The source of excitation used in the present work possesses some usefulness in band spectra research because of the two conditions of high gas pressure and low temperature that can be simultaneously met. The difference between two sources of excitation which do not have these two factors in common is illustrated in Fig. 4 in which some emission of oxygen containing a small amount of nitrogen is shown. The emission in the upper picture is from the ozonizer while in the lower picture it is from a high voltage, low current arc operated by the same type of transformer. In both cases the gas was at atmospheric pressure. In the lower picture the principal emission is the Runge bands of oxygen,[6] while from the ozonizer it is the second positive group of nitrogen. A further study is being made of the mechanism of excitation in the ozonizer.

The authors gratefully acknowledge the assistance given them by Mrs. L. S. Deming and Mr. R. T. O'Connor in the course of this work.

[6] C. Runge, Physica 1, 254 (1921).

APRIL 15, 1939 PHYSICAL REVIEW VOLUME 55

The Analysis of Nuclear Binding Energies

WALTER H. BARKAS
Institute for Advanced Study, Princeton, New Jersey

(Received February 25, 1939)

The fine structure of the mass defect curve has been studied, especially with reference to the nuclear symmetry character. From the analysis it has been found possible to obtain satisfactory empirical curves for the functions L and E_0 appearing in Wigner's theory. The behavior of these curves allows deductions to be made regarding nuclear shells, and binding energies of both known and as yet unknown, unstable, nuclei. About 150 masses have been computed in this way.

INTRODUCTION

THE light nuclei are well known to exhibit a marked periodicity of four, which may be interpreted as a consequence of the operation of the Pauli principle in systems containing two kinds of heavy particles. One may obtain an instructive representation of this behavior by plotting, as a function of the mass number A, the change in binding energy observed on the addition of successive particles. Such a plot (corresponding to the familiar graphs of ionization potentials for atomic electrons) is shown in Fig. 1. The effect is exhibited for two methods of building up atoms:

$$\nu+\mathrm{H}+\nu+\mathrm{H}+\nu+\mathrm{H}+\cdots,$$

and $$\nu+\nu+\mathrm{H}+\mathrm{H}+\nu+\nu+\cdots,$$

where ν and H represent, respectively, a neutron and a hydrogen atom. The following facts are made evident without a detailed study by this method of displaying the empirical information on atomic masses.

(1) Successive particles entering the same period or level are bound with energies increasing in a roughly linear way for small mass numbers, but later a depression of the proton points takes place.

(2) The heights of the peaks decrease as the mass number increases.

(3) The binding energy of the last proton in Ne^{20} is abnormally high compared to that in O^{16}. This is probably to be correlated with the unusually low total binding energy of Ne^{20} noticed by Hafstad and Teller.[1] (We shall later adduce evidence that Ne^{20} is at the end of the $2s$ sub-shell.)

E. Wigner[2] has studied the consequences of a detailed application of the Pauli principle to a many-body nuclear model in which the specifically nuclear forces are equal between all pairs of particles and do not depend on the spin ("1th approximation"). Although it is now well known, particularly from the neutron-proton scattering

[1] L. R. Hafstad and E. Teller, Phys. Rev. 54, 684 (1938).
[2] E. Wigner, Phys. Rev. 51, 947 (1937).

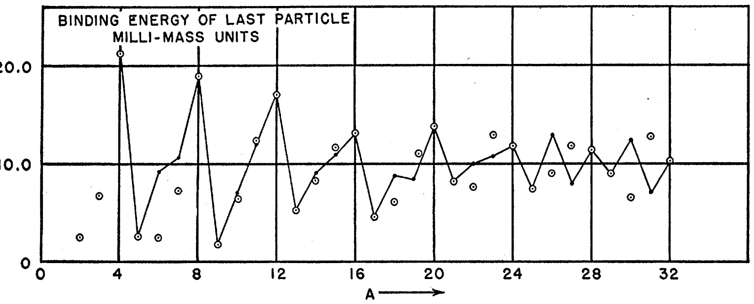

FIG. 1. Binding energy of successive nuclear particles as a function of the mass number. The connected points are determined from the series $\nu+\nu+\pi+\pi+\nu+\nu+\cdots$. The encircled points come from the sequence $\nu+\pi+\nu+\pi+\nu+\cdots$. The points affected by the mass of He^5 are uncertain.

experiments, and from the reported[3] quadrupole moment of deuterium, that spin dependent forces do actually exist, it seems that in most nuclei they play only a subordinate role so far as the binding energy is concerned.

