potassium is known to occur at 25,000 kg/cm^2;[7] a plot of the first differences of Table I suggests that the same phenomenon occurs at least with the alloys rich in potassium and probably at pressures lower than for the pure potassium.

Finally, it may be of interest to mention that certain indirect evidence indicates that alloys of about 50 percent K must possess highest compressibilities.

The author wishes to acknowledge the assistance of Professor Bridgman who suggested the problem and took an interest in its solution.

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Townsend Ionization Coefficients in Cs-Ag-O Photo-Tubes Filled with Argon*

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The attempt is made to account for the amplification of photo-currents at low current densities in argon-filled Cs-Ag-O tubes on the basis of Townsend's theory assuming secondary electron emission by positive ions. Special sealed-off tubes were used having parallel plates at various separations and a constant gas pressure. Values of the Townsend coefficients α and γ were determined at each value of field by simultaneous solution of the Townsend equations for two plate separations. The α/p_0 *vs.* E/p_0 plot has a maximum at $E/p_0 \cong 700$ volts/cm·mm. The γ-values for the two types of cathodes used are sensibly constant at low fields. Measured values of the striking potentials of the glow discharge are in quantitative agreement with the positive ion mechanism assumed. The photosensitivity of the compound caesium cathodes was found to increase with increase of bombarding ion current.

STUDIES of the amplification of photoelectric currents by gases are complicated by the circumstance that several auxiliary processes may act simultaneously to augment the primary ionization by electron impact. In commercial photo-tubes having compound low work-function cathodes, it is especially difficult to determine which of the possible secondary mechanisms are operative and to measure their relative effects. Although numerous measurements have been made in tubes in which metallic cathodes are used, little information exists as to the numerical value of the Townsend coefficients in tubes having activated photo-cathodes. An attempt has been made to obtain such information, and in this paper results are reported for Cs-Ag-O tubes filled with argon which indicate: (1) that the photosensitivity of the compound cathode increases with increase in positive ion current to the cathode; (2) that in gas-amplified currents at all potentials secondary mechanisms play a significant part; (3) that the Townsend coefficient α for argon increases with increasing field to a maximum, and then decreases; and (4) that the chief auxiliary mechanism at low current densities leading to high gas amplification, and accounting quantitatively for the low values of striking potential is the release of electrons at the cathode by positive ions.

These results hold for low current densities; the maximum values used range up to about 7×10^{-8} amp./cm^2. For larger currents space charges and other effects very definitely modify the mode of amplification by the gas and these combined with unavoidable changes in structure of the coating with continued bombardment make it extremely difficult to obtain an accurate determination of the factors contributing to gas amplification at high current densities.

Photo-Tubes and Apparatus

In view of the difficulty of maintaining uniform emissivity from a caesium cathode over a long

* A preliminary report of this work was made at the 222nd regular meeting of the American Physical Society, Toronto, Canada, on June 24, 1938. W. S. Huxford, Phys. Rev. 54, 313(A) (1938).

period of time while the tube is on the pumping system, several sealed-off tubes were prepared in a uniform way with the same gas pressure. Six tubes were used having parallel plates with separations of 2.0, 4.2, 6.0, 8.1, 12.3 and 16.0 mm. These tubes were made by the Research and Engineering Department of the RCA Manufacturing Company, following a special design. They were filled with pure argon at a pressure of 0.175 mm Hg at room temperature. The bulbs were tubular, about 2.5 inches in diameter, and the plates were plane square sheets of nickel and silver measuring 1 inch on a side. A circular aperture 0.5 inch in diameter was cut in the nickel sheet used as anode and the entire inside surface was covered with a nickel mesh having openings 0.4 mm square. The silver plate which served as cathode was oxidized in the usual way, coated with caesium, treated and baked in the manner employed in the preparation of commercial photo-tubes. The cathode could be illuminated by light sent from outside the tube through the aperture in the anode. The mesh transmitted approximately 50 percent of the light falling upon it.

