, and in all adjustments the discrepancy never exceeded 0.001 cm. The vertical adjustment located the suspended cylinder within 0.01 mm of the central position. The angular position of the suspended cylinder was adjusted so that the mirror was normal to center of the optical scale (comparator). The bell jar was put in place and rotated until the plane parallel window was almost parallel to the mirror.

Optical System

The optical system, consisting of a telescope and comparator, was mounted on a steel table firmly bolted to the concrete floor.

The telescope objective was a very high quality corrected lens 7.5 cm in diameter and 100 cm focal length. The micrometer eyepiece was the same as used by the Societe Genevoise in the microscopes on the high precision x-ray spectrometers. The parallel cross hairs are moved by a precision screw which was calibrated for both periodic error and error of run. The size of the light source was selected so that the width of its image formed by the telescope objective in the plane of the cross hairs was just less than the separation of the parallel cross hairs. This method permits a minimum error of setting for this type of measurement.

The graduated scale usually used was replaced in the present method by a high precision comparator calibrated to within 0.001 mm for both periodic error and error of run.

The light source, which was mounted on the comparator, consisted of a vertical wire with a neon lamp placed directly behind it. Thus the image in the telescope was a dark line on a uniformly lighted background.

In order for the angles to be easily computed from the comparator measurements, it is necessary to set the ways of the comparator parallel to the reflecting mirror at its zero position. A 30-mm objective telescope equipped with a Gauss eyepiece was set perpendicular to the mirror in its zero position and about nine meters away. A plane front silvered mirror 18×8 cm was placed about 10 cm from the comparator and adjusted normal to the telescope axis, which made it parallel to the suspended mirror within five seconds of arc. A sensitive indicator was attached to the comparator slide and touching the large silvered mirror. The base of the comparator was then adjusted until the indicator showed no variation as the carriage was moved through the entire length of the mirror (18 cm).

The distance from the vertical wire to the mirror was measured with the aid of a steel alloy measuring bar having parallel ends. This bar, which was calibrated by the National Bureau of Standards, was mounted horizontally in appropriate supports between the mirror and vertical wire. The remaining short distances, about 6 cm, were measured with calibrated inside micrometers. Instead of measuring the distance between the measuring bar and mirror directly, the distance between bar and mirror rod was measured, and to this was added one-half the diameter of the rod. This eliminated the difficulty caused by the oscillation of the suspended mirror. The distance was measured five times during the course of the experiments, and the results of these measurements are given in Table V.

The probable error in this distance is about 0.01 mm and certainly does not exceed 0.02 mm, which has been accepted as the probable error in calculations.

The measured distance given in Table V must be corrected for the glass window. The thickness of the window was 8.585 mm and the index of refraction was $\mu=1.5216$. The equation for the shortening of the optical path is

$$X=(\mu-1)d/\mu$$

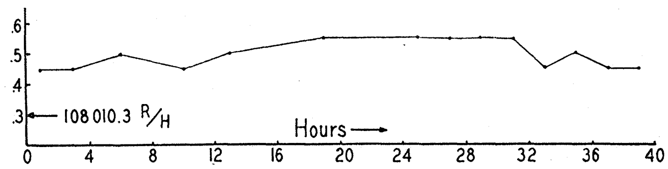

FIG. 4. A 40-hour test on the speed of the tuning fork controlled motor. The maximum variation in the motor speed is 0.1 revolution per hour or one part in 10^6.

where d is the thickness and μ the index of refraction of the glass plate. This gives 2.94 mm which must be subtracted from the result in Table V, and gives 2781.44 mm as the corrected mirror-to-wire distance.

After the above measurements were made, the torsion head of the suspension was rotated until the image of the vertical wire was visible in the field of the telescope. This established the zero point on the scale. Rotation of the inner cylinder then produced a deflection θ on either side of the zero depending upon the direction of rotation. If d represents the total deflection on the comparator, then

$$2 \tan 2\theta = d/2781.44.$$

Two methods of observation were used. In the first, the suspended cylinder was brought to rest either in its zero position or in the deflected position. The micrometer was set at its zero position and the comparator carriage moved until the vertical illuminated wire appeared midway between the cross hairs. This method required that the suspended cylinder be brought exactly to rest or that a large number of settings be made. Both of these required so much time that this method was used only as a check on the second procedure.

In the second method, the suspended cylinder was purposely oscillated at its natural period sufficiently to give a displacement of the telescope image of about ± 0.05 mm. The comparator carriage was then set approximately in position. The extreme positions of several oscillations were determined with the micrometer. The mean of these indicated the correction that had to be added to the comparator setting. The correction usually did not exceed 0.020 mm. Since different amplitudes were used and each setting of the micrometer head was made in a darkened room, it is obvious that this method reduces the human error which falsifies so many precision measurements.

DRIVING AND TIMING APPARATUS

The drive for the rotating cylinder has to maintain a very constant and known speed for long periods of time. In the torsional measurements a high precision printing clock is essential. A single powerful clock has been constructed which serves for both purposes. The clock is accurate to better than one part in 10^6 for both long and short intervals, and because of its possible use in other experimental work, it will be described in some detail.

