The *E* Region of the Ionosphere

E. O. HULBURT
Naval Research Laboratory, Washington, D. C.

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Comparison of E region observations from 1930 to 1938 of ionosphere stations scattered over the earth with the theory of ionization caused by solar radiation absorbed exponentially in a relatively quiet terrestrial atmosphere yielded the conclusions: (1) the diurnal variation of y_m, the maximum-with-height value of the equivalent electron density, during daylight is in close accord with the theory with the recombination coefficient α proportional to y_m^2 and equal to 2×10^{-8}, or perhaps greater; (2) y_m at night was not in accord with the theory, but the data do not preclude a night value of $\alpha=2\times10^{-8}$ or greater provided suitable hypotheses are added to the theory; (3) the morning increase in y_m was observed at Washington near midsummer to begin about 35 minutes before ground sunrise, whereas the theory put it about 20 minutes; (4) the seasonal variation of noon y_m agrees with the theory at stations in latitudes from 12° S to 80° N; (5) at any season noon y_m falls off with latitude more rapidly than the theoretical expectation. A relation between noon y_m and sunspots is given.

THE E region of the ionosphere is situated at about 100 to 130 km above sea level. It is the lowest in altitude and usually the least densely ionized of the three main ionospheric regions, and the most regular. E ionization experiences regular changes through the day, with the season and with sunspots; it is subject to erratic short period variations. It has often been shown[1] that average daily and seasonal changes in E ionization are in approximate accord with simple relations of the theory that the ionization is caused by solar radiations, probably ultraviolet light, absorbed exponentially in a relatively quiet terrestrial atmosphere. In this paper E region observations from ionosphere stations scattered over the earth are compared with fully developed theoretical derivations. In certain cases good agreement is found, in other instances discrepancies are noted.

Short period fluctuations in E ionization may or may not be associated directly with fluctuations in solar radiation, and therefore may or may not fall within the scope of the present theoretical comparisons. It has been established with considerable certainty by Dellinger[2] that the sudden increases in ionization near, and probably slightly below, E levels associated with radio "fade outs" are due to eruptions of solar radiations which move approximately with the velocity of light. The cause of the suddenly appearing E echoes,[3] known as "sporadic" E, is not yet clearly understood, although a reasonable suggestion has been made[4] that attributes the echoes to scattering by ionic clouds or irregularities of size comparable with the radio wavelengths. Sporadic E phenomena are local, not frequent near the equator, increase in frequency with latitude, and in temperate latitudes occur both night and day. The E region perturbation, described as "turbulence," [5] occurs during strong magnetic disturbance.

It is of interest to remark that, although an ultraviolet solar radiation origin of E ionization is reasonably probable, no detailed theory has yet been outlined due to lack of knowledge of photochemical reactions; no contribution to this aspect of the theory is offered here. Thus it is not known what radiations cause E ionization or what atmospheric gases are ionized. A calculation[6] based on equations of astrophysics indicated that the F regions were oxygen and nitrogen ionized by frequencies in the continua above the series limits at 910 and 860A, but gave no information about E region. Wulf and Deming[7] have given reasons for suggesting that E ionization is molecular oxygen ionized by wave-lengths from about 1100 to 1300A. Recent experiment[8]

[1] For example, S. S. Kirby and E. B. Judson, Proc. Inst. Rad. Eng. **23**, 733 (1935); E. O. Hulburt, Terr. Mag. **40**, 193 (1935).
[2] J. H. Dellinger, Proc. Inst. Rad. Eng. **25**, 1253 (1937).
[3] T. R. Gilliland, S. S. Kirby, N. Smith and S. E. Reymer, Nat. Bur. Stand. J. Research **20**, 627 (1938); L. V. Berkner and H. W. Wells, Terr. Mag. **42**, 73 (1937).
[4] H. G. Booker and H. W. Wells, Terr. Mag. **43**, 249 (1938).
[5] S. S. Kirby, N. Smith and T. R. Gilliland, Phys. Rev. **54**, 234 (1938).
[6] E. O. Hulburt, Phys. Rev. **53**, 344 (1938).
[7] O. R. Wulf and L. S. Deming, Terr. Mag. **43**, 283 (1938).
[8] H. Raether, Zeits. f. Physik **110**, 611 (1938).

has shown that ultraviolet wave-lengths, believed to be in the neighborhood of 800 to 1000A, from a spark can penetrate several cm of air and cause ionization. The thickness of the atmosphere above 100 km is equivalent to several cm of air.

