

and Skellett⁹ have reported the probability of process (1) to be low, with the very high intensity electron beams employed in this work the likelihood of the H_1 's liberated in the process being ionized before recombination takes place is good. This is especially true at low pressure. Consequently, process (1) may be responsible

⁹ A. L. Hughes and A. M. Skellett, *Phys. Rev.* **30**, 11 (1927).

for the observed high proton yields at low pressure and high electron intensity.

An analysis of the helium ion beam with the mass spectrograph showed the yield of He^{++} to be of the order of five percent of the total ion beam.

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On Radiative Corrections for Electron Scattering

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A relativistic treatment of the radiative correction of order $e^2/\hbar c$ to the elastic scattering cross section leads to the following results: (a) For the scattering in an electrostatic field of a particle described by the Pauli-Weisskopf theory, the correction is finite and is given by (3). (b) For the scattering of a Dirac electron in an electrostatic field, the correction diverges logarithmically and is positive. (c) The convergence or divergence of the correction depends critically on the type of scattering potential considered.

THE customary quantum-mechanical treatment of the scattering of electrons in a field of force involves the assumption that radiative reaction may be considered a small correction. However, when one attempts to calculate the contribution of radiative effects to the scattering cross section, certain characteristic difficulties are encountered.¹ Making an expansion in powers of $\alpha = e^2/\hbar c$, one finds that the probability of scattering with emission of a single quantum with frequency between q and $q+dq$ behaves as dq/q at low frequencies, resulting in an infinite cross section. This "infrared catastrophe" has been shown to arise from the illegitimate neglect, implied in the expansion, of processes involving the simultaneous emission of many light quanta. By taking these into account, and considering only frequencies so low that the light quantum energy and momentum

may be neglected in comparison with those of the electron, one finds, in complete analogy with the classical result, that the scattering probability is just that which is obtained by neglecting radiative effects entirely.

If we now consider the contribution of higher frequencies we might expect to find that as the light quantum energy is increased, a point is reached beyond which the expansion in powers of α is legitimate; this would imply the convergence at high frequencies of the successive terms in the expansion. The first-order terms in α are of two types, one giving the cross section for scattering with emission of a quantum, the other giving a correction to the elastic scattering cross section. For light quantum energies higher than the electron's kinetic energy, the radiative cross section vanishes and only the correction to elastic scattering remains. It is with the behavior of this correction for high light quantum energies that we shall be concerned.

According to Braunbek and Weinmann, if one takes a point charge for the electron and neglects

¹ N. F. Mott, *Proc. Camb. Phil. Soc.* **27**, 255 (1931); F. Bloch and A. Nordsieck, *Phys. Rev.* **52**, 54 (1937); W. Braunbek and E. Weinmann, *Zeits. f. Physik* **110**, 360 (1938); W. Pauli and M. Fierz, *Nuovo Cim.* xv, 3, 1 (1938).

all relativistic effects, this (negative) correction of order α diverges logarithmically at high frequencies. If this result were right it would again show that the expansion in powers of α could not be valid. This is the point of view adopted by Pauli and Fierz who, making a nonrelativistic calculation not involving the expansion in powers of α , show that with this treatment the effect of the high frequencies is actually to make the total cross section vanish. Pauli and Fierz see in this paradoxical result another illustration of the inadequacies of the quantum electrodynamics.

We want now to investigate to what extent the inclusion of relativistic effects modifies the conclusions of Pauli and Fierz. In the absence of a complete relativistic method not involving the expansion in powers of α , only the first-order terms in α are considered in the range $q > T$ (the kinetic energy of the electron) where one would expect relativity to be important.

A simple argument will illustrate why one would expect the relativistic treatment to be essential even when the electron velocity is not comparable with that of light. The emission of a high frequency quantum by the electron causes a recoil which may be large even compared to mc . The transverse momentum, on the other hand, is unaltered. Hence, the relativistic mass increase will reduce the transverse velocity, to which the coupling is proportional. One may therefore expect to obtain convergence at high frequencies in terms which, if calculated nonrelativistically, would diverge logarithmically.