Photosensitivity curves for all of the caesium oxide cathodes were obtained by means of a Van Cittert double monochromator with a glass dispersing system used with a tungsten filament

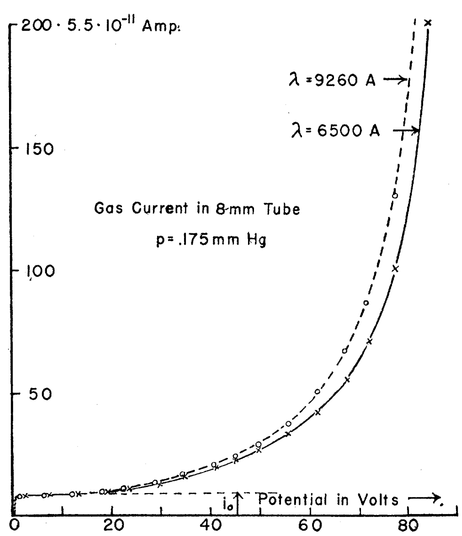

FIG. 1. Dependence of gas current upon the tube potential for two monochromatic beams of light in which the intensity of light has been adjusted to give the same value of i_0, the initial photo-current.

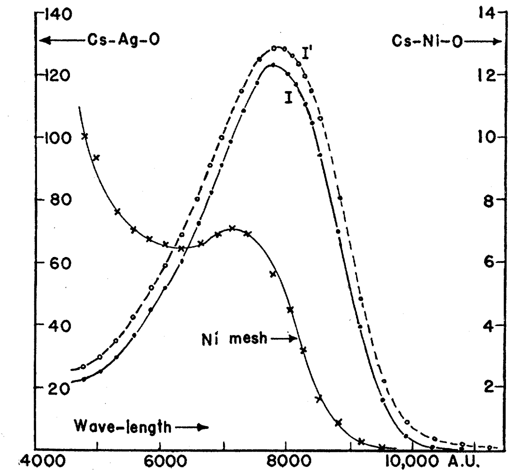

FIG. 2. Photosensitivity plots for the silver and nickel electrodes showing in arbitrary units the current per unit radiant energy as a function of wave-length. Curve I gives discharge currents in argon per unit energy in the 12-mm tube at 54 volts, and I' the currents due to the same monochromatic beams when a discharge current of 0.1 microampere due to an auxiliary white source was set up simultaneously in the tube. The other curve shows the relative sensitivity of the activated nickel electrode.

source. The wave-length spread of the monochromator with a 0.2-mm slit width was about 8A at 4000A and increased to 100A at 6000A. The relative amounts of energy in the light beams from the monochromator were determined by means of a vacuum thermocouple. Currents were measured by two Leeds and Northrup galvanometers each having a sensitivity of the order of 5×10^{-11} ampere per mm. Considerable care was taken in checking the linearity of response of these galvanometers. Some of the currents were measured by balancing the photocurrent with a potentiometer circuit, while the galvanometer was used as a null instrument.

Change of Photosensitivity with Ion Current

When measuring the photoelectric response of the cathodes while currents were passing in the gas, it was discovered that the amplification was noticeably greater for long wave-length radiations than for shorter wave-lengths. For large emission currents this effect was somewhat masked by the general increase in amplification which occurs with increase in intensity of illumination. In the low current range this effect is not due to a change in mechanical structure of the coating nor to the removal of a gas film as has often been observed when the cathode is heavily bombarded.

The increase of gas amplification with wave-length of incident light and with intensity of illumination are both thought to arise from the accumulation on the oxide surface of positive ions which have there a certain "mean life." Because of some local action in the coating, perhaps a change in the energy states of the free electrons in the vicinity of each ion, the temporarily adsorbed ions increase the likelihood of electron emission and probably cause local variations in the work function. This effect may be illustrated by two types of measurements. The curves of Fig. 1 show the amplification obtained when light of wave-lengths 9260A and 6500A falls on the oxide coating. At the higher potentials the amplification produced by the longer-wave-length is from 20 to 30 percent higher than for the shorter. This could be explained by a shift of the photoelectric threshold to the red.

A second type of critical experiment is illustrated in Fig. 2. The solid curve (I) shows the gas currents per unit light intensity when only resolved light is used. The dashed curve (I') shows the enhanced sensitivity produced in the cathode when a simultaneous current is set up in the gas (by an auxiliary source of white light) which is approximately seven times the maximum current obtained with the monochromator alone. The effect is reproducible and is present only during the bombardment.

Figure 3 shows in detail amplification curves in the low current range for all the tubes. Log i/i_0 is here plotted as a function of the accelerating potential, where i_0 is the saturation current obtained at potentials less than 18 volts (see Fig. 1). In all cases distinctly larger amplifications are obtained with infra-red light ($\lambda \cong 9000A$, dashed curves) than with red light ($\lambda \cong 6600A$, solid curves). These curves are for small current densities ($< 10^{-8}$ amp./cm^2) unless otherwise indicated.