The essential feature of the present apparatus is the synchronizing of a high quality 0.5-horsepower ball-bearing d.c. motor with a precision electrically driven tuning fork by a Thyratron circuit.[7] The maximum possible variation in synchronism from the mean position is 0.25 revolution of the motor, and this can occur only for large changes in the supply voltage (20 volts) or similar changes in load. In the present work a 240-volt battery supplied the voltage and the small load changed very little at any time. The variation in synchronism under these conditions was less than 0.02 of a revolution of the motor.

The time between any two events could be determined by recording the number of revolutions of the motor by the usual method of reduction gears and divided dials. The positions of the dials were recorded photographically by a Sept movie camera with positive film. The illumination was obtained from an evacuated mercury discharge tube, similar to those previously described.[8] A 10-mf condenser at 2500 volts was discharged through the tube, which gave ample light to photograph all dials. The duration of the flash was less than 10^{-5} second, making it possible to photograph the divisions of the most rapidly moving dial without blurring. Two methods were provided for setting off the discharge tube: a hand key which could be used for timing oscilla-

[7] J. A. Bearden and C. H. Shaw, Rev. Sci. Inst. **5**, 292 (1934).

[8] H. E. Edgerton and K. J. Germeshausen, Rev. Sci. Inst. **3**, 535 (1932).

tion periods, etc., and a connection to a time signal receiver which allowed the time signals from N.A.A. to set off the discharge automatically. This made it possible to record the time signals with an error not exceeding 0.001 second. If the tuning fork vibrated at exactly 60 cycles, then the motor should revolve 108,000.0 revolutions between two hourly time signals. The error in the time signals is about 0.01 second, so for precision checks one must obtain the correction sheets which show how much each hourly time signal was in error.

The constancy of the tuning fork frequency is the determining factor in how accurately the motor speed can be maintained. The first fork used was an Invar fork maintained at constant temperature ($\pm 0.01°C$) driven by the usual vacuum tube method, in which a sine wave current is applied to the magnetic driving coils. This method not only drives by magnetic coupling to the tines of the fork, but also uses a magnetic pick-up for the input to the grid of the first vacuum tube. It is practically impossible to avoid undesirable coupling between these coils. It was proved that this method usually drives the fork at a frequency other than its natural frequency. It may drive the fork either too fast or too slow, depending on the phase relations of the electrical couplings in the amplifier and pick-up system.

Two fundamental changes were made in the method of driving. The first was to use a condenser pick-up for the input to the first vacuum tube. The second was to arrange a Thyratron circuit such that a very short driving impulse could be applied to the tines of the fork as they passed through their normal position at maximum velocity. Small phase errors then have practically no effect on the frequency of the fork, and the fork is driven at exactly its natural frequency.

The new vacuum tube drive made it possible to maintain the frequency of the fork under good conditions to about one part in 10^5 for a few hours at a time. The frequency of the fork was very sensitive to amplitude changes and even though the amplitude was maintained consant, the fork occasionally drifted to a new frequency as if internal damping and stresses had been altered. If the fork stopped for a few hours, it took 12 to 36 hours to attain a constant frequency after restarting. When Elenvar became available for making large forks, one was obtained and this has proved very satisfactory. The tines of the fork were carefully balanced and length adjusted to give a natural frequency of 216,020 vibrations per hour. This fork has none of the bad characteristics of the Invar fork and its frequency has remained constant for 18 months. Fig. 4 shows a typical 48-hour run in which the number of revolutions of the motor per hour is plotted against time. This shows a deviation of less than one part in 10^6, which is more than adequate for the present experiments. Improvements in the circuit and voltage supply should improve the operation so that variations of less than one part in 10^7 should be attainable.

Determination of the Torsional Constant

The method used was the measurement of the period of oscillation of suspended objects of calculable moment of inertia. This is the dynamic method of determining the torsional constant, and in the viscosity measurements the torsion wire is used statically. It is possible that the two torsion constants are not the same, but if not, one would expect the torsional constant to be a function of the period of oscillation. Six inertia objects of different moment of inertia were used to test this point. The period of oscillation was varied from 11 to 67.5 seconds and in some wires a small difference was detected. The wire finally selected showed no difference, as is shown later in Table VII. The expression connecting the period of oscillation T, the moment of inertia I and the torsional constant F is

$$T=\frac{2\pi}{(F/I-K^2/4I^2)^{\frac{1}{2}}},$$

where K is determined from the logarithmic decrement λ by

$$\lambda=(K/2I)\cdot(T/2).$$

It is to be expected that the torsion constant of a given wire will depend on the mass it supports. The exact magnitude of this effect cannot be calculated, for the reduction in cross section of the wire which should lower the torsion constant, may be more than compensated for by the hardening of the wire due to stretching. Tests were made on several wires and in general the torsion constant increased with load. A steel wire of the type used by Harrington was tested with loads similar to those used in his experiments and it was found that the error made by neglecting this effect might easily account for his low results. In the present experiments the inertia objects were designed to have the same mass as that of the suspended cylinder, together with that of the spider, mirror E, and rod and supports H. The difference in mass did not exceed 0.1 gram in any case.

