The Delayed Neutron Emission which Accompanies Fission of Uranium and Thorium

In our previous letter[1] we suggested that the delayed neutrons produced by neutron bombardment of uranium might originate either in direct neutron emission (by one of the disintegration products), or in a photodisintegration process. Further evidence has now been obtained which indicates that direct neutron emission is responsible for the delayed neutrons which we observed.

All elements except uranium, carbon, and hydrogen (in paraffin) were eliminated as a source of the delayed neutrons by changing the neutron detector from a boron-lined brass chamber to one of aluminum lined with lithium. The detecting apparatus was located at a distance from the bombarding position at the high voltage target. Furthermore, cloud-chamber photographs of hydrogen recoils from the delayed neutrons showed no appreciable diminution in the number of recoils when one inch of lead was interposed between the chamber and the activated uranium, although the gamma-ray intensity was greatly decreased. The remaining possibility of photodisintegration in the uranium itself was eliminated by surrounding a small amount of the activated uranium with a large quantity of normal uranium. No observable increase in the number of delayed neutrons was produced by the additional uranium.

In comparing the periods of the delayed neutrons and gamma-rays from uranium several longer periods were observed for the gamma-rays but not for the neutrons. The complexity of the gamma-ray decay makes it difficult to determine any of the periods accurately, but there seems to be evidence for at least three periods considerably longer than the short period previously reported.

It was found that the delayed neutrons and all the gamma-ray periods were produced by both thermal and high energy neutrons but not by the medium energy neutrons from carbon. These conditions are the same as for the uranium fission process.

The cross section for the production of delayed neutrons by lithium-neutron bombardment of uranium (high energy neutrons) was measured by comparing the number of delayed neutrons with the number observed from a calibrated radon-beryllium source. By neglecting the asymmetry of the lithium-neutron source and using the yield curves of Amaldi, Hafstad, and Tuve,[2] the cross section was found to be about 4×10^{-26} cm^2 which is roughly one-half the reported cross section for fission when fast radon-beryllium neutrons are used.[3] This large cross section is further evidence that the 15-sec. gamma-rays cannot be the cause of the neutrons.

Cloud-chamber observations of the recoils from the delayed neutrons indicated that their energy is less than one million electron volts and probably near one-half million electron volts.

Delayed neutrons were also observed from thorium nitrate which had been activated by fast lithium neutrons. The intensity was roughly one-fourth of that observed from uranium. The period seemed to be roughly the same as that of the delayed neutrons from uranium.

We are greatly indebted to Professor J. R. Dunning for the radon-beryllium neutron source used in the cross section determination, and to Dr. F. Kracek for the loan of material which made the measurements on thorium possible.

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[2] E. Amaldi, L. R. Hafstad, and M. A. Tuve, Phys. Rev. **51**, 896–912 (1937).
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The Determination of Force Fields from Scattering in the Classical Theory

Although mutual scattering of nuclei provides one of the most important sources of information on nuclear force fields, no systematic method has so far been developed for the determination of these fields from the observed angular distributions. In a recent conversation Dr. O. Klein suggested to me that a method devised by him[1] for the determination of the potential curves of diatomic molecules from their band spectra might be useful in this connection. A slightly modified form of this method proves, indeed, to be applicable to the scattering problem, but only insofar as the results are interpreted according to classical mechanics. In spite of this serious limitation, a brief account of the method should be of interest.

For scattering by a fixed force center with potential function $V(r)$, the effective potential U for the radial motion of particles with angular momentum L and mass m is $U=V(r)+L^2/2mr^2$, where r is distance from the force center. The angular deflection θ of such particles is determined by their energy E, and for present purposes is most conveniently measured by the "deflection parameter" $\rho=\pi/2-\theta/2$. It is readily shown that ρ can be expressed in the form

$$\rho(E)=-[L/(2m)^{\frac{1}{2}}]\int_{\infty}^{r_0} dr/r^2(E-U)^{\frac{1}{2}},$$

where r_0 is the distance of closest approach, for which $E=U$. If the equation of the potential function be taken in the form $f(U)=1/r$ so that $dr/r^2=-f'(U)dU$, the above relation may be written

$$\rho(E)=\frac{L}{(2m)^{\frac{1}{2}}}\int_0^E\frac{f'(U)dU}{(E-U)^{\frac{1}{2}}}.$$

This functional equation connecting $\rho(E)$ and $f(U)$ is of the Abelian type encountered by Klein and can be solved for $f(U)$ to give the equation of the potential curve in the form

$$\frac{1}{r}=f(U)=\frac{(2m)^{\frac{1}{2}}}{\pi L}\int_0^U\frac{\rho(E)dE}{(U-E)^{\frac{1}{2}}}.$$

The function $\rho(E)$ (angular deflection as a function of the energy for a fixed angular momentum) can, in principle, be determined from the scattering data in the following way. From elementary mechanical considerations it can be shown that $\sigma(\theta)\sin\theta=(1/4mE)(d/d\theta)L^2$ where σ is the scattering cross section per unit solid angle. Integration gives

$$L^2=4mE\int_0^{\pi-2\rho}\sigma(\alpha)\sin\alpha d\alpha,$$

which establishes a functional relation between ρ, E and L from the angular distribution curves at different energies. From it the function $\rho(E)$ (for constant L) can be deduced, and the field can then be obtained by a graphical integration. The classical problem of determining the force field from the scattering data can thus be regarded as solved in principle.