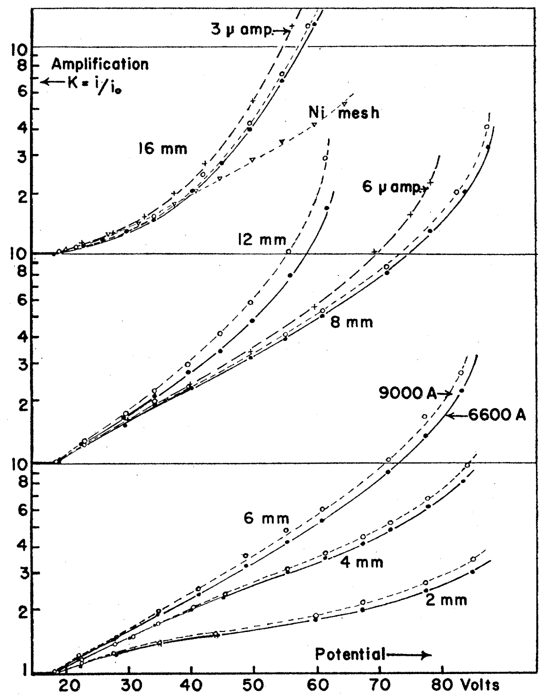

FIG. 3. These plots show the gas amplifications of low intensity photo-currents in argon-filled Cs-Ag-O tubes when the cathode is illuminated by two monochromatic beams of different color. The wave-lengths in each case are approximately those shown for the 16- and 8-mm tubes, with white light, which indicate the increase in amplification at high current densities.

Discharge Mechanisms Involved

An examination of the plots of Fig. 3 on the basis of Townsend's theory shows that the amplification depends to a considerable degree upon processes other than simple ionization of the gas by electron impacts. It is well known that in such discharges cathode phenomena play a significant role. If we assume the argon to be very pure, the processes which merit consideration include photoemission by radiation generated in the gas, release of electrons by metastable argon atoms at the cathode and secondary emission of electrons by impact of positive ions.

For pressures of the order of 0.1 mm Hg the number of photoelectrons released by radiation from excited gas atoms is negligible by comparison to those released by the light admitted to the cathode and by other agencies. To what extent electrons which are released from a cathode surface by metastable atoms contribute to the discharge in argon has never been determined. Experiments have been carried out in this laboratory on the time required for the discharge in argon-caesium photo-tubes to reach a state of equilibrium after light is flashed upon the cathode[1] which indicate that a part of the measured lag may be attributed to the relatively slow diffusion of metastable atoms. It is well known that these atoms effectively ionize minute quantities of vapor or gaseous contamination when the energy of the metastable state exceeds that of the ionization potential of the impurity. In the absence of such contamination the only contribution to the discharge would be the release of electrons from the cathode surface. A number of investigators believe that the time lag in Townsend currents in rare gases is to be ascribed chiefly to the action of metastable atoms. An exception to this interpretation is the recent measurements by Skellett[2] of the change in amplification with frequency of the incident light pulse in the case of a spherical photo-tube filled with argon. He finds no evidence that metastable atoms contribute to ionization processes in the gas, and accounts for the observed dynamic response of the photo-currents in terms of the transit time of argon ions which release secondary electrons upon arrival at the cathode.

Kingdon and Thomson[3] among others have demonstrated that secondary emission of electrons by positive ion bombardment of Cs-Ag-O cathodes exists, and that it plays a major role in gas amplification in argon-filled photo-tubes. This mechanism was first proposed by J. J. Thomson and by J. S. Townsend to account for the abnormally large current amplifications observed when an alkali cathode was employed in the discharge tube. That the effect is particularly important in compound cathodes is shown by the work of Guntherschulze and Betz[4] on the glow discharge. For example, these investigators found that in a glow discharge in pure argon, cathode fall 1000 volts, each ion released on the average 1.88 electrons from a cathode of oxidized magnesium; while for a pure magnesium surface the value was 0.37 electrons per ion. In a recent report on the effect of cathode material on the second Townsend coefficient in pure and contaminated N_2 gas Bowls[5] concludes that emission of electrons by positive ion bombardment is one of the chief mechanisms contributing to the discharge.