Theoretical

The fact that the variation of the virtual height of E region with the solar zenith angle is in agreement[5] with deductions from the ultraviolet theory indicates that diurnal and seasonal temperature changes in levels from the stratosphere to about 100 km are not great. Therefore, as a working hypothesis, an atmosphere is assumed relatively undisturbed by winds and at a temperature 219°K constant with altitude from 20 to 100 km for day and night at all latitudes. It is assumed that the atmospheric gases are completely mixed. The density n of the air molecules is expressed as a function of the height z by

$$n=n_0\epsilon^{-pz}, \quad (1)$$

in which z is measured from the height where $n=n_0$; for $z=20$ km, $n=1.86\times 10^{18}$. The term $p=mg/kt$, where m is the mass of the average air particle and t the temperature. For $t=219$°K, $p=1.54\times 10^{-6}$.

It is assumed that the atmosphere is ionized by the absorption of solar ultraviolet frequencies of effective average molecular or atomic absorption coefficient β, and that all of the absorbed energy causes ionization. The number q cm^{-3} sec.$^{-1}$ of electron-ion pairs produced is

$$q=ni\beta/\chi, \quad (2)$$

where i is the intensity of the light at a level z where the molecular density is n and χ is the ionizing energy per molecule or atom. If i_0 is the intensity outside of the atmosphere and $\bar{n}$ is the total number of absorbing particles traversed by the light in its path from outside of the atmosphere to the level z,

$$i=i_0\exp(-\beta\bar{n}). \quad (3)$$

The evaluation of $\bar{n}$ for various zenith angles ζ was accomplished by working out two cases, a flat earth and a curved earth, the one exact at $\zeta=0°$ and the other at $\zeta=90°$, which approached each other at ζ about equal to 75°. It is assumed that there is no refraction of the solar ionizing rays in the atmosphere. For the case of a flat earth consider an element dx of a solar ray at angle ζ with the vertical. Then $dx=dz/\cos\zeta$, and $\bar{n}$ for a point in the atmosphere at an altitude z and molecular density n is

$$\bar{n}=\int_z^\infty ndx=\int_z^\infty (n_0\epsilon^{-pz}dz)/\cos\zeta=n/p\cos\zeta. \quad (4)$$

When applied to a curved earth (4) is exact only for $\zeta=0°$ and becomes too large by an amount that increases to ∞ as ζ increases to 90°. However, as shown later, the amount is less than one percent for $\zeta<75°$.

To determine $\bar{n}$ for a curved earth consider the ray AP, Fig. 1, which passes within a distance z_1 of the surface of the earth SS. Let n_1 and n refer to A and P, respectively; let the altitudes of A and P be z_1 and $z+z_1$, the zenith angle of the ray AP at P be ζ, the radius of the earth be $r=3670$ km and the distance AP be x. From Fig. 1,

$$x^2=(r+z_1+z)^2-(r+z_1)^2. \quad (5)$$

Let $r_1=r+z_1$. Then (5) is

$$x^2=2r_1z(1+z/2r_1). \quad (6)$$

Also
$$\cos\zeta=[1-r_1^2/(r_1+z)^2]^{\frac{1}{2}}. \quad (7)$$

For $z\ll r_1$ (6) is approximately

$$x=(2r_1z)^{\frac{1}{2}}(1+z/4r_1). \quad (8)$$

By differentiation (8) yields

$$dx=(r_1/2)^{\frac{1}{2}}\{z^{-\frac{1}{2}}+3z^{\frac{1}{2}}/4r_1\}dz, \quad (9)$$

and if (1) is put into (9) we obtain

$$ndx=n_1(r_1/2)^{\frac{1}{2}}\{z^{-\frac{1}{2}}\epsilon^{-pz}+3z^{\frac{1}{2}}\epsilon^{-pz}/4r_1\}dz. \quad (10)$$