The calculations have been carried through both for a Dirac electron and for a particle of charge e and mass m described by the scalar relativistic wave equation of Pauli and Weisskopf. In each case, the procedure is as follows: one calculates the probability of transition from a state $\Psi(\mathbf{k})$ to a state $\Psi(\mathbf{p})$, where $k^2 = p^2$, induced by an electrostatic potential, V . Ψ represents a stationary state of a free particle interacting with its own radiation field; as the interaction (or charge on the particle) is permitted to vanish, Ψ approaches a free particle wave function u_0 with vanishing light quantum numbers. Higher terms in the expansion of Ψ in powers of e represent states with one or more

quanta in the field and with an electron and, in general, some pairs present. "Renormalization terms," the first of which occur in order e^2 , give the depletion of the state u_0 .

The matrix element of V between states $u_0(\mathbf{k})$ and $u_0(\mathbf{p})$ —called $V_{p\mathbf{k}}$ —leads to the elastic scattering cross section. We then pick out the terms in $\Psi(\mathbf{k})$ and $\Psi(\mathbf{p})$ which combine, through the scattering, pair production, or pair annihilation parts of V , to give corrections to $V_{p\mathbf{k}}$ of order e^2 . This results in a correction to the elastic scattering cross section, $\delta\sigma/\sigma$, of order α .

We will make the following grouping of terms: (A) Those representing a relativistic modification of terms occurring in the nonrelativistic theory. (B) Those involving pairs in initial and final wave functions, but scattering of electron or positron by the scattering potential (even in V). (C) Those involving pair production and annihilation by the scattering potential (odd in V). Terms involving matrix elements of the square of the vector potential may appear in (C) but not in (A) or (B).

Terms (A)

The effect of the relativistic mass increase is most immediately evident on the Pauli-Weisskopf theory. Here we obtain:

$$\left(\frac{\delta\sigma}{\sigma}\right)_A = \frac{\alpha}{4\pi^2 E_{\mathbf{k}}} \int_{q \geq T} \frac{dq}{q^3} \times \left\{ \frac{(E_{\mathbf{k}-\mathbf{q}} + E_{\mathbf{p}-\mathbf{q}})(\mathbf{k} \times \mathbf{q}) \cdot (\mathbf{p} \times \mathbf{q})}{E_{\mathbf{k}-\mathbf{q}} E_{\mathbf{p}-\mathbf{q}} \Delta^+(\mathbf{k}) \Delta^+(\mathbf{p})} - \frac{|\mathbf{k} \times \mathbf{q}|^2}{E_{\mathbf{k}-\mathbf{q}} |\Delta^+(\mathbf{k})|^2} - \frac{|\mathbf{p} \times \mathbf{q}|^2}{E_{\mathbf{p}-\mathbf{q}} |\Delta^+(\mathbf{p})|^2} \right\}. \quad (1)$$

Here \hbar , m and c are chosen unity, $E_{\mathbf{k}} = (1 + k^2)^{1/2}$, and $\Delta^+(\mathbf{p}) = E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{q}} - q$. The lower limit of q , the propagation vector of the quantum, can be taken in the neighborhood of the particle's kinetic energy, T .

The first term in the bracket comes from terms linear in e in $\Psi(\mathbf{k})$ and $\Psi(\mathbf{p})$. The second and third are the corresponding renormalization terms.

We note that when q is large $E_{\mathbf{k}-\mathbf{q}}$, $E_{\mathbf{p}-\mathbf{q}}$, $\Delta^+(\mathbf{p})$, and $\Delta^+(\mathbf{k})$ are all proportional to q ; hence each

of the three terms converges at high light quantum energy.

If we now take $k, p \ll 1$, we see that the non-relativistic formula differs from this only in having $(1+q^2)^{\frac{1}{2}}$ replaced everywhere by 1. This is particularly important in the matrix elements of the radiative coupling corresponding to the emission and absorption of light of momentum q in scattering, in the one case from \mathbf{k} to $\mathbf{k}-q$, in the other from $\mathbf{p}-q$ to \mathbf{p} . Such matrix elements have the form $e(1+q^2)^{-\frac{1}{2}}(\mathbf{k} \times \mathbf{q})/q$ on the present theory and simply $e(\mathbf{k} \times \mathbf{q})/q$ nonrelativistically. The presence of the factor $(1+q^2)^{-\frac{1}{2}}$ corresponds to the use for the effective mass of the particle the harmonic mean of its mass before and after recoil.² It is to this circumstance that convergence is due.