The binding energy $-E$ was found to consist, under these assumptions, of the sum of four terms as follows:

$$-E=-E_0(A)-\Xi' L(A) +\tfrac{1}{2}K_1A^{-\frac{1}{3}}(A-T_\zeta-1)T_\zeta-K_2T_\zeta^2/A. \quad (1)$$

These terms, in this order, are generally decreasing in magnitude. The first, $-E_0$, is a smoothly increasing function of A. The factor Ξ' is a number which represents the symmetry character of the configuration, while L is again a smooth function of A, decreasing as A^{-1} for large A. $T_\zeta=\frac{1}{2}(N-Z)$ is the isotopic spin of the nucleus, and K_1 is a constant determined by the Coulomb energy under the assumption that the nuclear volume is proportional to A. The last term, which gives the dependence of kinetic energy on isotopic spin, while perhaps not strictly applicable to actual nuclei, is the best estimate available. The last two terms for many light nuclei are to be regarded as hardly more than small corrections. In this paper we shall undertake to examine the experimental findings by comparing them with this formula. The central problem will then be to investigate the behavior of the functions L and E_0.

Equation (1) may for the purpose of this study be written

$$-E_0-\Xi' L=-E -\tfrac{1}{2}K_1A^{-\frac{1}{3}}(A-T_\zeta-1)T_\zeta+K_2T_\zeta^2/A, \quad (2)$$

[3] Kellogg, Rabi, Ramsey and Zacharias, Phys. Rev. **55**, 318 (1939).

where known or measurable terms appear on the right. The method of analysis will be to obtain as accurate estimates as possible for these terms, and then to graph the function $-E_0-\Xi' L$ for each value of Ξ'. (The value of Ξ' corresponding to a given nucleus provides a convenient and significant property for its classification.) Several curves are thus obtained, the ordinate separations of which determine L as a function of A. After one has the function L, $-E_0$ may be derived most accurately by using the $k\alpha$ type[4] nuclei, but where a rapid change of slope takes place other nuclear types have also been used to obtain points at other mass numbers. In the next section the various terms on the right side of Eq. (2) will be evaluated and examined as critically as present information permits.

NEGLECTED EFFECTS

The formula (1) is simple and unambiguous, so that when compared with the measured binding energies any important influences neglected in the derivation are detectable. As will be

[4] A configuration $k\alpha$ (an alpha-particle type nucleus or one having $2k$ protons and $2k$ neutrons) may for convenience be referred to as a nuclear "core," because such a structure is known to bind individual protons and neutrons strongly. Other nuclei will be represented by $k\alpha$ plus additional neutrons and protons; e.g. $4\alpha+3\nu+\pi=F^{20}$. Such a "core" will only have the usual significance of the word if the additional particles are loosely bound, and if the "core" has no low lying excited states. These conditions are sometimes fulfilled in light nuclei. (O^{16} seems to have its first excited state at over 10 Mev.) In such cases the core will produce something very similar to a central field in which we may have some confidence that the orbital angular momentum will have properties similar to those studied extensively in connection with atomic spectra. The circumstance that spins of light nuclei may be predicted with some confidence suggests also that $L-S$ coupling is a valid assumption.

shown presently, for light nuclei it is feasible to correct for these. On the other hand, for heavy atoms this treatment is not adequate. Among these possible additional effects the following may be listed: (a) Additional spin dependent forces between the nuclear particles, (b) an effect on the symmetry character of the lowest state due to large Coulomb energy, (c) a strong tendency for shell or group structure, (d) a different dependence of kinetic energy on isotopic spin than that adopted, and (e) the electrostatic energy not so simply dependent on mass number and charge as supposed.

(a) When the mass values are studied it is observed that, if the assumption of a symmetrical interaction not depending on spin between all pairs of nuclear particles is made, the nuclei $(k\alpha+\nu+\pi)$ are more tightly bound compared to $(k\alpha+2\nu)$ than would be anticipated merely from the mass difference between the proton and neutron, after correction for the Coulomb energy. An accurate estimate of the electrostatic energy may be obtained from the isobaric pairs $(k\alpha+\nu)$, $(k\alpha+\pi)$, if the sole difference in binding energy for these isobars is this Coulomb energy.[5]

One is led to believe also that the kinetic energies are not different in the isobars $(k\alpha+2\nu)$, $(k\alpha+\pi+\nu)$ because the Pauli principle in each case permits both extra particles to enter the same level. (For nuclei with high Coulomb energy this situation may, however, be altered.) Hence any difference in the binding energies, corrected for Coulomb energy, appears to be attributable to a difference in the interaction between the "extra" particles. Since the spins of $(k\alpha+2\nu)$ nuclei are zero, and those of $(k\alpha+\pi+\nu)$ nuclei unity in every case investigated, we might designate these interactions by ${}^1\nu\nu$ and ${}^3\pi\nu$. That such a relatively large spin dependence exists has already been deduced from experiments on slow neutron scattering by hydrogen. Table I exhibits the present evidence for the effect.[6] The Coulomb energies were derived from the inspection limits of beta-ray spectra when available. Otherwise they were computed by the usual method.[2] At mass numbers greater than 17 the Coulomb energy formula has not been adequately verified, so no great reliance should be placed on the differences at higher mass numbers. Indeed, the experiments of Pollard, Schultz and Brubaker,[7] and Brubaker[8] make it appear that considerable deviations from a constant ratio of nuclear volume to mass number may exist. Another interpretation of these experiments may be that the binding energy of a helium nucleus to A^{40} is particularly great because the addition of an alpha-particle to A^{40} carries one from a point somewhat off the bottom of the Heisenberg valley back to it. Thus the virtual levels of Ca^{44} may be reached with comparatively low α-particle energies, and this may be responsible for the anomalous penetration and scattering.