By writing $s=pz$ and integrating, one obtains from (10)

$$\int ndx=n_1\left(\frac{r_1}{2p}\right)^{\frac{1}{2}}\left\{\left(1+\frac{3}{8r_1p}\right)\times\int s^{-\frac{1}{2}}\epsilon^{-s}ds-\frac{3s^{\frac{1}{2}}\epsilon^{-s}}{4r_1p}\right\} \quad (11)$$

$$=D\left\{E\int s^{-\frac{1}{2}}\epsilon^{-s}ds-F\right\}, \quad (12)$$

where D, E and F are defined by identifying (11) and (12). Now

$$\bar{n}=\int_A^\infty ndx\mp\int_A^P ndx, \quad (13)$$

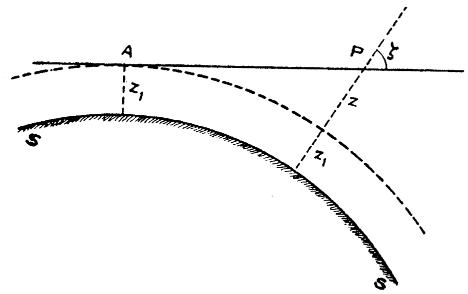

FIG. 1. Solar ray AP in the terrestrial atmosphere.

Here the $-$ and $+$ signs refer, respectively, to P and the sun on the same side of A, and on opposite sides. At A and ∞, s is 0 and ∞, respectively; in each case F is zero. Then from (12) and (13)

$$\bar{n}=D\left\{E\int_0^\infty s^{-\frac{1}{2}}\epsilon^{-s}ds\mp E\int_0^s s^{-\frac{1}{2}}\epsilon^{-s}ds\pm F\right\}. \quad (14)$$

The first integral of (14) is $\Gamma(\frac{1}{2})=\pi^{\frac{1}{2}}$. The second integral is an incomplete gamma-function; upon substitution of a new variable $\sigma=s^{\frac{1}{2}}$ it is transformed[9] to an error function, the values of which are given in mathematical tables.[10] Eq. (14) becomes

$$\bar{n}=D\left\{E\Gamma(\tfrac{1}{2})\mp ZE\int_0^{s^{\frac{1}{2}}}\epsilon^{-\sigma^2}d\sigma\pm F\right\}. \quad (15)$$

Eq. (15) is exact for $\zeta=90°$ and merges asymptotically into (4) at about $\zeta=75°$. The asymptotic approach of the two functions may be shown analytically by expanding (15) in a semi-convergent series. The ratio of $\bar{n}$ of (15) to $\bar{n}$ of (4) is 0.46, 0.87, 0.94, 0.98 and 0.998 at $\zeta=89°$, 86°, 83°, 80° and 77°, respectively. For ζ from 0° to 75° $\bar{n}$ was calculated from (4) and for ζ from 75° to 105° from (15). The results are accurate within one percent. It does not seem probable that the assumption of no atmospheric refraction of the solar ionizing rays introduces an important error.

From the values of $\bar{n}$ thus calculated q was determined by means of (2) and (3) as a function of ζ, or the time t, for each altitude z; the (q, t) curves are not shown here. For each value of z above a station the (q, z) curve is symmetrical about local noon.

Recombination of electrons and positive ions is assumed to take place at the rate of αy^2, where α is the recombination coefficient and y is the electron density. Then at any point in the atmosphere

$$dy/dt=-\alpha y^2+q, \quad (16)$$

where q is a function of z and the time t. At night $q=0$, and (16) becomes

$$y=1/(1/y_1+\alpha t), \quad (17)$$

where y_1 is the value of y at nightfall.

Since q was determined in the form of (q, t) curves for each value of z (16) could not be integrated explicitly. Therefore the (y, t) curves were obtained graphically by following the course of dy/dt, which could be done with desired precision.[11] The (y, t) curves are not shown here. The envelope of the family of (y, t) curves gave the curve of (y_m, t), where y_m denotes the maximum-with-height value of y for any t. (y_m, t) curves were computed for various values of α for comparison with observation.

A special case is of interest, namely, when recombination is very rapid or $\alpha=\infty$. Then in (16) $dy/dt=0$ and

$$q=\alpha y^2. \quad (18)$$

From (2), (3), (4) and (18)

$$y=\{(ni_0\beta/\alpha\chi)\epsilon^{-\beta n/p\cos\zeta}\}^{\frac{1}{2}}. \quad (19)$$

The maximum-with-height value of y of (16) occurs at a value of n, corresponding to an altitude z, obtained by equating the exponent of ϵ to unity. Therefore

$$y_m=(pi_0/\alpha\chi\epsilon)^{\frac{1}{2}}(\cos\zeta)^{\frac{1}{2}}, \quad (20)$$

at

$$n=(p\cos\zeta)/\beta. \quad (21)$$

Eq. (20) may be written

$$y_m=y_0(\cos\zeta)^{\frac{1}{2}}, \quad (22)$$

where

$$y_0=(pi_0/\alpha\chi\epsilon)^{\frac{1}{2}}, \quad (23)$$

is the value of y for $\zeta=0$. Eqs. (19) to (23), are

[9] The general transformation is discussed by E. T. Whittaker and G. N. Watson, *Modern Analysis* (1927), fourth edition, p. 341.