During the course of the present study it became apparent that many of the effects observed could be explained in a straightforward manner in terms of the interaction of argon ions with the activated surface layer of the compound cathode. The enhanced photosensitivity and observed shift of the threshold with increased bombarding current is readily accounted for on this basis. Also the marked increase of amplification at large current densities,[6] an effect which we have so far been unable to explain in terms of a simple increase of positive ion space charge in the neighborhood of the cathode, may be thought of as due to an increase in charge density of argon ions temporarily adsorbed on the cathode surface and resulting in an enhance-

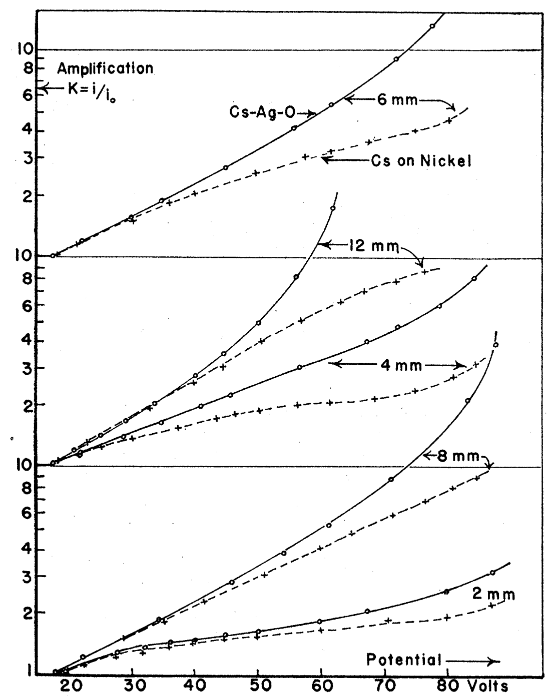

FIG. 4. Amplification plots at low current densities in each tube for the caesium oxide cathode (solid curves) and for the activated nickel mesh (dashed curves). The corresponding plots for the 16-mm tube are shown in Fig. 3.

[1] See W. S. Huxford and R. W. Engstrom, Rev. Sci. Inst. **8**, 385–390 (1937) for details of the circuit used in measuring time lags in argon photo-tubes.

[2] A. M. Skellett, J. App. Phys. **9**, 631–634 (1938).

[3] K. H. Kingdon and H. E. Thomson, Physics **1**, 343–351 (1931).

[4] A. Guntherschulze and H. Betz, Zeits. f. Physik **108**, 780–785 (1938).

[5] W. E. Bowls, Phys. Rev. **53**, 293 (1938).

[6] The apparent increase in amplification at current densities of 3μ amp./cm^2 and at 6μ amp./cm^2 is shown in Fig. 3.

ment of the photoelectric efficiency of the emitting layers. All of these effects are small at low current densities, and it seemed worth while to carry out measurements of the Townsend coefficients under the simplest possible conditions at very low light intensities in order to determine, if possible, whether the hypothesis of release of electrons from the cathode by positive ions provides an auxiliary mechanism which adequately accounts for the amplifications observed in the absence of other complicating phenomena.

Photo-currents with Nickel Mesh Cathodes

Studies of secondary emission of electrons by positive ions show that this process is highly sensitive to changes in structure of the emitting surface. Fortunately it was possible in the present work to vary the cathode surface simply by reversing the role of the two electrodes. Preliminary tests had shown that the nickel mesh anodes were photosensitive to visible light; this sensitivity arose from Cs deposited during the process of forming the cathode. A comparison of the photosensitivity of the meshed electrode and that of the regular cathode for the 12.3-mm tube is shown in Fig. 2. All of the anodes exhibited about the same photoactivity, and for a given light intensity, currents set up in the gas with the electrodes reversed were from twenty to fifty times smaller than when the tube was operated in the usual way.

Gas-amplification plots for five of the tubes operated with the nickel mesh as cathode are shown in Fig. 4, together with plots obtained in normal operation. The corresponding plots for the 16.0-mm tube are shown in Fig. 3. White light was used in both cases and the intensity was adjusted so that the average currents were of the same order of magnitude ($<10^{-8}$ amp./cm^2). The effect of the circular aperture in the anode plate upon the amplification produced was slight.