Although the spin dependent component of the nuclear forces is here shown to be sufficiently large to be detected definitely in the binding energies, it produces, after all, rather a small effect and will not cause any great difference in the behavior of actual nuclei as compared to the nuclear model based on a symmetric spin-independent Hamiltonian. The average of the

TABLE I. *Effect of spin dependent forces.*

Isobaric pair	Binding energy difference mMU	Coulomb energy difference mMU	2γ
He^6, Li^6	−3.2	1.4	4.6
Be^{10}, B^{10}	0.2	2.4	2.2
C^{14}, N^{14}	0.6	3.3	2.7
O^{18}, F^{18}	2.7	3.9	1.2
Ne^{22}, Na^{22}	2.5	4.6	2.1
Mg^{26}, Al^{26}	3.9	5.2	1.3
Si^{30}, P^{30}	5.1	5.7	0.6

[5] The assumption that such homologous nuclei have binding energies differing only by the Coulomb energy appears to depend just on the premise that the average specific interaction between pairs of protons in the nucleus is the same as the similar average interaction between neutrons. This is accepted as a principal assumption, the approximate validity of which may be established by a study of Fig. 1 or otherwise.

[6] The effect is expected to decrease with increasing nuclear radius as the average separation of the particles increases with the size of the nucleus. Even the irregularities seem to be real, however. A more complete study of this phenomenon seems to require an understanding of, or at least detailed assumptions regarding, the forces between individual elementary particles. Feenberg and Phillips (Phys. Rev. **51**, 597 (1937)) have observed also that possibly the order of levels in B^{10} and N^{14} is not that given by the Hartree model. Another matter causes difficulty. Not all the beta-activities for these isobars lie on the same Sargent curve, nor are the deviations systematic. For these reasons we avoid using the mass values of atoms of types $(k\alpha+2\nu)$ and $(k\alpha+\nu+\pi)$ in the determination of the functions L and E_0 where small discrepancies might produce spurious kinks in the curves.

[7] Pollard, Schultz and Brubaker, Phys. Rev. **53**, 351 (1938).

[8] G. Brubaker, Phys. Rev. **54**, 1011 (1938).

$^3\pi\nu$ and the $^1\nu\nu$ interaction is approximately the average specific interaction between nuclear particles. If we write $^3\pi\nu - {}^1\nu\nu = 2\gamma$, then a reasonable procedure seems to be merely to correct for the effect in Eq. (1) by adding a term γ to the right-hand side of the equation for nuclei of type $(k\alpha+\nu+\pi)$, and subtracting it for nuclei $(k\alpha+2\nu)$ (and also presumably for $(k\alpha+2\pi)$).

(b) Disintegration energies are known very accurately for heavy, naturally radioactive atoms, and differences of binding energies are readily deduced. Consistent values for the constants appearing in the binding energy formula were not obtained, however, in many attempts at their determination. The most promising of these methods, that of utilizing the energy balance in the two branches of ThC, RaC and AcC proved no more successful than other devices. The conclusion to be drawn from this result is doubtless that the symmetry character of the ground state is affected by the large electrostatic energy, and that probably other terms, such as the Coulomb energy itself, are affected also.

(c) It might be expected on general grounds that the behavior of the function $-E_0(A)$ should yield information regarding possible "shells" in nuclei, for, if no such structures exist, the curve should be smoothly increasing with A. When the form of this function is obtained the evidence for such shells will be exhibited. On the other hand, an estimate of the amount of correlation between groups of particles such as those making up an α-particle is difficult. This is especially so, since according to Hafstad and Teller[1] no change of symmetry is introduced by postulating an α-particle structure. An α-particle model which assumes a correlation of *unity* between the positions of groups of four particles has very limited validity, however. This is demonstrated, for example, by the occurrence with a fairly large cross section of the type of reaction

$$k\alpha+\alpha\rightarrow(k+1)\alpha^*\rightarrow(k\alpha+2\nu+\pi)+\pi$$

when the energy balance permits. It seems that further information relative to the nature and probability of such grouping effects may be obtained from the several possible disintegration experiments in which the compound nucleus is of the $k\alpha$ type. α-particle projectiles are now available with sufficiently high energies so that almost all possible products of disintegration may be anticipated. It is probable, however, that the α-particle model is a better approximation for a nucleus in the ground state than in an excited state. The number of possible states of excitation should also depend on the amount of particle grouping.