The torsion measurements were made in a separate bell-jar vacuum system designed for this purpose. The base was a steel plate $2.5\times 50\times 60$ cm to which was attached a yoke platform for supporting the torsion head assembly as used in the viscosity apparatus. The height of the platform was such that the vertical adjusting nuts would permit lowering the tapered joint until connection could be made to the tapered joints of the inertia objects which were resting on the base. The nuts were used to raise or lower the inertia objects, which avoided any sudden tension being applied to the wire. A high speed oil diffusion pump, connected through the base, was used in conjunction with a Megavac pump for evacuating the system to about 10^{-4} mm. A flexible connection was made through the base which permitted the amplitude of vibration to be adjusted without disturbing the vacuum. The steel bell jar contained an observation window and the jar was raised and lowered by a pulley and counterbalance arrangement.

A small thin mirror $(0.5\times 1.\times 2.$ mm) was attached to the tapered joint wire clamp Fig. 3. The oscillations of the inertia object were then observed by the usual telescope and scale method. The time was photographically recorded for 10 initial oscillations of the torsion pendulum and then repeated at eight to fifteen times as the amplitude slowly decreased. Recording 10 initial vibrations and 10 more one hour later, permitted a determination of the period for that amplitude within 1 part in 10^5. The amplitude dropped to one-half in times ranging from 3 hours to 18 hours, depending on the moment of inertia of the object. The effect of damping on the period of oscillation was neglected, since in the worst case it caused an error in the torsion constant of less than one part in 10^6.

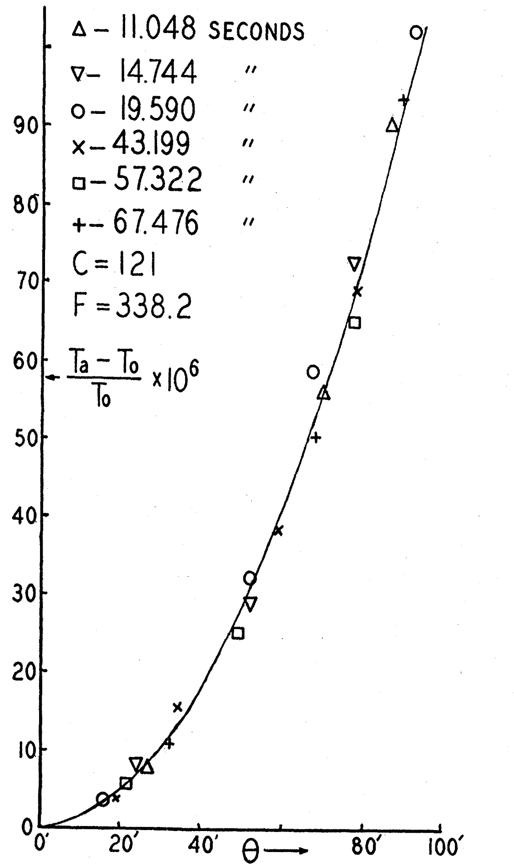

FIG. 5. The variation in the period of oscillation as a function of the amplitude.

The period of oscillation was found to be a function of the amplitude of vibration. For a one-degree amplitude the period was about one part in 25,000 greater than for zero amplitude. This means a deviation from Hooke's law and the oscillatory motion is anharmonic. Since the deviation from Hooke's law and the amplitude of oscillation are small, one needs to retain in the differential equation of motion only terms of the first and third power of the displacement. The solution of this equation gives

$$T_a = T_0\left(1 - \frac{3c}{8F_0}\theta^2 \cdots\right), \qquad (2)$$

where T_a is the period of oscillation for an amplitude θ, T_0 the period for $\theta = 0$, F_0 the usual torsion constant as calculated from T_0 and the moment of inertia, c is a constant of the torsion wire and is independent of the period of oscillation. Several values of c were determined for different values of T_a-T_0 and for different periods. The average value of c was

$$c=+121.$$

A graph of $(T_a-T_0)/T_0$ as a function of θ is shown by the solid line in Fig. 5. The points plotted are representative of all those taken for the different inertia objects. It was found that c was a sensitive function of the temperature and had to be determined for the same temperature as was used in the viscosity apparatus.

The period of oscillation increased with θ which made c positive. This means that the torsion constant decreases with increasing deflection and since the torsion wire is used statically in the viscosity measurement, a correction must be subtracted from the torsion constant as determined from T_0 for each angle of deflection used. It is easily seen that the effective value of the torsion constant for an angle of deflection θ is

$$F_\theta=F_0-c\theta^2, \qquad (3)$$

where F_0, c and θ have their usual meanings. The maximum correction applied to any measurement was less than one part in 7000.