Table I. *Values of α at $E=50$ volts/cm. K_1 and K_2 are the amplifications at V_1 and V_2 volts, respectively.*

Cathode	x_1	x_2	V_1	V_2	K_1	K_2	α	α_{av}	α/p_0	E/p_0
Cs-Ag-O	0.89	0.47	61.5	40.5	16.0	2.35	0.84			
"	0.47	0.26	40.5	30.0	2.35	1.55	1.00			
"	0.26	0.08	30.0	21.0	1.55	1.14	0.88	0.91	5.7	312.5
Cs-Ni	0.89	0.47	61.5	40.5	6.0	2.20	1.16			
"	0.47	0.26	40.5	30.0	2.20	1.50	1.12			
"	0.26	0.08	30.0	21.0	1.50	1.13	1.00	1.09	6.7	312.5

Sensibly the same results were obtained when light was focused on an area of the mesh backed by the solid nickel plate as when it was focused on the mesh over the half-inch hole. Some differences were found when the whole mesh was diffusely illuminated and when the light was concentrated on a small area.

The branching of the two curves for each plate separation furnishes a most interesting and instructive comparison of the effect of reversing the role of the two electrodes. Obviously the efficiency of ionization of the gas by electron impact is the same in any tube at a given potential, so that the striking differences in amplification which are observed must depend upon an auxiliary mechanism associated with the cathode (or possibly the anode). A crucial test of the hypothesis proposed to account for the amplification in these tubes is furnished by applying the equation of Townsend and determining the value of the ionization coefficient for electrons at each value of the electric field for both normal and reversed polarity of the electrodes. The value of this coefficient should be sensibly constant for both cases at each field value, while the secondary Townsend coefficients should exhibit systematic differences for the two types of electrodes. Such calculations have been carried out and are presented in the sections which follow.

Calculation of α-Values

The equation of Townsend for gas-amplified photo-currents in which secondary electrons are produced by positive ions at the cathode is given by

$$i=i_0\frac{e^{\alpha x}}{1+\gamma-\gamma e^{\alpha x}}, \tag{1}$$

in which α represents the number of pairs of ions produced by each electron per cm of path, γ is the average number of electrons released at the cathode by each positive ion reaching it, and $x=d-\delta=d-V'/E$ is the effective plate separation for a parallel plate arrangement. The current i_0 is the photo-current generated by radiation at the cathode and is measured at potentials so low that ionization of the gas by electrons does not occur (see Fig. 1); and δ is the average distance an electron moves from the cathode in order to

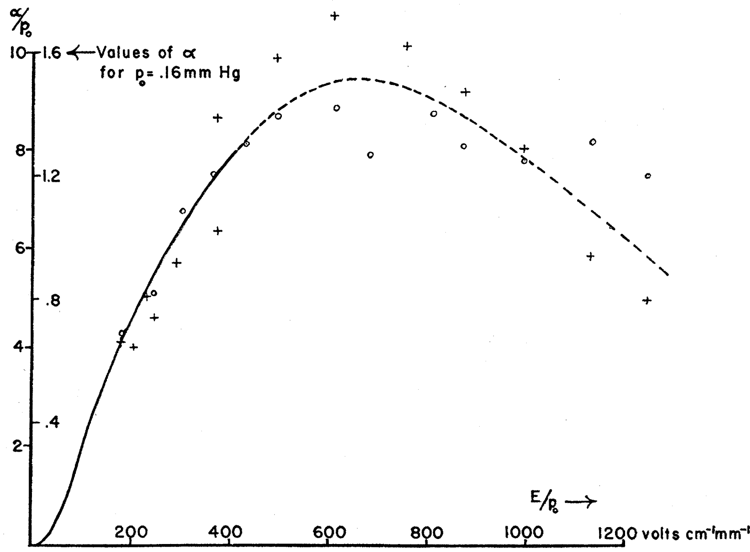

FIG. 5. Values of α/p_0 for the nickel cathodes are shown by small circles, those for the Cs-Ag-O cathodes by crosses. The solid curve represents α/p_0 for pure argon as obtained by Kruithof and Penning.

acquire enough energy to ionize a molecule of the gas upon impact. An approximate value is given by $\delta = V'/E$, where V' is the effective ionizing potential of the gas in volts,[7] and E is the electric field in volts per cm.