(d) The expression $K_2T_\zeta^2/A$ for the dependence of kinetic energy on T_ζ is for most light nuclei a small and doubtful term. The form of this expression erroneously implies a difference in kinetic energy for the nuclear types $(k\alpha+2\nu)$, $(k\alpha+\pi+\nu)$, which has already been discussed. This term will therefore be omitted for nuclei $(k\alpha+2\nu)$. For the larger values of T_ζ the value 26.4 mMU for K_2 will be used.[2] Another effect which occurs at the end of the $3d$ shell has been evaluated by Wigner.[2] Particles forced outside the closed shell appear to lose binding energy of approximately 1.6 mMU. A similar effect may be expected also at the end of other shells.

(e) The expression for the electrostatic energy of the last proton might be expected to vary with the radial extension of its wave function and therefore on its tightness of binding. A formula for this energy allowing for such an effect has been derived by Bethe.[9] For reasons of simplicity, however, this more refined formula will not be used; the error thus introduced is very slight. It will be shown next that a very satisfactory constant value is obtained for K_1 by using the formula for this constant derived from Eq. (1) on the lighter elements.*

TABLE II. *Determination of coefficient of Coulomb energy term.*

A	ΔE mMU	K_1
3	0.86	1.241
7	1.87	1.193
11	3.14	1.397
13	3.27	1.281
15	3.66	1.289
17	3.91	1.257
	Average	1.276

[9] H. A. Bethe, Phys. Rev. **54**, 436 (1938).

* *Note added in proof.*—As remarked earlier in this paper the Coulomb energy formula has not been tested adequately for $A>17$. This should perhaps be emphasized even more, particularly since the rather poorly known beta-disintegration energy of Si^{27} was not used in the evaluation of K_1 because it gave a value differing widely from the other numbers obtained, so that it was not considered wise to include this figure in averaging.

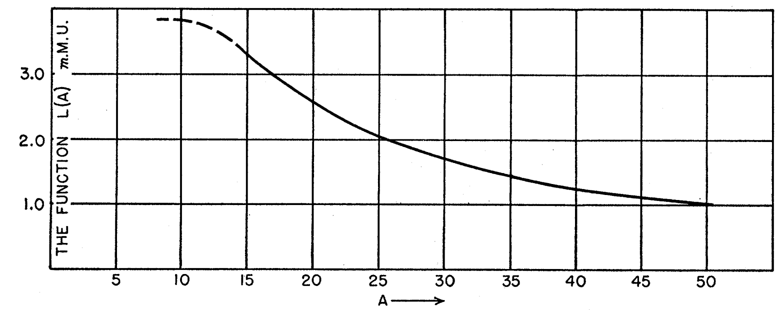

FIG. 2. The function $L(A)$ determined from the empirical masses.

From the isobaric pairs $(k\alpha+\nu)$, $(k\alpha+\pi)$, and $(k\alpha+\nu+2\pi)$, $(k\alpha+\pi+2\nu)$ the Coulomb energy may be determined, for on subtracting their binding energy equations, one is left with the expression

$$K_1 = 2A^{\frac{1}{3}}\Delta E/(A-1),$$

where ΔE is the binding energy difference for the isobars. The data from which K_1 is determined are given in Table II.

THE FUNCTIONS $L(A)$ AND $E_0(A)$

Figure 2 is a curve for $L(A)$. For $A>36$ very incomplete mass data are available, but in many cases L values may be obtained from the inspection upper limits of β-ray spectra when the absence of γ-rays has been established. The curve in this interval is to be regarded as considerably more tentative than for intermediate mass numbers. Likewise for very small mass numbers some approximations become poor. For example, it is difficult to be certain that the expression which has been used will give the correct kinetic energy difference between the isobars Li^8-Be^8, so that the behavior of the curve for small mass numbers may be misleading. Indeed, the values for L obtained by different methods do not check well below $A=12$; it is probable that the kinetic energy term, which here becomes quite large for certain nuclear types, is a very poor approximation in this region. For this reason the curve has not been carried below $A=8$. It is found, however, that almost the entire useful range of the curve $L(A)$ is represented rather accurately by the hyperbola $AL=50$, in agreement with theory.[2]

The values of Ξ' may be obtained according to the rules given in Wigner's paper.[2] Table III gives the numerical values taken on by Ξ' for some nuclear types.

