ing way. From elementary mechanical considerations it can be shown that $\sigma(\theta)\sin\theta=(1/4mE)(d/d\theta)L^2$ where σ is the scattering cross section per unit solid angle. Integration gives

$$L^2=4mE\int_0^{\pi-2\rho}\sigma(\alpha)\sin\alpha d\alpha,$$

which establishes a functional relation between ρ, E and L from the angular distribution curves at different energies. From it the function $\rho(E)$ (for constant L) can be deduced, and the field can then be obtained by a graphical integration. The classical problem of determining the force field from the scattering data can thus be regarded as solved in principle.

It is obvious that such a classical method can give no information on the internal "potential wells" that are assumed for nuclear scattering. Attention has been called to it in the hope that a similar method may be devised which is valid in quantum mechanics to within the limits of the WKB method.

F. C. Hoyt

Ryerson Physical Laboratory,
University of Chicago,
Chicago, Illinois,
March 15, 1939.

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Electric Quadrupole Moment of the Deuteron

Kellogg, Rabi, Ramsey, and Zacharias[1] have pointed out that their observations on the magnetic moments of H_2, D_2 and HD molecules can be accounted for if the deuteron is assumed to have an electric quadrupole moment of magnitude

$$Q=(3z^2-r^2)_{\mathrm{Av}}\sim 2\times 10^{-27}\ \mathrm{cm}^2.$$

If this interpretation is correct, the interaction between the proton and the neutron must contain, besides the usual Majorana and Heisenberg forces, a force which mixes some D state wave function with that of the normal S state of the deuteron. Neither the Thomas precession, which gives no mixing of S with higher states, nor the interaction between the magnetic moments of neutron and proton, which gives much too small a mixing, can account for the observed quadrupole moment. However, a spin-orbit interaction of the form

$$J(r)(\boldsymbol{\sigma}_p\cdot\mathbf{r}_1)(\boldsymbol{\sigma}_n\cdot\mathbf{r}_1),\qquad(1)$$

where $\boldsymbol{\sigma}_p$, $\boldsymbol{\sigma}_n$ are the spins of proton and neutron, respectively, $\mathbf{r}_1$ is a unit vector along their relative coordinate, and $J(r)$ is the radial dependence of the interaction, similar to that which arises in the first-order terms of the Yukawa[2] mesotron theory of nuclear forces, gives a quadrupole moment of the observed magnitude if the strength of this force is assumed to be about equal to that of the Majorana force.

Since the quadrupole moment is small compared to the area of the deuteron, the change in the wave function by this interaction must be small and the use of perturbation theory for the calculation is justified. The usual interaction between the proton and the neutron was taken as a rectangular potential hole of depth D and width a, and $J(r)$ in (1), as a rectangular hole of the same width a but of depth K.[3] The integration over the positive energy of the D state of the deuteron had to be carried out numerically. By taking the binding energy of the deuteron as $\epsilon=2.15$ Mev, and the range of the nuclear forces as $a=2.2\times 10^{-13}$ cm we find

$$Q=(3z^2-r^2)_{\mathrm{Av}}=(K/D)\times 1.9\times 10^{-27}\ \mathrm{cm}^2.$$

By taking the value of Q given by Rabi and his associates,[4] we obtain

$$K/D\sim 1.$$

Thus the strength of the spin-orbit interaction must be about equal to that of the ordinary interaction.

A rough idea of the magnitude of the spin-orbit interaction and its dependence on a can be obtained by taking some suitably chosen average energy $\bar{E}$ for the D state in the denominator of the energy integral and then using the completeness relation for the wave functions. It gives

$$Q=(2/75)(K/D)(D/(\epsilon+\bar{E}))a^3/(a+\alpha),$$

where $\alpha=4.36\times 10^{-13}$ cm is the "radius" of the deuteron. This estimate shows that K/D varies approximately as $1/a^3$ so that if a were taken to be 2.7×10^{-13} cm we would get $K/D\sim\frac{1}{2}$.

It is interesting to note that the ratio between the strength of the spin-orbit interaction, calculated in this way, and the strength of the Heisenberg force between the proton and neutron, determined from the position of the singlet S state of the deuteron, is in rough agreement with the same ratio derived from Yukawa's theory. The experiments quoted above, however, indicate that Q is positive, whereas the Q calculated from Yukawa's spin-orbit interaction (which is repulsive) is negative.

If it is true that there is a spin-orbit interaction of strength comparable with that of the ordinary interaction, then there is an important consequence that the total spin of the particles in a nuclear system will not be, even approximately, a constant of the motion. There can thus be no selection rules for spin.

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R. F. Christy
S. Kusaka

University of California,
Berkeley, California,
March 7, 1939.

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[3] This schematic form for K was chosen for convenience since divergences render questionable the form of the terms given by Yukawa's theory (reference 2).
[4] In this formula, z and r are coordinates of the proton in the reference frame with origin at the center of gravity of the deuteron; and, in using the value of the quadrupole moment given by Rabi and his associates, we assumed that their value is given in terms of these coordinates. If, on the contrary, their z and r are coordinates of the distance of the proton from the neutron, then the value of K/D given here should be divided by 4.