[10] For example, B. O. Pierce, *A Short Table of Integrals* (1929), third edition p. 116.

[11] M. V. Wilkes, Proc. Phys. Soc. London **51**, 138 (1938) used a Bush Harmonic Analyser to determine (y, t) curves; S. Chapman, Proc. Phys. Soc. London **43**, 26 and 483 (1931), determined (y, t) curves by a method of successive trials.

valid only for ζ less than about 80° since they are based on (4). During the daylight hours the variation of y_m of (22) with ζ or t is symmetrical about local noon. As α decreases from ∞ the (y_m, t) curve is distorted and its maximum is displaced into the afternoon; the amount of displacement increases with decreasing α.

The Diurnal Variation of y_m of E

Routine ionosphere data are usually reported as critical frequencies f_c for each ionospheric region for each hour of the day averaged over a month. Unless otherwise stated monthly average values are used throughout this paper. From f_c the equivalent electron density y_m was calculated from the dispersion formula[12] which, for the ordinary component of polarization, leads to

$$y_m = \pi m f_c^2 / e^2 c^2, \qquad (24)$$

where e and m refer to the electron and c is the velocity of light. In Figs. 2 and 3 are plotted y_m values for the first three months of 1938 from the ionosphere station at Huancayo, Peru, lat. 12° 2.7′ S, long. 20′.4 W, of the Carnegie Institution.[13] New automatic recording equipment was placed in operation at the station in November, 1937. The daytime data of Fig. 2

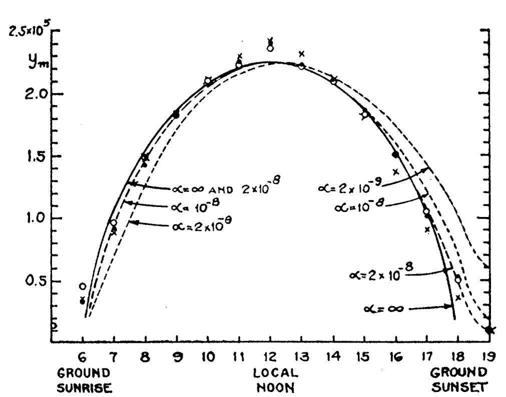

Fig. 2. E data (reference 13) at Huancayo, Peru, for January, February and March, 1938, shown by circles, dots and crosses, respectively. Theoretical curves from (16).

[12] The discussion continues concerning whether the value of a in the H. A. Lorentz dispersion theory is 0 or $\frac{1}{3}$ or some intermediate value: see H. G. Booker and L. V. Berkner, Terr. Mag. **43**, 427 (1938); D. F. Martyn and G. H. Monro, Nature **142**, 1159 (1938). The difficulty arises from ignorance of the exact nature of collision between ionized and neutral particles. In (24) a is 0; if a were $\frac{1}{3}$, values of y_m mentioned here would be increased by 50 percent, but all conclusions would remain unaltered.

[13] H. W. Wells and H. E. Stanton, Terr. Mag. **43**, 467 (1938).

are symmetrical about noon within the error of variation.[14] The same conclusion holds for E region data at Washington, D. C., U. S. A., lat. 38° 50′ N, long. 77° W, of the United States National Bureau of Standards,[15] which are continuous from 1931. One set of Washington values are plotted in Fig. 4. The (y_m, t) curves for four