It is anticipated that the curve for $L(A)$ may be found useful in the study of excited states.[10] It is notable in this connection that excited states of $k\alpha$ type nuclei which may be put in correspondence with the ground states of the isobaric nuclei $((k-1)\alpha+\pi+3\nu)$ do seem to exist. The present paper is, however, concerned principally with the normal configurations.

By using the curve derived for L one may add $\Xi'L$ to $-E_0-\Xi'L$ and obtain the curve for $-E_0$. This was done for $A=4k\pm1$ by using $(k\alpha+\nu)$, $(k\alpha+\pi)$, $(k\alpha+2\nu+\pi)$ and $(k\alpha+\nu+2\pi)$. Above $A=40$ the points were determined by using Sc^{45}, Ti^{48} and V^{51}.

The curve, Fig. 3, for E_0 makes a very definite dip in the region 16–20. Beyond 20 it rises smoothly to 40 where another break in the curve occurs. The points beyond 40 do not lie on a continuation of the previous section. Other circumstances likewise indicate that a change[11] takes place in the curve at this point.

TABLE III. *The ground state symmetry character for light nuclei.*

A	$T_\zeta=0$	$\pm1/2$	±1	$\pm3/2$	±2	$\pm5/2$	±3	$\pm7/2$	±4
$4k$	5/2		13/2		17/2		29/2		37/2
$4k+1$		35/8		59/8		91/8		131/8	
$4k+2$	5		5		25/2		13		20
$4k+3$		35/8		59/8		91/8		131/8	

[10] L. Motz and E. Feenberg, Phys. Rev. **54**, 1055 (1938); J. Bardeen and E. Feenberg, Phys. Rev. **54**, 809 (1938).

[11] It was remarked earlier that the change in kinetic energy taking place at $A=40$ can be detected. It may be noticed, also, that on a T_ζ *vs.* A diagram the locus of stable nuclei turns definitely to positive values of T_ζ beyond 40, while up to that point zero and even negative values were possible.

TABLE IV. *Computed and observed atomic masses. The experimental*[13] *value appears above the computed mass. Parentheses indicate a possible uncertainty* $\lesssim$ *1 mMU. P.E. in* 10^{-5} *MU.*