The quantities α and γ may be calculated from Eq. (1) for a particular value of field by a simultaneous solution involving two plate separations, x_1 and x_2. This method would be expected to be particularly effective in cases, such as the present, where γ is large even at low fields. For the six tubes used in this study, however, the results are not particularly impressive, because γ does not have the same value for presumably identical cathodes at a given field intensity. There is the additional complication that the photosensitivity of the activated silver cathodes changes with the magnitude of the bombarding ion current. One important result of the present calculations is that the values of the Townsend coefficients at high fields ($E/p > 500$) are the first to be published for argon in discharges of this type.

If we can assume that at a given field strength γ is constant for cathodes prepared in the same way, the simultaneous solution of Eq. (1) for two plate separations yields the following expression involving α:

$$K_1(K_2-1)+(K_1-K_2)e^{\alpha x_1} = K_2(K_1-1)e^{\alpha(x_1-x_2)}, \quad (2)$$

in which the K's are the amplifications, $K_1 = i_1/i_0$ for a plate separation of x_1 cm, and $K_2 = i_2/i_0$ at a separation of x_2 cm. The coefficient α is determined by choosing values for it which satisfy Eq. (2). For low fields it was possible to obtain three independent values of α, for intermediate fields two values, and for high fields at small plate separations only one value. Table I gives sample computations of α for both types of cathodes at $E = 50$ volts/cm ($p_0 = 0.16$ mm Hg).

The mean values of α obtained by applying Eq. (2) are shown graphically in Fig. 5. The small circles indicate α's found when the Cs−Ni electrodes were used as cathodes. Electron coefficients for normal operation of the tubes at low current densities are indicated by crosses. The fluctuations of these values from the mean are large and arise chiefly from variations in structure of the activated silver oxide. There is a definite correlation between these fluctuations and the activation treatment given the different cathodes. The surface structure in the 12.3-mm tube, for example, is distinctly different from that in the 8.1-mm tube.

[7] A value of $V' = 17$ volts was used in the present calculations and is an estimate based on the best agreement between data on the various tubes. A. A. Kruithof and F. M. Penning (Physica **3**, 524 (1936)) used $V' = 15.7$ volts in their measurements of α in pure argon.

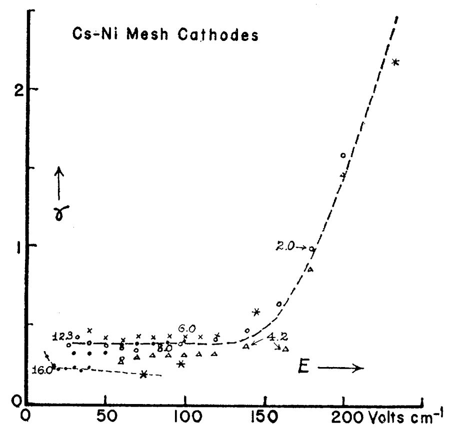

FIG. 6. Graphical summary of calculated values of the second Townsend coefficient γ for the nickel cathodes. The numbers indicate electrode separations in mm. The points marked (*) are γ's calculated from sparking potentials.

The solid curve in Fig. 5 gives values for argon obtained by Kruithof and Penning.[8] These investigators give no data beyond $E/p_0=429$ volts/cm·mm. The dashed curve represents the trend of α/p_0 as a function of E/p_0 for higher fields as determined in the present investigation. It is evident that computations of α in discharges with the nickel cathode show more uniformity than those obtained with normal polarity of the electrodes. Although α values beyond $E/p_0=1000$ volts/cm·mm are unreliable, since these depend solely upon one computation involving the 4.2-mm and 2.0-mm tubes, it is believed that α/p_0 for argon reaches a maximum at about $E/p_0=700$ and then decreases at higher fields. This conclusion requires confirmation under conditions in which the pressure of the gas can be varied. The marked constancy of the γ values in this region, as shown in the following section, tends to support the reality of this variation of α with field.

VALUES OF THE SECOND TOWNSEND COEFFICIENT, γ

The coefficient γ was calculated at various fields for each of the twelve electrodes used as cathode. Solving Eq. (1) for γ we have

$$\gamma=\frac{K-e^{\alpha x}}{K(e^{\alpha x}-1)}, \tag{3}$$

where K represents the amplification at a particular value of the field. The values of α used in these determinations were taken from the curve plotted in Fig. 5. The results are shown in the plots of Figs. 6 and 7. There is considerable variation between the individual cathodes of each type. The Ni cathode of the 16-mm tube and the Cs-Ag-O cathode of the 8.1-mm tube each exhibits especially low values of γ. It is significant that over a considerable range of field