Six inertia objects were used whose mass and dimensions could be accurately measured, making it possible to calculate their moment of inertia with high precision. The smallest moment of inertia was obtained with a Dow metal rod 3.6 cm diameter and 35.3 cm long, and the largest with a Dow metal ring whose outside and inside diameters were 18.7 and 16.7 cm. A steel sphere and three Dow metal disks were also used. Fig. 3 shows how the disks and rods were constructed. The steel sphere was a high precision ball 5.4 cm in diameter, used in ball bearings, and uniformly hardened throughout. The construction of the

TABLE VI. *Inertia measurements.*

Object	Diameter	Moment of Inertia c.g.s. units
Rod	3.61392 cm	1,045.61
Sphere	5.39612	1,862.17
Cylinder 1	6.39820	3,287.89
Cylinder 2	14.26903	15,986.31
Cylinder 3	18.95308	28,148.03
Ring	18.67921 16.67943	39,002.91

other objects was carried out with the same high degree of accuracy as was used in machining the rotating cylinders and other precision parts of the apparatus. The dimensions and masses were determined by the National Bureau of Standards. Table VI contains these measurements of the diameters together with a calculation of the moment of inertia for each object.

The most important characteristics of a torsion wire are that it have no zero drift in either its normal or deflected position and that the static and dynamic torsion constant be exactly the same. It should also have a high tensile strength in order that a small diameter wire can be used which will give measurable deflections for the small viscous force available. Several wires were tested and it soon became evident that tungsten wires were of much higher quality than any other type. The failure to maintain a constant zero and a difference between the torsion constant as measured by short and long periods were the most common faults. Tungsten wires were not perfect, but it was thought that suitable heat treatments might improve them sufficiently to make them usable. Dr. Irving Langmuir of the General Electric Company had some tungsten wires of the appropriate diameter and length selected and treated at high temperatures in very dry hydrogen. These wires gave decidedly better results than untreated wires and in particular those treated at 1200°C gave the best results. The late Dr. Archer Hoyt of the Gulf Research Laboratories found that in constructing a precision gravity meter, the roundness of the wire was important and it should not be coiled during the drawing process and should be kept under tension at all times.

Through the cooperation of Dr. W. E. Forsyth of the General Electric Laboratories in Cleveland, some tungsten wires were obtained which were drawn through selected round diamond dies and not coiled in the drawing process. Two of these were treated at 1200°C for five minutes in pure dry hydrogen. The wires were under slight tension during the process. The results on these wires were superior to any previous ones and the one which had had the greatest tension during the processing was the better of the two. This suggested the use of greater tension. It was found that a tension of about one-half that required for rupture at 1200°C gave the best results. The time of heating was varied and about 10 minutes seemed the best.

The wire selected for the present measurements gave excellent results on all tests except when the tension was removed. When a change was made in the apparatus which required removing the tension on the wire for several hours, it was found that when reassembled the zero drifted in one direction for a period of 12 to 30 hours. The total amount of the drift never exceeded 0.1 mm on the comparator scale, which was 2781.44 mm from the mirror, or a total angular shift of the suspended cylinder of less than 3 seconds. After this readjustment took place, tests for a shift of zero were made when the cylinder was also deflected, and none could be detected. This was done by two methods. In the first the deflected position was recorded as soon as possible after starting the rotation of the inner cylinder. The position was rechecked by the eyepiece micrometer several times over a period of an hour or more. The rotating cylinder was then stopped, which allowed the suspended cylinder to swing almost an equal angle from the normal zero in the opposite direction. Just prior to the suspended cylinder's attaining its maximum deflection, the inner cylinder was started rotating in the opposite direction. The deflection in this opposite direction was recorded and rechecked as before for an hour or more. Failure to detect any drift of greater than one part in 30,000 of the total deflection was conceded satisfactory. The second method of checking the zero was to take the average of the comparator readings for the deflections to each side of the normal zero. This average always agreed with the normal zero position within the error of observation.

The wire used has retained the same torsion constant for over a year. The results of all measurements made on this wire are shown in Table VII.

The average and probable errors were calculated in the usual manner assuming each determination to be an independent one and of equal weight. To assume the resulting values of the torsion constant to be independent is not quite correct, because the same dimensional measurements are used in calculating all values of F for each object. However, the error in determining the period is about the same as that in the dimensions of the suspended bodies, and to this extent constitutes an independent determination. The precision of these measurements is such that the probable error calculated by any method is so small as to be negligible.