A	$T_\zeta=-1$	$-1/2$	0	1/2	1	3/2	2	5/2	3	7/2	4
1		↑H^{1} 1.00812±2		↑n^{1} 1.00893±5							
2			↑H^{2} 2.01472±2								
3		↑He^{3} 3.01701±12		↑H^{3} 3.01704±7							
4			↑He^{4} 4.00388±7								
6			↑Li^{6} 6.01690±20		↑He^{6} 6.02090±30						
7		↑Be^{7} 7.01911±30		↑Li^{7} 7.01804±20							
8			↑Be^{8} 8.00777±20 (8.0078)		↑Li^{8} 8.0249±30 (8.0250)						
9		↑B^{9} ...... (9.0164)		↑Be^{9} 9.01497±25 (9.0151)		↑Li^{9} ...... (9.0313)					
10			↑B^{10} 10.01605±30 (10.0144)		↑Be^{10} 10.01671±30 (10.0148)						
11		↑C^{11} 11.01544±35 11.0151		↑B^{11} 11.01286±20 11.0130		↑Be^{11} ...... (11.0277)					
12	↑N^{12} 12.0233		↑C^{12} 12.00398±10 12.0040		↑B^{12} 12.0193±70 12.0188						
13		↑N^{13} 13.01005±15 13.0101		↑C^{13} 13.00766±15 13.0077		↑B^{13} ...... (13.0207)					
14	↑O^{14} ...... (14.0136)		↑N^{14} 14.00750±8 (14.0080)		↑C^{14} 14.00763±12 (14.0083)						
15		↑O^{15} 15.0078±40 15.0080		↑N^{15} 15.00489±20 15.0051		↑C^{15} ...... (15.0165)					
16	↑F^{16} ...... (16.0175)		↑O^{16} 16.00000 16.0000		↑N^{16} 16.011±200 16.0114						
17		↑F^{17} 17.0076±30 17.0075		↑O^{17} 17.00450±7 17.0044		↑N^{17} ...... (17.0136)					
18	↑Ne^{18} ...... (18.0114)		↑F^{18} 18.0066±100 (18.0066)		↑O^{18} 18.0047±100 (18.0047)						
19		↑Ne^{19} 19.0077		↑F^{19} 19.00452±17 19.0042		↑O^{19} ...... (19.0139)					
20	↑Na^{20} ...... (20.0160)		↑Ne^{20} 19.99881±11 19.9988		↑F^{20} 20.0087±100 (20.0066)		↑O^{20} ...... (20.0168)				
21		↑Na^{21} 21.0035		↑Ne^{21} 20.99968±23 20.9996		↑F^{21} ...... (21.0059)		↑O^{21} ...... (21.0176)			
22	↑Mg^{22} ...... (22.0062)		↑Na^{22} 22.0002±50 (21.9999)		↑Ne^{22} 21.99864±36 (21.9983)		↑F^{22} ...... (22.0157)				
23		↑Mg^{23} ...... (23.0004)		↑Na^{23} 22.9961±35 22.9961		↑Ne^{23} 23.0013		↑F^{23} ...... (23.0114)			
24	↑Al^{24} ...... (24.0059)		↑Mg^{24} 23.9924±60 23.9920		↑Na^{24} 23.9974±45 23.9974		↑Ne^{24} ...... (24.0012)		↑F^{24} ...... (24.0160)		
25		↑Al^{25} 24.9970		↑Mg^{25} 24.9938±90 24.9926		↑Na^{25} ...... (24.9967)		↑Ne^{25} ...... (25.0054)		↑F^{25} ...... (25.0186)	
26	↑Si^{26} ...... (25.9993)		↑Al^{26} 25.9929±200 (25.9935)		↑Mg^{26} 25.9898±50 (25.9905)		↑Na^{26} ...... (26.0044)		↑Ne^{26} ...... (26.0069)		
27		↑Si^{27} 26.9931±150 26.9944		↑Al^{27} 26.9899±80 26.9896		↑Mg^{27} 26.9921±150 26.9929		↑Na^{27} ...... (27.0003)		↑Ne^{27} ...... (27.0119)	
28	↑P^{28} 28.0001		↑Si^{28} 27.9866±60 27.9870		↑Al^{28} 27.9903±70 27.9904		↑Mg^{28} ...... (27.9926)		↑Na^{28} ...... (28.0041)		
29		↑P^{29} 28.9919		↑Si^{29} 28.9866±60 28.9869		↑Al^{29} 28.9904±200 28.9892		↑Mg^{29} ...... (28.9956)		↑Na^{29} ...... (29.0060)	
30	↑S^{30} ...... (29.9944)		↑P^{30} 29.9882±150 (29.9891)		Si^{30} 29.9832±90 (29.9841)		↑Al^{30} ...... (29.9954)		↑Mg^{30} ...... (29.9964)		
31		↑S^{31} 30.9892		↑P^{31} 30.9842±50 30.9841		↑Si^{31} 30.9862±60 30.9858		↑Al^{31} ...... (30.9911)		↑Mg^{31} ...... (31.0002)	
32	↑Cl^{32} 31.9950		↑S^{32} 31.9823±30 31.9823		↑P^{32} 31.9841±50 31.9841		↑Si^{32} 31.9849		↑Al^{32} ...... (31.9939)		
33		↑Cl^{33} 32.9872		↑S^{33} 32.9816		↑P^{33} 32.9826		↑Si^{33} ...... (32.9870)		↑Al^{33} ...... (32.9950)	
34	↑A^{34} ...... (33.9898)		↑Cl^{34} 33.981±300 (33.9839)		↑S^{34} 33.978±200 (33.9784)		↑P^{34} ...... (33.9874)		↑Si^{34} ...... (33.9873)		
35		↑A^{35} 34.9851		↑Cl^{35} 34.9803±60 34.9792		↑S^{35} 34.9796		↑P^{35} ...... (34.9833)		↑Si^{35} ...... (34.9920)	
36	↑K^{36} 35.9907		↑A^{36} 35.9780±100 35.9783		↑Cl^{36} 35.978±100 35.9788		↑S^{36} 35.978±100 35.9784		↑P^{36} ...... (35.9870)		
37		↑K^{37} 36.9830		↑A^{37} 36.9769		↑Cl^{37} 37.9779±120 36.9767		↑S^{37} ...... (36.9812)		↑P^{37} ...... (36.9889)	
38	↑Ca^{38} ...... (37.9860)		↑K^{38} ...... (37.9795)		↑A^{38} 37.974±250 (37.9735)		↑Cl^{38} 37.981±300 (37.9821)		↑S^{38} ...... (37.9826)		
39		↑Ca^{39} ...... (38.9811)		↑K^{39} ...... (38.9747)		↑A^{39} ...... (38.9755)		↑Cl^{39} ...... (38.9794)			
40			↑Ca^{40} 39.974±100 (39.9738)		↑K^{40} 39.975±100 (39.9748)		↑A^{40} 39.97504±26 (39.9750)		↑Cl^{40} ...... (39.9819)		
45				↑Ti^{45} ...... (44.9711)		↑Sc^{45} 44.9689±100 44.9689		↑Ca^{45} ...... (44.9693)			
48							↑Ti^{48} 47.9651±50 47.9651				
51						↑Cr^{51} 50.9613±100 (50.9611)		↑V^{51} 50.9598±100 50.9598		↑Ti^{51} 50.9612±100 (50.9610)	