Corresponding to (1), we have for a Dirac electron:

$$\left(\frac{\delta\sigma}{\sigma}\right)_A = \frac{\alpha}{4\pi^2} \int_{q \geq T} \frac{dq}{q^3} \times \left\{ \frac{2 \text{Tr}[\lambda_p^+ \lambda_k^+ (\boldsymbol{\alpha} \times \mathbf{q}) \cdot \lambda_{k-q}^+ \lambda_{p-q}^+ (\boldsymbol{\alpha} \times \mathbf{q})]}{\text{Tr}[\lambda_p^+ \lambda_k^+] \Delta^+(\mathbf{k}) \Delta^+(\mathbf{p})} - \frac{1}{2} \frac{\text{Tr}[\lambda_{k-q}^+ (\boldsymbol{\alpha} \times \mathbf{q}) \cdot \lambda_k^+ (\boldsymbol{\alpha} \times \mathbf{q})]}{|\Delta^+(\mathbf{k})|^2} - \frac{1}{2} \frac{\text{Tr}[\lambda_{p-q}^+ (\boldsymbol{\alpha} \times \mathbf{q}) \cdot \lambda_p^+ (\boldsymbol{\alpha} \times \mathbf{q})]}{|\Delta^+(\mathbf{p})|^2} \right\}. \quad (2)$$

Here λ is the projection operator,

$$\lambda_k^\pm = \frac{1}{2} [1 \pm (\boldsymbol{\alpha} \cdot \mathbf{k} + \beta) / E_k]$$

and $\boldsymbol{\alpha}$ and β are the customary Dirac matrices.

In contrast to (1), the three terms in (2) are separately divergent, converging only upon combination. Expanding each term in powers of p/q and k/q , the leading integrals, of form $\int dq/q$ subtract out. In the limit of low k and p , (2) can

² It may be remarked that the effect of the $(1+q^2)^{-\frac{1}{2}}$ in the above matrix elements of the current is *formally* equivalent to the introduction, into a nonrelativistic calculation, of an extended current distribution of the form

$$\frac{e\mathbf{k}}{8\pi^3} \int d\mathbf{x} (1+x^2)^{-\frac{1}{2}} \exp(i\mathbf{x} \cdot \mathbf{r})$$

with \mathbf{r} the distance from the particle in units \hbar/mc .

be approximately evaluated, and yields

$$\left(\frac{\delta\sigma}{\sigma}\right)_A = -\frac{2\alpha}{3\pi} (\mathbf{p}-\mathbf{k})^2 \ln g/T, \quad (3)$$

where g is a number of order unity. This result is identical with that obtained from the Pauli-Weisskopf treatment in the same limit.³

Terms (B)

The terms (B) arise from first-order corrections in $\Psi(\mathbf{k})$ and $\Psi(\mathbf{p})$ involving pairs, and from the corresponding renormalization terms. Here only the scattering (even) parts of the force field are involved. The terms may be combined and expressed as follows.

For the Pauli-Weisskopf particle: the resulting formula is the negative of (1), if in that formula the Δ^+ are replaced by Δ^- , where $\Delta^-(\mathbf{k}) = E_k + E_{k-q} + q$.

For the Dirac electron: the resulting formula is equal to (2), if there λ_{p-q}^+ and λ_{k-q}^+ are replaced by λ_{p-q}^- and λ_{k-q}^- , respectively, and if the Δ^+ are replaced by Δ^- .

As in terms (A), these terms converge independently in the P-W case and converge only upon combination in the Dirac case. However, the occurrence of the Δ^- replacing the Δ^+ renders them negligible in comparison for low particle velocities, since then the q integral builds up logarithmically with decreasing T for terms (A), but not for terms (B).