TABLE VII. *Constant of torsion wire.*

Date	Object	Period sec.	Torsion Constant
2/3/38	Cylinder (3.6)	11.0477	338.209
4/9/38	Cylinder (3.6)	11.0475	338.221
3/18/38	Sphere (5.4)	14.7438	338.189
11/24/38	Sphere (5.4)	14.7434	338.208
10/13/38	Cylinder (6.4)	19.5910	338.191
11/22/38	Cylinder (6.4)	19.5906	338.206
11/30/38	Cylinder (6.4)	19.5907	338.203
12/6/38	Cylinder (6.4)	19.5909	338.196
11/21/38	Cylinder (14.3)	43.1995	338.192
12/5/38	Cylinder (14.3)	43.1999	338.185
1/7/39	Cylinder (14.3)	43.1986	338.207
1/10/39	Cylinder (14.3)	43.1989	338.201
10/10/38	Cylinder (19.)	57.3228	338.184
11/20/38	Cylinder (19.)	57.3211	338.204
12/4/38	Cylinder (19.)	57.3225	338.187
1/5/39	Cylinder (19.)	57.3209	338.206
3/16/38	Ring (18.7–16.7)	67.4766	338.182
10/9/38	Ring (18.7–16.7)	67.4763	338.185
		Average	338.198 ± 0.002

FURTHER TEST ON APPARATUS

If the cylinders had not been perfectly cylindrical the suspended cylinder should have possessed forced vibrations of the frequency of the speed of rotation of the inner cylinder. As noted earlier, no such effect was observed for the speeds then used. The amplitude of the forced vibrations should be a maximum at an inner cylinder speed corresponding to the natural period of the suspended cylinder, which was 61.620_1 seconds. The closest speed to this that could be obtained with the gears was one revolution in 53.3 seconds. No detectable vibration was observable and similar tests were made at speeds ranging from one rotation in 6.67 seconds to 240 seconds.

If the cylinders are not coaxial then the velocity gradient will not be the same in all angular directions in a plane perpendicular to the axis of the cylinders. Couette[9] made a theoretical estimate of the error introduced by the failure of the cylinders to be coaxial. In this he assumed a

[9] M. Couette, Ann. Chim. Phys. **21**, 433 (1890).

uniform velocity distribution in every angular element and integrated the usual expression for the torque on the deflecting cylinder under this condition. This theory was tested experimentally by Kellström, who found that theory and experiment disagreed not only in magnitude but also in the sign of the correction. Inglis[10] has derived an expression for the torque on the deflecting cylinder which includes the pressure gradient term neglected by Couette and also did not assume the spacing between the cylinders to be small compared to the radii of the cylinders. His expression for the torque $F\phi$ on the deflecting cylinder is

$$F\phi=\frac{2\pi\eta U(a+g)}{g}\left[1-\frac{g}{2a}-c^2+\frac{9gc^2}{4a}\cdots\right], \quad (4)$$

where U is the linear velocity of the rotating surface, $g=b-a$, c is the ratio of the separation of the axes of the cylinders to the spacing g, and the remaining quantities are the same as in Eq. (1). It is seen that Eq. (4) reduces to Eq. (1) if only the first two terms of the bracket are retained. The last two terms are negative, giving a reduced torque as observed by Kellström.[2] The calculated reduction is greater than the observed, which is probably due to the large value of c (theory assumed c small) used, and also to the end effects introduced when the deflecting cylinder was not concentric with the guard cylinders. In the present experiment the eccentricity of the cylinders was about 0.0005 cm, which gives a negative correction for the viscosity less than one part in 10^6 and has been neglected.

In the present method where the inner cylinder rotates, it is not permissible to use as large angular velocities as in the case where the outer cylinder rotates. Taylor[11] has shown theoretically and experimentally that when the space between concentric cylinders is filled with water, laminar flow ceases at angular velocities of the inner cylinder greater than ω where ω is defined by the relation

$$\omega^2=\frac{\pi^4\eta^2(a+b)}{2\rho^2(a-b)^3b^2P},$$

$$P=0.0571\left[1-\frac{0.652(a-b)}{b}\right]-0.00056\left[1-\frac{0.652(a-b)}{b}\right]^{-1}. \quad (5)$$

ρ is the density of the liquid and the remaining symbols are the same as used in Eq. (1). The same relation should be valid for gases, and if the constants of the present apparatus are substituted in the above equation, one finds $\omega=1.89$ radians per second, which is about twice the maximum velocity attainable with the present gear system. An examination of Tables XI and XII shows that there was no consistent change of the results as the speed of the rotating cylinder was varied which proves that no turbulence existed at any of the speeds used in the present experiments.

Theoretically[3] it should be expected that for the present construction the effective length of the suspended cylinder should be the average of the actual length of the cylinder and the distance between the guard cylinders. Since no test of this theory exists, three sets of measurements were made in which the distance between the guard cylinders was varied. This was accomplished by placing parallel spacers between the top base and the shoulders on the posts J. A constant value of viscosity was assumed and the effective length of the suspended cylinder calculated. The calculated lengths and the directly measured lengths assuming the correctness of the theory are compared in Table VIII. The differences are within the experimental errors. The errors are larger than those of the final measurements because the adjustments were not made with the highest accuracy. These results are in complete accord with theory and justify the use of the effective length of the suspended cylinder as the average of the distance between the guard cylinders and the length of the suspended cylinder. From Table IV the distance between the guard cylin-

TABLE VIII. *Effective length of suspended cylinder.*

Calculated length	Measured length
26.0610 cm	26.0610 cm
26.0755	26.0749
26.0902	26.0891

[10] D. R. Inglis, following paper.
[11] G. I. Taylor, Trans. Roy. Soc. London **A223**, 289 (1923).