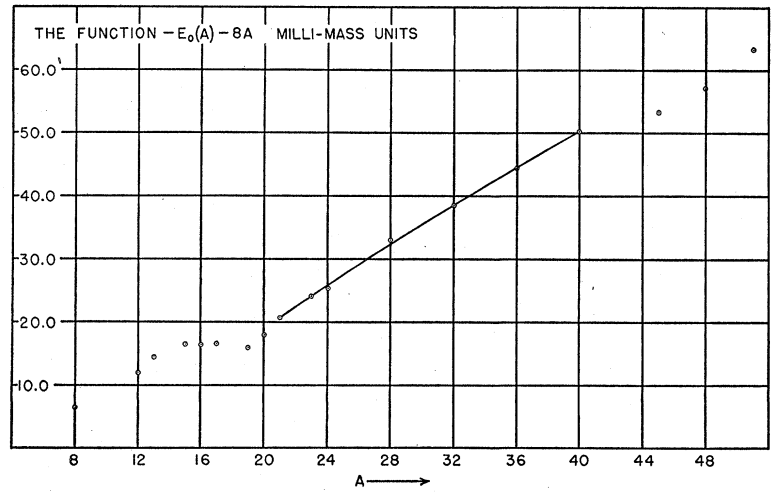

FIG. 3. $-E_0-8A$ as a function of the mass number. The term $8A$ is subtracted in order to reduce the ordinate values and bring out the structure of the curve. The function is expected to be smooth through the range of mass numbers where the $3d$ shell is being built up. Such a curve is drawn through this interval.

It has been considered fairly certain that O^{16} completes the $2p$ shell. The present evidence points in that direction, too. One may note in Fig. 1, for example, that the last neutron in O^{17} is bound very loosely. Such an effect is not, however, as evident in the case of Ne^{21}, but the

[13] References for experimental data: For the masses n^1, H^1, H^2, H^3, He^3, He^4, Li^6, Li^7, Be^8, Be^9, B^{10}, B^{11}, C^{12} and C^{13} I am indebted to Professor H. A. Bethe who kindly communicated this list to me. This combination at the moment seems most reliable. In arriving at these figures the disintegration data of Allison *et al.* (Phys. Rev. **54**, 171 (1938) and **55**, 107 (1939)) were utilized along with the best value for the binding energy of the deuteron. (H. A. Bethe, Phys. Rev. **53**, 314 (1938)). Other disintegration data used were $H^2(d, n)He^3$, 3.26 Mev and $Li^6(d, \alpha)He^4$, 22.21 Mev. The uncertainties given were estimated by the writer. With these data the mass difference $Be^{10}-B^{10}$ is, however, suspiciously large in view of the relative stability of Be^{10}.

He^6 comes from the present value of Li^6 and the beta-disintegration energy of Bjerge and Brostrom, Danske Videns. Selskab, Math. fys. Med. XVI, 8.

He^5 is not listed, for although Joliot and Zlotowski (J. de phys. et rad. **9**, 403 (1938)) report that it is stable with mass 5.0106 ± 0.0005, this is very difficult to interpret, for such a figure implies the instability of Be^9. By examining Fig. 1, one might be led to conclude that the stability of He^5 is improbable.

Li^8 is derived from the disintegration energy of Rumbaugh, Roberts and Hafstad (Phys. Rev. **54**, 657 (1938)) taken with the present value of Li^7.

Be^7 is derived from the energy limits determined from a study of the K-electron capture phenomenon by Breit and Knipp (Phys. Rev. **54**, 652 (1938)), taken with the adopted mass of Li^7.

C^{11} is obtained from the adopted value for B^{11} and the mass difference $C^{11}-B^{11}$, taken from the article of Livingston and Bethe (Rev. Mod. Phys. **9**, 373 (1937)).

N^{13} is obtained by using the value for C^{13} and the positron energy of N^{13} given by Lyman (Phys. Rev. **55**, 234 (1939)).

C^{14} comes from Bethe (Phys. Rev. **53**, 314 (1938)).

O^{18} was obtained by averaging the values of Mattauch and Aston quoted in Livingston and Bethe. F^{18} is given relative to this value.

F^{20} comes from the results of Burcham and Smith (Proc. Roy. Soc. **A168**, 176 (1938)). The uncertainty was estimated by the writer.

Cl^{36} and S^{36} are derived as follows: Grahame (Phys. Rev. **54**, 972 (1938)) reported that the radioactivity of Cl^{36} was undetectable. Hence it must have very nearly the same mass as A^{36}. Likewise its mass cannot be greater than that of S^{36} by more than $c\cdot 1.5$ mMU, since S^{36} is stable. (See A. O. Nier, Phys. Rev. **53**, 282 (1938).) The argument for the isobars at mass 40 is somewhat the same. Henderson (Phys. Rev. **55**, 238 (1938)) puts the upper limit on the beta-rays from K^{40} at 1.3 Mev. K^{40} also goes to A^{40} by K capture. No positrons are observed. From the known mass of A^{40} the other masses are thus obtained.