Terms (C)

There are twelve terms which involve pair production and annihilation by the scattering potential; the first two, (a) and (a') below, combine corrections of order e in $\Psi(\mathbf{k})$ and $\Psi(\mathbf{p})$ while the remainder combine corrections of order e^2 in one Ψ with the uncorrected u_0 in the other. (f) and (f') below result from the corrections of order e^2 arising from the terms in the Hamiltonian in the square of the vector potential; (f) and (f') will occur in the P-W treatment, but not in the Dirac.

The terms are indicated by the following transition schemes:

³ These proofs of the convergence of terms (A) are due to Professors Bloch and Oppenheimer who, because of these results, suggested the remainder of this investigation.

$$\begin{aligned}
E(\mathbf{k} \rightarrow \mathbf{l}\mathbf{q}) \cdot V(\mathbf{l}\mathbf{q} \rightarrow \mathbf{l}\mathbf{q}\mathbf{p}\mathbf{u}) \cdot A(\mathbf{p}\mathbf{l}\mathbf{q}\mathbf{u} \rightarrow \mathbf{p}), & \quad (a) \\
E(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{u}\mathbf{q}) \cdot V(\mathbf{l}\mathbf{q}\mathbf{k}\mathbf{u} \rightarrow \mathbf{l}\mathbf{q}) \cdot A(\mathbf{l}\mathbf{q} \rightarrow \mathbf{p}), & \quad (a') \\
E(\mathbf{k} \rightarrow \mathbf{l}\mathbf{q}) \cdot A(\mathbf{l}\mathbf{q} \rightarrow \mathbf{l}\mathbf{p}\mathbf{u}) \cdot V(\mathbf{p}\mathbf{l}\mathbf{u} \rightarrow \mathbf{p}), & \quad (b) \\
V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{u}) \cdot E(\mathbf{l}\mathbf{k}\mathbf{u} \rightarrow \mathbf{l}\mathbf{q}) \cdot A(\mathbf{l}\mathbf{q} \rightarrow \mathbf{p}), & \quad (b') \\
E(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}\mathbf{q}) \cdot A(\mathbf{p}\mathbf{u}\mathbf{k}\mathbf{q} \rightarrow \mathbf{p}\mathbf{u}\mathbf{l}) \cdot V(\mathbf{p}\mathbf{l}\mathbf{u} \rightarrow \mathbf{p}), & \quad (c) \\
V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{u}) \cdot E(\mathbf{k}\mathbf{u}\mathbf{l} \rightarrow \mathbf{k}\mathbf{u}\mathbf{p}\mathbf{q}) \cdot A(\mathbf{p}\mathbf{k}\mathbf{u}\mathbf{q} \rightarrow \mathbf{p}), & \quad (c') \\
E(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{u}\mathbf{q}) \cdot A(\mathbf{k}\mathbf{u}\mathbf{l}\mathbf{q} \rightarrow \mathbf{k}\mathbf{u}\mathbf{p}) \cdot V(\mathbf{p}\mathbf{k}\mathbf{u} \rightarrow \mathbf{p}), & \quad (d) \\
V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}) \cdot E(\mathbf{p}\mathbf{u}\mathbf{k} \rightarrow \mathbf{p}\mathbf{u}\mathbf{l}\mathbf{q}) \cdot A(\mathbf{p}\mathbf{l}\mathbf{u}\mathbf{q} \rightarrow \mathbf{p}), & \quad (d') \\
E(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}\mathbf{q}) \cdot A(\mathbf{k}\mathbf{p}\mathbf{u}\mathbf{q} \rightarrow \mathbf{k}\mathbf{p}\mathbf{v}) \cdot V(\mathbf{p}\mathbf{k}\mathbf{v} \rightarrow \mathbf{p}), & \quad (e) \\
V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}) \cdot E(\mathbf{k}\mathbf{p}\mathbf{u} \rightarrow \mathbf{k}\mathbf{p}\mathbf{v}\mathbf{q}) \cdot A(\mathbf{p}\mathbf{k}\mathbf{v}\mathbf{q} \rightarrow \mathbf{p}), & \quad (e') \\
A^2(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}) \cdot V(\mathbf{p}\mathbf{k}\mathbf{u} \rightarrow \mathbf{p}), & \quad (f) \\
V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{p}\mathbf{u}) \cdot A^2(\mathbf{p}\mathbf{k}\mathbf{u} \rightarrow \mathbf{p}). & \quad (f')
\end{aligned} \tag{4}$$

The propagation vectors of the pair particles (e.g., electrons and positrons) are here indicated by Latin and Greek letters, respectively. E and A represent the matrix elements of the current-vector potential coupling for emission and absorption, respectively, of a quantum \mathbf{q} . V indicates the matrix element of the scattering potential (odd for all terms (C)).