TABLE IX. *Sample viscosity determination. Initial zero*=243.480 *cm. Final zero*=243.483 *cm.* t=10.90803 *seconds. Pressure*=76.2 *cm.**

Temperature	Left, mm	Right, mm	Difference	$\eta\times10^7$	$\eta\times10^7$† 20.00°C
20.032	314.530	172.440	142.090	1819.49	1819.35
20.034	314.535	172.440	142.095	1819.55	1819.40
20.032	314.535	172.435	142.100	1819.62	1819.48
20.036	314.530	172.435	142.095	1819.55	1819.39
20.036	314.530	172.445	142.085	1819.43	1819.27
20.036	314.525	172.445	142.080	1819.36	1819.20
20.040	314.525	172.430	142.095	1819.55	1819.37
20.038	314.530	172.430	142.100	1819.62	1819.45
20.040	314.530	172.440	142.090	1819.49	1819.31
20.042	314.540	172.440	142.100	1819.62	1819.43
20.042	314.540	172.435	142.105	1819.68	1819.49
			Average		1819.38
		Correction for $F_\theta=F_0-c\theta^2$			0.11
					1819.27

* Viscosity measurements have been made at pressures of 38 cm, 7.6 cm and 0.8 cm, and when corrected for slip, these pressures indicate an increase in viscosity with pressure. From 7.6 cm to 76 cm, the increase is about one part in 2000. All measurements recorded have been made with a pressure of 74.5±0.5 cm.

† The slip correction of 1.26 parts in 10^5 has been applied to the results in this column. See below.

ders is 26.07634±0.00020 cm. The length of the suspended cylinder is 26.04583±0.00008 cm. The average of these gives for the effective length of the suspended cylinder

$$L=26.0611\pm0.0002 \text{ cm.}$$

and the largest of the two errors has been retained as the probable error of the average.

Apparatus Constant and Probable Error of Measurements

The apparatus constant K in Eq. (1) is

$$K=(b^2-a^2)/La^2b^2.$$

The error ΔK can be obtained by taking partial derivatives. ΔK is then

$$\Delta K=[(\partial K/\partial b)^2(\Delta b)^2+(\partial K/\partial a)^2(\Delta a)^2+(\partial K/\partial L)^2(\Delta L)^2]^{\frac{1}{2}}. \quad (6)$$

From Tables II and III $b=7.47654\pm0.000025$ cm. The effective length of the suspended cylinder is $L=26.0611\pm0.0002$ cm. Two inner cylinders a were used, and from Table I

$$a_I=6.22112\pm0.00001 \text{ cm}$$
$$a_{II}=6.78517\pm0.00002 \text{ cm.}$$

From these values one obtains

$$K_I=3.04988\times10^{-4} \text{ and } [\Delta K/K]_I=2.01\times10^{-3} \text{ percent,} \quad (7)$$

$$K_{II}=1.46999\times10^{-4} \text{ and } [\Delta K/K]_{II}=4.5\times10^{-3} \text{ percent.} \quad (8)$$

The torsion constant and speed of rotation enter to the first power in the numerator of the viscosity equation. From Table VII the torsion constant $F_0=338.198\pm0.002$ or the error is 0.59×10^{-3} percent. The error in the speed of the driving motor is about one in 10^6 or 0.1×10^{-3} percent. Combining these errors with those of Eqs. (7) and (8), the factor $KF_0/8\pi^2$ becomes for the two rotating cylinders I and II

$$(KF_0/8\pi^2)_I=13.06363\pm2.10\times10^{-3} \text{ percent,} \quad (9)$$

$$(KF_0/8\pi^2)_{II}=6.29645\pm4.56\times10^{-3} \text{ percent.} \quad (10)$$

The only other instrumental error is that due to θ which involves the ratio of the comparator scale measurements and the effective distance of the mirror from the vertical wire on the comparator carriage. The error in the comparator measurements does not exceed 0.002 mm for any measurement and no angles giving less than 10 cm comparator distance have been used. The error due to the comparator then does not exceed 2.10^{-3} percent. This does not include the random error of setting the image of the vertical wire between the parallel cross hairs, which is taken into account by the deviations of the independent measurements. The effective mirror to wire distance as given in Table V is 2781.44±0.02 mm $\pm0.72\times10^{-3}$ percent. The combination of these gives for the instrumental error in θ 2.13×10^{-3} percent. The viscosity equations may then be

TABLE X. *Temperature coefficient.*

Author	Date	Result
Millikan[12]	1913	$4.93\times10^{-7}/c^0$
Rigden[5]	1938	$4.93\times10^{-7}/c^0$
Banerjea and Plattanaik[4]	1938	$4.95\times10^{-7}/c^0$
	Average	$4.94\times10^{-7}/c^0$

[12] R. A. Millikan, Ann. d. Physik **41**, 759 (1913).

