LETTERS TO THE EDITOR

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On Stability in the Sense of Poisson for Orbits of Cosmic Rays, and Magnetic Storms

The question of stability of the orbits of cosmic rays presents great physical interest. In a forthcoming paper to appear in a mathematical journal we shall prove that all bounded orbits of charged particles in the field of a magnetic dipole are not stable in the sense of Poisson.[1] This conclusion contradicts a theorem already announced by Schremp,[2] without thereby affecting either his theorems 1 and 2, from which he thought the theorem in question could be deduced, or his other theorems.

To begin with we prove that the regions of stability[3] around stable periodic orbits are infinitesimal. This is a consequence of the fact that the potential energy[4] of the motion in the meridian plane $-P(x, \lambda; \gamma_1)$ contains positive and negative powers of the distance to the dipole. The convergence of the Hamiltonian function K which enters into the variational equations of the periodic orbit therefore holds only in an annular region. The study of the successive approximations of these equations shows that this region should be divided by n when one stops at the nth approximation. If n is taken greater and greater, that is, if the time increases, the region of stability becomes smaller and smaller. When the distance is infinitesimal, the time required for all particles to leave the vicinity of the periodic orbit must be infinite.

In the second place our problem may be reduced to the problem of the motion of molecules of an incompressible liquid in an appropriate force field and in an *infinite* container. We shall than have an integral invariant[5] $\int\int\int dx d\lambda d\eta$ (in the notation used by Lemaitre and Vallarta[4]). Stability in the sense of Poisson has been proved[1] in this case, but only for a finite vessel, that is, when the integral invariant extended to the entire domain of variation of x, λ, η remains finite. Is it possible to introduce a change of independent variable defined by

$$d\sigma/d\sigma' = M(x, \lambda, \eta) \gtrless 0$$

in such a way that the integral invariant

$$\int\int\int M dx d\lambda d\eta$$

remains finite throughout the infinite domain of variation of x, λ, η? If so, the analytic continuations of the orbits, in terms of the old variable, will be stable in Poisson's sense. But it is impossible to find such a function, for the same reason that reduced the size of the region of stability of the periodic orbits to an infinitesimal domain. Nevertheless, the integral invariant

$$\int\int\int M dx d\lambda d\eta$$

remains finite for all bounded orbits which do not reach the dipole. If such orbits exist in a region of stability then they are stable in Poisson's sense, with the exception of orbits of zero measure such as asymptotic orbits. Isolated bounded orbits do not satisfy this condition of stability, because if they returned near their starting point they could be considered as new orbits, since the time origin is arbitrary. But in such a case the bounded orbit would no longer be isolated. It is therefore clear that Poisson's stability exists only in regions of stability which do not contain stable periodic orbits. Such regions, if they exist, must possess on their boundary at least one unstable periodic orbit.

It seems therefore that stable periodic orbits in the case of terrestrial magnetism will play an insignificant role as far as the external magnetic field is concerned. But a weak perturbation which keeps unaltered the qualitative characteristics of the problem (axial symmetry, etc.) may, in accordance with Poincaré's theorem of analytic continuation, enlarge to a great extent the regions of stable periodic orbits, and thus cause magnetic perturbations. Further this weak perturbation may bring and retain for a brief time charged particles on unstable periodic orbits,[6] and in regions of stability of which such orbits may be on the boundary. This would account for the fact that a magnetic storm begins abruptly and disappears slowly. But the perturbation on the other orbits, due to the presence of charged particles on these periodic orbits, does considerably affect the angle η (zenith angle) while scarcely changing the angle θ (angle with the meridian plane). Since, by Liouville's theorem, the intensity of cosmic radiation depends essentially on the shape of the cone of allowed directions (η, θ), it is seen that weak magnetic perturbations may cause relatively large changes in the intensity of cosmic radiation.

This paper arose from discussions with Professor M. S. Vallarta, to whom I wish to express my indebtedness.

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[1] H. Poincaré, *Méthodes Nouvelles de la Mécanique Céleste*, Vol. 3 (1899) Chap. 26; hereafter referred to as M. N.

[2] E. J. Schremp, Phys. Rev. **54**, 154 (1938).

[3] H. Poincaré, M.N., Vol. 3, p. 156, Sec. 298.

[4] G. Lemaitre and M. S. Vallarta, Phys. Rev. **49**, 719 (1936), to which reference may also be made for other necessary definitions.

[5] H. Poincaré, M.N., Vol. 3, Chap. 22; G. D. Birkhoff, Mem. Pont. Acad. Scient. Novi Lyncaei, Ser. 3, Vol. 1, p. 91.

[6] A preliminary examination of this question has been made by S. Kusaka, S.M. Thesis, M.I.T., 1938.