One reads (b), for example, as follows: in $\Psi(\mathbf{k})$ we consider the term through which \mathbf{k} is scattered to \mathbf{l} with emission of a quantum \mathbf{q} , then \mathbf{q} is absorbed by the creation of the pair \mathbf{p} and \mathbf{u} , \mathbf{l} remaining in the field; this combines, by means of the term in the scattering potential annihilating \mathbf{l} and \mathbf{u} with $u_0(\mathbf{p})$ in $\Psi(\mathbf{p})$.

Inserting the matrix elements and energy denominators corresponding to the above schemes, we see that the primed terms are the (\mathbf{k} , \mathbf{p}) interchange of the unprimed.

The values of \mathbf{l} , \mathbf{u} , \mathbf{v} above, the propagation vectors for intermediate states, are determined by conservation of momentum in each process involving the vector potential. For example in (b), $\mathbf{l} = \mathbf{k} - \mathbf{q}$; $\mathbf{u} = \mathbf{p} - \mathbf{q}$.

PAULI-WEISSKOPF PARTICLE

We consider first the P-W case. We note that for terms (a), (d), (e), and their primes, as well as for the terms (f) and (f') in the square of the vector potential, the scattering potential V is involved through creation or annihilation of a pair both of whose members have the same energy. Consequently these terms vanish, since the matrix elements for pair creation or annihilation by an electrostatic potential are here proportional to $(E_1 - E_u)(E_1 E_u)^{-\frac{1}{2}}$, where \mathbf{l} and \mathbf{u} are the propagation vectors of the pair members.

There remain (b), (c), and their primes. These

prove to be much more rapidly convergent at high q and of higher order in v/c than either terms (A) or (B), and are therefore of little interest.

This concludes the proof that the radiative corrections of order α for elastic scattering of a P-W particle by an electrostatic potential are finite.

DIRAC ELECTRON

Terms (a), (b), (c), and their primes are found to converge separately and to be of order $(\Delta v^2/c)^2$. They give a smaller contribution for low particle velocities than terms (A) because of the less singular behavior of their energy denominators for low q .

Terms (d) and (e) are as follows:

$$\left(\frac{\delta\sigma}{\sigma}\right)_{4d} = -\frac{\alpha}{4\pi^2 E_k} \int_{q \geq T} \frac{dq}{q^3} \times \frac{\text{Tr}[\lambda_p^+ \lambda_k^+ \lambda_p^- (\boldsymbol{\alpha} \times \mathbf{q}) \cdot \lambda_{p-q}^+ (\boldsymbol{\alpha} \times \mathbf{q})]}{\text{Tr}[\lambda_p^+ \lambda_k^+] \Delta^-(\mathbf{p})}, \tag{4d}$$

$$\left(\frac{\delta\sigma}{\sigma}\right)_{4e} = +\frac{\alpha}{4\pi^2 E_k} \int_{q \geq T} \frac{dq}{q^3} \times \frac{\text{Tr}[\lambda_p^+ \lambda_k^+ \lambda_p^- (\boldsymbol{\alpha} \times \mathbf{q}) \cdot \lambda_{p-q}^- (\boldsymbol{\alpha} \times \mathbf{q})]}{\text{Tr}[\lambda_p^+ \lambda_k^+] \Delta^-(\mathbf{p})}. \tag{4e}$$