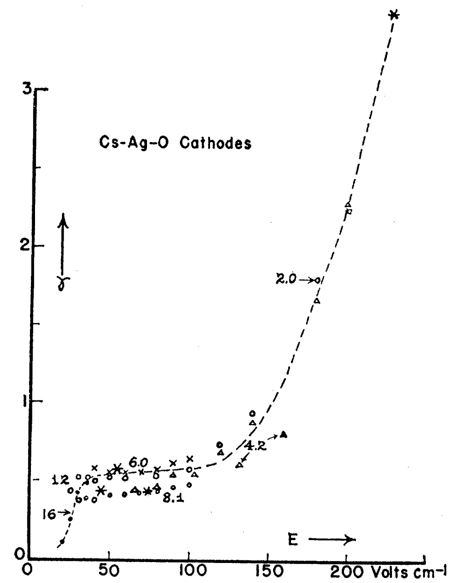

FIG. 7. Graphical summary of calculated values of the second Townsend coefficient γ for the Cs-Ag-O cathodes. The numbers indicate electrode separations in mm. The points marked (*) are γ's calculated from sparking potentials.

strengths the values of γ for most of the cathodes are sensibly constant. For the nickel cathodes this range includes the region in the neighborhood of $E=110$ volts/cm at which α reaches its maximum value. The present results cannot be taken as indicating accurately the form of the upward trend of γ at high fields, since the values in this region are least reliable. It is to be expected, however, that at fields sufficiently high the efficiency of the ions in releasing secondary electrons should increase rather rapidly as the ion is able to gain appreciable amounts of energy from the field in one mean free path. This sudden increase in γ is verified by the following data obtained from measurements of the potentials at which a glow discharge is set up in the gas.

[8] A. A. Kruithof and F. M. Penning, Physica 3, 529 (1936); 4, 447 (1937).

γ-Values from Striking Potentials

In the neighborhood of the minimum sparking potential of a gas it is to be expected that the nature of the cathode will noticeably alter the value of potential at which a glow discharge is initiated. In measurements of the striking potentials in the tubes studied with normal and reversed electrodes this was found to be the case, for the potential required to set up a glow was considerably higher when the nickel electrode was the cathode; in one tube it was nearly double that required for the silver cathode.

In these measurements a resistance of 460 megohms was inserted in the circuit so that the glow current could not exceed a fraction of a microampere. When the tubes were carefully shielded so that no light could enter the bulb, the error in determining the glow-initiating potential was about 2 volts. The values of the potentials so determined are shown in Table II for both types of cathodes.

According to the Townsend theory of sparking, if Eq. (1) represents correctly the manner in which the gas currents grow, the condition for disruptive discharge is the increasing of i to an infinite value. In this case we have

$$1+\gamma-\gamma e^{\alpha x}=0, \quad \text{or} \quad \gamma=(e^{\alpha x}-1)^{-1}. \qquad (4)$$

Values of the positive ion coefficient calculated by means of this equation from the α corresponding to the value of field at which striking of the glow occurred, are shown in the columns marked γ_s of Table II. For the 4.2- and 2.0-mm tubes γ's cannot be computed because the values of α are not known at very high fields.

The values of γ_s given in Table II are indicated by an asterisk (*) in the plots of Figs. 6 and 7, and are in good agreement with the ion coefficients obtained from measurements of the Townsend currents. These findings confirm the original ideas of Holst and Osterhuis[9] concerning the lowering of the sparking potential when low work-function cathodes are used, and support their original hypothesis concerning the action of positive ions.

Table II. *The γ-values from striking potentials.*

Electrode separation	Nickel Cathode V_s	Nickel Cathode γ_s	Silver Cathode V_s	Silver Cathode γ_s
16.0 mm	118.0 v	0.185	72.4 v	0.43
12.3	120.3	0.26	63.5	0.58
8.1	117.0	0.60	89.5	0.58
6.0	140.0	(2.20)*	92.5	1.20
4.2	129.0		95.0	(3.50)*
2.0	147.6		97.0	

* These estimates were obtained by extrapolating the α/p_0 curve of Fig. 5.