TABLE XI. *Final results rotating cylinder I. Rotating cylinder radius* = 6.2211 *cm.*

t(SECONDS)	$\eta\times10^7$
6.67	1819.18
6.67	1819.23
10.91	1819.14
10.91	1819.21
10.91	1819.24
10.91	1819.12
10.91	1819.19
10.91	1819.27
16.00	1819.31
16.00	1819.24
16.00	1819.09
16.00	1819.16
16.00	1819.20
16.00	1819.28
40.00	1819.35
40.00	1819.29
146.65	1819.17
146.65	1819.00
239.98	1819.11
239.98	1819.39
Average	1819.21 ±0.02

written

$$\eta_I = 13.063638\theta t \pm 3.0\times10^{-3} \text{ percent}, \qquad (11)$$

$$\eta_{II} = 6.29645\theta t \pm 5.0\times10^{-3} \text{ percent}, \qquad (12)$$

where the errors include all instrumental errors.

A Typical Viscosity Determination

The system was evacuated to 10^{-4} mm and tested for leaks by closing the stopcock C. Air from outside the building was admitted through a cylindrical brass trap 3×20 cm filled with copper shavings which was immersed in a mixture of dry ice and alcohol. The rate of flow of air through the trap was about 500 cm^3 per minute, which allowed adequate time for removal of the water vapor. The apparatus was usually filled to about 74.5 cm pressure. Several hours were allowed for temperature equilibrium to be established. The measurements then consisted in recording the zero position, a deflection with the rotating cylinder turning in one direction and then an equal but opposite deflection with the direction of rotation of the rotating cylinder reversed. The temperature was recorded by a calibrated Beckman thermometer whose bulb was very close to the suspended cylinder inside the bell jar. This process of taking readings was repeated about 10 times for each run. The total time taken for such a run was from 45 to 60 minutes. Table IX gives the result of a typical run. The correction of the viscosity to 20.00°C was made by taking an average of the results in Table X on the temperature coefficient. It makes practically no difference which value of the coefficient one uses because measurements were made slightly above and below 20°C, so that any small error in the correction term is canceled. This average value of the temperature coefficient has been used with the present results to calculate a value of the viscosity at 23°C which can be compared with previous results.

A small correction has to be applied to the above values of the viscosity which is due to the influence of slip. This has been studied in detail by Millikan,[13] Stacy,[14] States,[15] and Blankenstein[16] with apparatus very similar to that used in the present experiments. The expression for the true viscosity including the slip correction may be written in the form

$$\eta = \frac{F}{8\pi^2}\,\frac{b^2-a^2}{La^2b^2}\,\varphi t\left[1+2\rho\frac{a^3+b^3}{ab(b^2-a^2)}\right], \qquad (13)$$

where ρ is the slip coefficient and the other symbols represent the same quantities as in Eq. (1). The value of ρ for the present surfaces should not be greater than that for brass which is 75.4×10^{-7}. Substituting this in the bracket term of Eq. (13) together with the values of a and b, one finds the true value of the viscosity is greater than the normally calculated value by only 1.26 parts in 10^5 for the smallest rotating cylinder and 2.19 in 10^5 for the largest.

TABLE XII. *Final results rotating cylinder II. Rotating cylinder radius* = 6.7817.

t(SECONDS)	$\eta\times10^7$
10.91	1819.13
10.91	1819.25
16.00	1819.16
16.00	1819.16
16.00	1819.11
16.00	1819.09
146.65	1819.31
146.65	1819.10
Average	1819.16 ±0.02

[13] R. A. Millikan, Phys. Rev. **21**, 217 (1923).
[14] L. J. Stacy, Phys. Rev. **21**, 239 (1923).
[15] M. N. States, Phys. Rev. **21**, 662 (1923).
[16] E. Blankenstein, Phys. Rev. **22**, 582 (1923).

The average of the results in Table IX must be corrected for the change in torsion constant with angle of deflection. This correction can be easily made by the use of Eq. (3), which for the deflections of Table IX amounts to about one part in 17,000. This correction has been applied to the average of the 11 determinations as indicated at the end of the table.

Results

The results obtained in 28 independent runs of the type recorded in Table IX are given in Tables XI and XII. Six different angular velocities of the rotating cylinders have been used and in each table the results have been grouped according to the velocity used. The maximum velocity of one revolution in 6.67 seconds was used primarily to test for possible turbulence. The agreement of the first two results in Table XI with the results for lower velocities indicates a complete absence of turbulence at all velocities used. The best results were obtained with velocities corresponding to one revolution in 10.91 and 16 seconds. The statistical fluctuation of the results in this group is least in both Tables XI and XII, but the average of this group is practically the same as that for the remaining results. In the final average all values were given equal weight.