From $\lambda_{p-q}^- = 1 - \lambda_{p-q}^+$, it follows that (d) and (e) are equal. Performing the spin sums, we find that for low p and k the sum of (d), (e), and their primes is simply

$$\left(\frac{\delta\sigma}{\sigma}\right)_c = \frac{2\alpha}{3\pi} (\mathbf{p} - \mathbf{k})^2 \int_0^\infty dq/q, \tag{5}$$

a positive logarithmic divergence, indicating an infinite cross section.*

* Dr. R. Serber has pointed out that corrections to the scattering cross section of order α resulting from the Coulomb interaction with the virtual pairs in the field of the scattering potential should properly be considered here. A typical term is represented by the scheme $V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{u}) \cdot C(\mathbf{l}\mathbf{u}, \mathbf{k} \rightarrow \mathbf{p})$, where C indicates the Coulomb matrix element for annihilation of \mathbf{l} and \mathbf{u} and scattering of \mathbf{k} to \mathbf{p} . While not a "radiative" process, the above will give corrections of the same structure and magnitude as those already considered.

Calculation of the above term plus its prime (in the sense of (4)) yields, for low velocities, on either theory: (a) a divergent term, $-2\alpha/3\pi \int_0^\infty dl/l$, and (b) convergent terms vanishing with $(\Delta v/c)^2$ and comparable in magnitude with the terms (B). These results also follow directly from formulae for the "polarization of the vacuum." The logarithmic divergence occurs in zero order of $\Delta v/c$, and may be removed by a suitable renormalization of charge density.

There are also two exchange terms (which do not

CONCLUSION

If the P-W theory is applied to the calculation of the radiative correction of order α to the elastic scattering cross section in an electrostatic field, a finite negative result is obtained, given for low particle velocities by (3). This corresponds to the fact that logarithmically divergent integrals which occur in the nonrelativistic treatment are reduced by factors which spring from the relativistic mass variation. New terms of order α (terms (B) and (C)) involving pair creation and annihilation give contributions small compared to (3) in the low velocity limit.

For a Dirac electron, on the other hand, while terms (A) converge and lead to (3), terms (C) contribute a *positive* logarithmic divergence, (5); it is to be remembered that nonrelativistically the divergence was negative, indicating an infinite cross section.

The convergence or divergence of these results could not have been predicted on the basis of the nonrelativistic treatment alone. The somewhat unexpected result that the radiative correction of order α diverges for the Dirac theory, but not for the P-W⁴ is due to features of the expressions

follow from the customary polarization calculation where the pair producing charge is not an electron, namely $V(\mathbf{k} \rightarrow \mathbf{k}\mathbf{l}\mathbf{y}) \cdot C(\mathbf{k}\mathbf{y}, \mathbf{l} \rightarrow \mathbf{p})$ and its prime. This calculation leads again to a convergent correction, vanishing with $(\Delta v/c)^2$ and comparable with the terms (B).

It follows that the conclusions drawn below are unaffected by the presence of the Coulomb interaction.

⁴In calculations of the proper energy, etc., where divergences appear, they are usually more extreme on the P-W theory than on the Dirac.

for charge and current which have no classical analog. One cannot therefore draw too close an analogy between these convergence questions and those which arise in the classical theory of radiation reaction for a point charge and vanishing collision time.

One gets a striking example of the fortuitous nature of the results by considering, instead of the electrostatic, a world scalar scattering potential (a potential invariant under Lorentz transformations). If one now examines terms (C) on the P-W theory, one finds that (4a), (4d), (4e), (4f), and their primes no longer vanish. On the contrary, they must be included, and the terms in A², (4f) and (4f'), in particular diverge as $\int^\infty q dq$. For the Dirac treatment, on the other hand, convergence arguments are essentially unaltered by the substitution of such a potential. Again, if we consider a purely even scattering potential, the correction is finite for both kinds of particles.

The results obtained are valid only to order e^2 in the radiative correction. It seems certain that calculations to order e^4 and higher would diverge on either theory. It is uncertain whether corrections of order e^2 , even when convergent, can be consistently included, and whether they will provide the correct prediction for the result of a given scattering experiment.

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The Liquid-Drop Model and Nuclear Moments

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The magnetic and electric quadrupole moments of charged spinning drops have been compared with experimental nuclear moments and the agreement found to be very poor.

DISCUSSION OF PROBLEM AND RESULTS

IN view of the renewed interest in the liquid-drop model aroused by the recent fission

experiments, it seems worth while to compare experimental nuclear moments, magnetic and electric quadrupole moments, with those of a uniformly charged spinning drop. Such a drop,

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