Sc^{45} is deduced from the results of Pollard (Phys. Rev. **54**, 411 (1938)). When the value given in this reference was put on the $-E_0-\Xi'L$ graph it became clear that something was amiss, because it came out about 9 mMU above its predicted position. The source of the discrepancy was found to be an erroneous quotation of Dempster's value for Ti^{48} in the computation of Sc^{45}. The figure was therefore recomputed and an estimate of the uncertainty made.

Ti^{48} was obtained from the mass-spectrographic measurement of Dempster (Phys. Rev. **53**, 64 (1938)).

V^{51} comes from the transmutation experiment of Davidson and Pollard (Phys. Rev. **54**, 408 (1939)), the uncertainty being estimated by the writer.

Ti^{51} and Cr^{51} come from privately communicated level schemes obtained by H. J. Walke to whom the writer is much obliged for this information.

All the remaining masses were obtained from Livingston and Bethe (Rev. Mod. Phys. **9**, 373 (1937)).

The actual work of this paper was carried through with the earlier value of 1.00813 ± 2 for the mass of H^1. This affects the curve for E_0 slightly (about 0.2 mMU for the heaviest atoms), but has no appreciable effect on the masses.

mass of F^{20} indicates a loss of binding energy of 2.1 mMU for its last neutron, presumably because it is excluded by the Pauli principle from a more tightly bound configuration.

The form of the curve for $-E_0$ suggests the hypothesis that the nuclei from O^{16} to Ne^{20} represent $2s$ configurations.[12] Further information which supports this view is the observed spin $(\frac{1}{2}\hbar)$ for F^{19}, and its magnetic moment, which is approximately that of the proton. These are to be compared with the spin $(\frac{3}{2}\hbar)$, and the somewhat smaller magnetic moment of Na^{23}.

Atomic Masses

From the curves for $-E_0(A)$ and $L(A)$ it is possible to compute the masses of the various isobaric nuclei at each mass number with considerable accuracy. We use the following formula

$$M=1.00853A+0.0008T_\zeta+E_0+\Xi'L -0.000638A^{-\frac{1}{3}}(A-T_\zeta-1)T_\zeta+0.0264T_\zeta^2/A,$$

where E_0 and L are taken in mass units. Table IV gives a list of masses thus computed. The uncertainties in the computation become large for very low mass numbers, and for large absolute values of the isotopic spin. Additional uncertainties arise in connection with the corrections at the ends of closed shells, for mass numbers above 40 where the curve for E_0 is not known very well, and for the nuclei of mass $4k+2$ where the effect of the spin dependent forces was demonstrated to influence the binding energy by c·2 mMU. The computed masses, therefore, for which one or more of these effects seems to introduce an uncertainty of more than 1 mMU have been bracketed.

The computed masses are derived directly from the functions $-E_0$ and L, in order that all masses, known and unknown, might be deduced from one systematic scheme. One could, however, reduce the uncertainty in a few of the computations somewhat if the masses were derived relative to accurately known isobars. This is because the function $E_0(A)$, except at the end of a shell, depends on A only; its choice does not affect comparisons between isobars. In the computation of the masses for $A>16$ the value of L has been taken from the simple relationship $L=50/A$ mMU.

Above mass number 40 the data are very scarce and not consistent enough to make computations of masses feasible except at mass numbers 45 and 51 where good agreement is obtained with the observed values.

An effort has been made to collect the best available experimental masses with which to make a comparison. These are also listed in Table IV above the computed mass.

The writer wishes to record his large debt to Professor E. Wigner who was consulted on many occasions for advice in connection with this work

[12] In the single particle models long range forces tend to favor low orbital angular momentum. On the other hand, the sequence of levels for very short range forces is $1s$, $2p$, $3d$, $2s$, $4f$, $3p$, $5g$, $\cdots$. The present information gives the sequence $1s$, $2p$, $2s$, $3d$ for actual many-body nuclei. Although the level filling up after the $3d$ shell is completed at Ca^{40} is probably $4f$, one may not exclude $3p$, $4d$ or $3s$ levels without further information. Because of the nearly symmetrical interaction between nuclear particles one may speak of "nuclear shells" for the light elements. Beyond Ca^{40}, however, the isotopic spin is no longer zero for stable nuclei, and one must speak of "neutron shells" and "proton shells" separately, because they do not close simultaneously. In addition, for very heavy nuclei, many of the striking features of light nuclei which depend on the statistics of the proton and neutron become less evident.