Angle Dependence and Range of Nuclear Forces

The discovery[1] of the electric quadrupole moment of the deuteron has indicated that the specific nuclear interaction between neutrons and protons must contain a considerable angle-dependent term coupling spin and orbit[2] (as suggested by the meson theory), so as to make the ground state of the deuteron a mixture of $^3S_1$ and $^3D_1$. Without this new term, the ground state would be an S state, and the D states would lie in the continuum. The admixture of D to S depresses the final state below the original S state,[3] down to the observed level $-\epsilon=-4.1mc^2$. The spherically symmetric terms (principally the space-exchange term suggested by Majorana) will thus have to account for less deuteron binding than previously. The range of nuclear forces has been determined mainly by the necessity of reconciling the small deuteron binding with the much greater alpha-binding, Wigner having showed that a short range and great depth of interaction favor the stability of the alpha-particle.[4] The angle-dependent term would probably cause comparatively little admixture of higher states (mainly $^3P_0$) to the very deep $^1S_0$ in the ground state of the alpha-particle, because of its isolation so we may tentatively assume that the spherically symmetric terms account for practically all of the alpha-binding. If they are to account for a smaller deuteron binding than previously, we are even more dependent on Wigner's device and require a still shorter range of the forces. Since analysis[5] of the new and rather extensive proton-proton scattering data shows that the earlier determination of the range[6] was correct for the like-particle interactions, a considerably reduced range may well be assumed only for the different-particle interactions.

How much the range might be reduced depends on what part ($\delta\epsilon$) of the deuteron binding (ϵ) is due to D admixture. It would be simplest to assume that the $^3S_1$ and $^1S_0$ are originally degenerate at about zero energy, and that their entire separation (ϵ) is due to D admixture, or $\delta\epsilon\approx\epsilon$, for one could thus completely eliminate the space-spin-exchange and the spin-exchange terms suggested by Heisenberg and by Bartlett.[3] Writing the wave function of the ground state as $\psi_0=(\psi_S+c\psi_D)/(1+c^2)$ and taking a rough average energy $\bar{E}$ of the admixed D states, one has by perturbation theory $\delta\epsilon\approx c^2\bar{E}$. If $\bar{E}$ were about $20mc^2$ or less, as seems consistent with recent computations of Christy and Kusaka,[7] a value $c^2\approx\frac{1}{5}$ or more would be required to make $\delta\epsilon\approx\epsilon$.

For the quadrupole moment $Q=(3\cos^2\theta-1)r^2$ of the proton (about the center of mass of the deuteron) we may write

$$(O|Q|O)=\{2c(D|Q|S)+c^2(D|Q|D)\}/(1+c^2)\approx-0.02(O|r^2|O)$$

the latter experimentally,[1] taking $(O|r^2|O)\approx10^{-25}$ cm. Without calculation based on further specification of the forces, it may at least be considered plausible that the diagonal matrix element $(D|Q|D)$ is larger than the nondiagonal element $(D|Q|S)$, and that neither is much smaller than $(O|r^2|O)$ in order of magnitude. The coefficient c is then approximately either $-0.01(O|r^2|O)/(D|Q|S)$ or $-2(D|Q|S)/(D|Q|D)$. The first root is probably too small, but the second quite large enough to be compatible with putting $\delta\epsilon\approx\epsilon$.

For the magnetic moment corresponding to ψ_0 we may write

$$\{\mu_S+(3\mu_N-2\mu_S)c^2/4\}/(1+c^2)\approx0.85\mu_N,$$

where the latter member is experimental, μ_N is the nuclear magneton and μ_S the sum of the neutron and proton spin moments.[2] For simplicity in a field theory of the forces and spin moments,[8] one would like to have μ_S about equal to or slightly less than μ_N, which implies $c^2\approx\frac{1}{5}$. This also agrees with the possibility that $\delta\epsilon\approx\epsilon$.

To estimate the possible change of range, we therefore assume that the interaction responsible for the alpha-binding gives no deuteron binding. Writing this interaction $J_d=-B_d\exp(-\alpha_dr^2)P^q$ between different particles (proton and neutron), where P^q indicates space exchange, and similarly J_l between like particles, which should also give rise to a state near zero, we then have approximately[9]

$$B_i=2.7\alpha_i,\qquad i=d,\,l.$$

The binding energy of the alpha-particle is[6]

$$E_{\mathrm{He}^4}=(9/4)\alpha_d\sigma-4B_d[\sigma/(\sigma+2)]^{\frac{3}{2}}-2B_l[\sigma/(\sigma+2\alpha_l/\alpha_d)]^{\frac{3}{2}}$$

(in the units of references 6 and 9, neglecting Coulomb energy and the second-order corrections to the central model, which are almost equally small and of opposite sign,[6] even if $\alpha_l\ll\alpha_d$), minimized by variation of σ. Eliminating the B_i,

$$E_{\mathrm{He}^4}/\alpha_d=(9/4)\sigma-10.8[\sigma/(\sigma+2)]^{\frac{3}{2}}-5.4(\alpha_l/\alpha_d)[\sigma/(\sigma+2\alpha_l/\alpha_d)]^{\frac{3}{2}}.$$

If we assume a symmetrical interaction with $\alpha_l=\alpha_d=\alpha$, this has a minimum -1.23 at $\sigma=1.9$. Since E_{He^4} is -55 in units mc^2, we have $\alpha=55/1.23=45$, about twice as large as in previous theory. The range is thus reduced by a factor $2^{-\frac{1}{2}}$, to a value $\alpha^{-\frac{1}{2}}=0.15\hbar/[(mM)^{\frac{1}{2}}c]=0.48e^2/mc^2=1.35\times10^{-13}$ cm. If, instead, we assume $\alpha_l=22$, as determined by proton-proton scattering,[5] we find that the value $\alpha_d\approx130$ gives the correct minimum energy (at $\sigma\approx1.0$). This corresponds to a proton-neutron range of 0.8×10^{-13} cm. The range is thus so sensitive to $\delta\epsilon$ that range measurements may help to delimit the possible spin and angle dependence of the interactions.

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