It is obvious that such a classical method can give no information on the internal "potential wells" that are assumed for nuclear scattering. Attention has been called to it in the hope that a similar method may be devised which is valid in quantum mechanics to within the limits of the WKB method.

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Electric Quadrupole Moment of the Deuteron

Kellogg, Rabi, Ramsey, and Zacharias[1] have pointed out that their observations on the magnetic moments of H_2, D_2 and HD molecules can be accounted for if the deuteron is assumed to have an electric quadrupole moment of magnitude

$$Q=(3z^2-r^2)_{\text{Av}}\sim 2\times 10^{-27}\text{ cm}^2.$$

If this interpretation is correct, the interaction between the proton and the neutron must contain, besides the usual Majorana and Heisenberg forces, a force which mixes some D state wave function with that of the normal S state of the deuteron. Neither the Thomas precession, which gives no mixing of S with higher states, nor the interaction between the magnetic moments of neutron and proton, which gives much too small a mixing, can account for the observed quadrupole moment. However, a spin-orbit interaction of the form

$$J(r)(\boldsymbol{\sigma}_p\cdot\mathbf{r}_1)(\boldsymbol{\sigma}_n\cdot\mathbf{r}_1),\quad (1)$$

where $\boldsymbol{\sigma}_p$, $\boldsymbol{\sigma}_n$ are the spins of proton and neutron, respectively, $\mathbf{r}_1$ is a unit vector along their relative coordinate, and $J(r)$ is the radial dependence of the interaction, similar to that which arises in the first-order terms of the Yukawa[2] mesotron theory of nuclear forces, gives a quadrupole moment of the observed magnitude if the strength of this force is assumed to be about equal to that of the Majorana force.

Since the quadrupole moment is small compared to the area of the deuteron, the change in the wave function by this interaction must be small and the use of perturbation theory for the calculation is justified. The usual interaction between the proton and the neutron was taken as a rectangular potential hole of depth D and width a, and $J(r)$ in (1), as a rectangular hole of the same width a but of depth K.[3] The integration over the positive energy of the D state of the deuteron had to be carried out numerically. By taking the binding energy of the deuteron as $\epsilon=2.15$ Mev, and the range of the nuclear forces as $a=2.2\times 10^{-13}$ cm we find

$$Q=(3z^2-r^2)_{\text{Av}}=(K/D)\times 1.9\times 10^{-27}\text{ cm}^2.$$

By taking the value of Q given by Rabi and his associates,[4] we obtain

$$K/D\sim 1.$$

Thus the strength of the spin-orbit interaction must be about equal to that of the ordinary interaction.

A rough idea of the magnitude of the spin-orbit interaction and its dependence on a can be obtained by taking some suitably chosen average energy $\bar{E}$ for the D state in the denominator of the energy integral and then using the completeness relation for the wave functions. It gives

$$Q=(2/75)(K/D)(D/(\epsilon+\bar{E}))a^3/(a+\alpha),$$

where $\alpha=4.36\times 10^{-13}$ cm is the "radius" of the deuteron. This estimate shows that K/D varies approximately as $1/a^3$ so that if a were taken to be 2.7×10^{-13} cm we would get $K/D\sim\frac{1}{2}$.

It is interesting to note that the ratio between the strength of the spin-orbit interaction, calculated in this way, and the strength of the Heisenberg force between the proton and neutron, determined from the position of the singlet S state of the deuteron, is in rough agreement with the same ratio derived from Yukawa's theory. The experiments quoted above, however, indicate that Q is positive, whereas the Q calculated from Yukawa's spin-orbit interaction (which is repulsive) is negative.

If it is true that there is a spin-orbit interaction of strength comparable with that of the ordinary interaction, then there is an important consequence that the total spin of the particles in a nuclear system will not be, even approximately, a constant of the motion. There can thus be no selection rules for spin.

We wish to thank Professor J. R. Oppenheimer for suggesting this problem and for much helpful advice, and Dr. L. I. Schiff for many illuminating discussions.

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[3] This schematic form for K was chosen for convenience since divergences render questionable the form of the terms given by Yukawa's theory (reference 2).
[4] In this formula, z and r are coordinates of the proton in the reference frame with origin at the center of gravity of the deuteron; and, in using the value of the quadrupole moment given by Rabi and his associates, we assumed that their value is given in terms of these coordinates. If, on the contrary, their z and r are coordinates of the distance of the proton from the neutron, then the value of K/D given here should be divided by 4.