Discussion

It is important to emphasize the conditions under which these experiments have been conducted. The limitations imposed included the use of small light intensities, the use of plane parallel electrodes with emission restricted to a small area in order to obtain uniform fields and to avoid effects of charges on the bulb walls, and the employment of well tested procedures in formation of the cathode and purification of the filling gas. It is believed that the precautions taken were justified in view of the excellent reproducibility of photosensitivity and gas amplification which was obtained throughout the course of the investigation. No attempt has been made to secure a complete analysis of gas amplification at high current densities in argon photo-tubes as employed in technical work.

The agreement of the values of α/p_0 at low fields with those of Kruithof and Penning indicate that the argon used was relatively free from contamination by foreign gases or vapors. The large variation in values of the electron ionization coefficient obtained at higher fields, however, indicates that not all of the complicating effects previously mentioned have been eliminated. Some of the fluctuations undoubtedly arise from unavoidable variations in cathode activation and gas pressure in the six tubes used. Differences in γ-values for the same type of cathode surface in different tubes indicate considerable fluctuation in the secondary emission efficiencies of the individual cathodes. Part of the lack of agreement in α-values obtained with normal and reversed polarity of the electrodes may be caused by the ionization of traces of Cs vapor or by release of electrons from the cathodes by metastable argon atoms. That positive ions play an important role in the amplification process, however, seems to be well established by the results

[9] G. Holst and E. Oosterhuis, Physica **1**, 78 (1921): Phil. Mag. **46**, 1117 (1923).

obtained in this attempt to apply Townsend's theory to a type of discharge which is complicated by many factors.

It is believed that the enhancement of cathode photosensitivity by ion bombardment indicates a lowering of the cathode work-function by the joint action of adsorbed ions and applied field. This "cathode" effect becomes increasingly important at high current densities where distortion and hysteresis occur in normal operation of commercial tubes and where time lag effects are very large. The study of gas currents in these tubes at higher current densities is being continued with particular reference to the dynamical characteristics of the discharge.

In conclusion the writer wishes to acknowledge the help of the Research and Engineering Department of the RCA Manufacturing Company, Inc., Harrison, New Jersey and also support for the project from the Graduate School of Northwestern University in the form of a grant. Mr. Dwight Wennersten gave valuable assistance in taking data for the photosensitivity plots and Dr. A. M. Glover of the RCA Manufacturing Company was responsible for the preparation of the tubes.

APRIL 15, 1939 PHYSICAL REVIEW VOLUME 55

The Propagation of Electric Currents in Terminated Lines

Solutions of the Telegraphic Equation*

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Aberdeen Proving Ground, Maryland
(Received February 17, 1939)

By a transformation of Heaviside's general solution, a special solution for the current having the value $e^{-\iota t}f(t)$ at $x=0$ is obtained. From this solution, an expression is obtained for the potential difference across the line at $x=0$ in terms of $f(t)$ and its successive integrals. From this and the terminal conditions a differential equation of negatively infinite order for $f(t)$ is obtained. This equation can be solved to any given degree of approximation by a method of integral operators. An illustrative problem is solved. An integral solution of the telegraphic equation and a corresponding expression for the potential difference at $x=0$ are given. An alternative method to Heaviside's and Carson's is suggested for obtaining the general solution for the terminated line.

THERE have been discovered several methods of computing the current in a terminated line when an electromotive force is applied in one of the terminal networks. Of these the principal ones are Heaviside's[1] operational methods which have been given a rigorous deductive foundation by Bromwich[2] and Carson.[3] Heaviside's solutions hold formally only for a unit electromotive force applied at time $t=0$ but Carson[3] has shown that the response to an arbitrary e.m.f. may be computed from Heaviside's results by the application of Duhamel's superposition theorem. Methods similar to those employed by Bromwich in his deduction of Heaviside's solutions have also been applied to the solution of circuit problems by Wagner[4] and Fry.[5]

The method about to be described is of the classical type, in that a special solution of the telegraphic equation is obtained. This solution involves a function $f(t)$ which is conveniently determined from the boundary conditions with an arbitrary e.m.f. by operational methods.

* Published by permission of the Chief of Ordnance, U. S. A.

[1] O. Heaviside, *Electromagnetic Theory* (1893), Vol. 1, 2 and 3. See also V. Bush, *Operational Circuit Analysis with an Appendix by N. Wiener* (1929); and E. J. Berg, *Heaviside's Operational Calculus* (1936).

[2] T. J. I'A. Bromwich, Proc. Lond. Math. Soc. (2) **15**, 401–448 (1916). See also H. Jeffries, *Operational Methods in Mathematical Physics* (1931).

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