The results of Tables XI and XII were taken over a period of about 14 months and during this time the rotating cylinders were changed three times, which necessitated a complete realignment of the apparatus. The torsion wire was removed for calibration on six other occasions and when replaced in the apparatus, the suspended cylinder was readjusted for height and axial centering. The deviations in the results of Tables XI and XII are due then not only to statistical errors of observation but also to possible errors in adjustment which may not have been sufficiently allowed for in the estimation of errors. The probable errors of Tables XI and XII calculated in the usual manner are not sufficiently large to permit an overlapping of the two averages. This can be explained by the fact that two rotating cylinders were used, and the errors involved in the measurement of their diameters easily accounts for the difference between the average results of the two tables.

Table XIII. *Comparison of recent viscosity measurements.*

Observer		$\eta\times 10^{-7}$ c.g.s. unit at 23°C
Kellström[2]		1834.9±2.7*
Houston[3]		1829.2±4.5
Banerjea and Plattanaik[4]		1833.3±2.2
Bond[1]	1834.8	
Rigden[5]	1830.3+	1832.6±2.3
Weighted average		1833.1±1.3

* Kellström makes no mention of applying a correction for the gap between the deflecting cylinder and guard rings. If this was not done then his value should be reduced to 1832.9×10^{-7} c.g.s. unit.

The final probable error of the results in Table XI can be obtained by combining the probable error indicated with that of Eq. (11). This gives for the smallest rotating cylinder

$$\eta_{20.00^\circ\mathrm{C}} = (1819.21\pm 0.06)10^7 \text{ c.g.s. units.}$$

Likewise for Table XII and Eq. (12) the largest rotating cylinder gives

$$\eta_{20.00^\circ\mathrm{C}} = (1819.16\pm 0.10)10^7 \text{ c.g.s. units.}$$

The larger probable error for the latter result is due to the smaller value of $(b-a)$ in Eq. (1) and to the greater inaccuracy of cylinder 11 as shown in Fig. 2. If the two values are weighted inversely proportional to square of the probable errors, then

$$\eta_{20.00^\circ\mathrm{C}} = (1819.20\pm 0.06)10^7 \text{ c.g.s. units.}$$

Since the results are dependent except for the rotating cylinder, the probable error of the average cannot be calculated by the usual method, which would yield a value of ±0.02. The probable error obtained for the results with the smallest rotating cylinder has been adopted as the probable error of the final weighted average. Assuming the temperature factor of Table X, the viscosity of air at 23.00°C is $(1834.12\pm 0.06)\times 10^7$ c.g.s. units.

Recent Determinations of the Viscosity of Air

All previous viscosity measurements which have been made in an effort to solve the problem of the discrepancy between the x-ray and oil-drop evaluations of e are recorded in Table XIII. The only notable discrepancy in the results is that of the capillary tube method of Bond and Rigden. Rigden used Bond's apparatus in which some improvements were made and obtained a much lower value of the viscosity than Bond. The probable error given by Bond is ± 0.8 and by Rigden is ± 0.7, or statistically there is only one chance in 2×10^4 of Rigden's value being as high as Bond's. From a study of the two reports, it seems unlikely that the improvements introduced by Rigden should have lowered Bond's results by one part in 400. In view of this discrepancy, which is not discussed, an average of the results is given in Table XIII and a probable error has been assigned which includes both values. The weighted average was obtained by the usual method of assigning weights inversely proportional to the square of the probable errors. The probable error from external consistency is 0.6 and from the internal is 1.3. In accordance with usual practice, the probable error of the average has been taken as ± 1.3. This probable error is more than 20 times that of the present experiment. A weighted average of the result from Table XIII and the author's value thus gives a final average identical with the present experimental value.

Final Value of the Viscosity of Air

In view of the above discussion the final value of the viscosity of air at 20.00°C is adopted as

$$\eta = (1819.20\pm 0.06)\times 10^{-7} \text{ c.g.s. unit.}$$

The Electronic Charge

If Millikan's[17] oil-drop data are used with the above value of the viscosity of air, one finds $e=4.815\times 10^{-10}$ e.s.u. Correspondingly for Backlin and Flemberg's[18] data $e=4.797\times 10^{-10}$ e.s.u. and from Ishida, Fukushima and Suetsugu's[19] data $e=4.852\times 10^{-10}$ e.s.u. The latter results differ by more than one percent and the first two by almost 0.4 percent. The x-ray value of e as recalculated by Dunnington[20] is $(4.8025\pm 0.0004)\times 10^{-10}$ e.s.u. and lies within the spread of the above values obtained by the oil-drop method. New oil-drop measurements are in progress in this laboratory from which it is hoped that a value of e can be obtained which is comparable in accuracy with the present viscosity results. In this work pure argon is being used instead of air, and the viscosity of the argon has been determined with an accuracy at least as great as the above results on air.

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[17] R. A. Millikan, *Electrons, Protons, Photons, Neutrons and Cosmic Rays* (University of Chicago Press, 1935).
[18] E. Backlin and H. Flemberg, Nature **137**, 655 (1936).
[19] Y. Ishida, I. Fukushima and T. Suetsugu, Sci. Papers of the Inst. of Phys. and Chem. Research **32**, 57 (1937).
[20] F. G. Dunnington, Rev. Mod. Phys. **11**, 65 (1939).
* Now at the University of North Carolina.