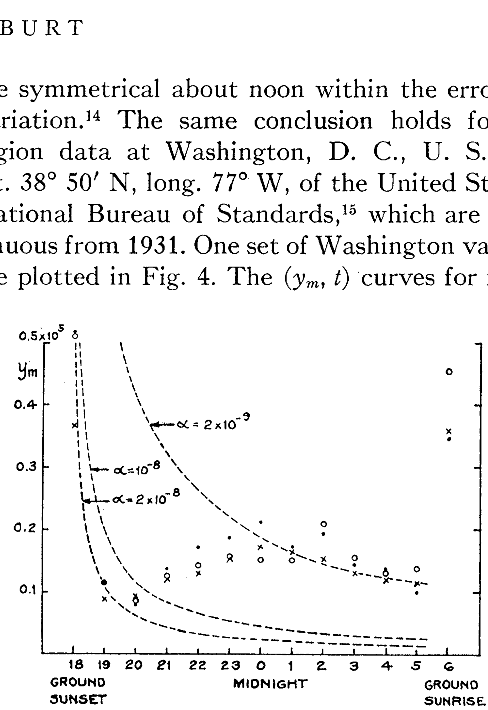

Fig. 3. E data (reference 13) at Huancayo, Peru, through the night, continued from Fig. 2. Theoretical curves from (16).

values of α, namely, ∞, 2×10^{-8}, 10^{-8} and 2×10^{-9}, were passed through the points of Figs. 2 and 3. The curves were based on values of ζ for February 15, 1938; since Huancayo is near the equator the use of ζ for January 15 and March 15 would lead to but slight deviation from the curves of Fig. 2.

It is seen that the (y_m, t) curve of Fig. 2 for $\alpha = 2\times10^{-8}$ is in best agreement with the observations. During most of the daylight hours the difference between the curves for $\alpha = 2\times10^{-8}$ $\alpha = \infty$ is imperceptible on the scale of Fig. 2; $\alpha = \infty$ is, of course, impossible physically. We conclude that available data favor $\alpha = 2\times10^{-8}$ but do not preclude an even higher rate of recombination. With $\alpha = 2\times10^{-8}$ and $y_0 = 2.25\times10^5$ at 100 km we obtain from (19) to (22) $\beta = 2.23\times10^{-19}$ and $i_0/\chi = 1.8\times10^9$. For an energy of ionization $\chi = 14$ electron-volts, $i_0 = 4\times10^{-2}$ erg cm^{-2} sec.$^{-1}$, which, on the assumption that the sun radiates as a blackbody at 6000°K, is the

[14] One cannot speak of "error of observation" for such a quantity is not reported. The irregularity in an average datum is due to error of observation and to natural fluctuation of the ionization.

[15] Now published each month by T. R. Gilliland, S. S. Kirby and N. Smith in the Proc. Inst. Rad. Eng. Their papers are in the Nat. Bur. Stand. J. Research.

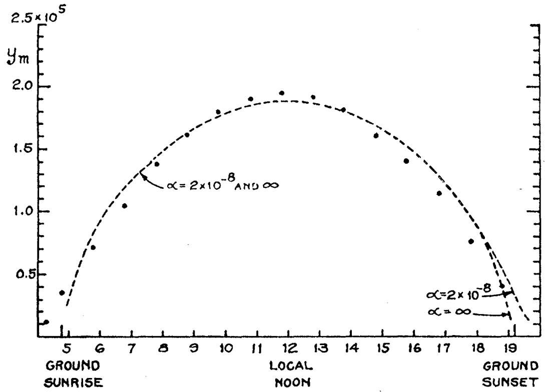

FIG. 4. *E* data (reference 15) at Washington for May 1938 shown by dots. Theoretical curves from (16).

solar energy falling on the top of the atmosphere in the wave-length region from about 960A to 0, or in a band about 10A wide at 1000A, or in a narrower band at a longer wave-length.

In turning to the night *E* values of Fig. 3 it is manifest that a simple theory of ionic recombination during the night, as (17), with no other changes cannot account for the facts. The value of y_m is observed to increase after sunset to a mild maximum around midnight, whereas (17) calls for a steady decrease in y_m through the night. Whether the nocturnal increase of y_m at Huancayo is peculiar to that station or occurs elsewhere is not known. Whether it is similar to "sporadic" *E* ionization[3] at higher latitudes is not yet reported; Kirby and Judson[1] expressed the opinion that most night-time *E* reflections at Washington were of a sporadic type. Whether the increase is caused by effects in the atmosphere, as ionic clouds and scattering centers due to winds ("white cap" theory), generation of ionization by collision of excited or combining atoms ("spontaneous generation" theory), night cooling of an *E* region warmed during the day ("contraction" theory), or by effects external to the atmosphere, as meteors ("interplanetary debris" theory), or by some other effect ("angel" theory), cannot be said. Be this as it may, the facts at present do not necessarily imply a value of α during the night which is different from its day value.

It has long been known[15, 16] that *E* ionization begins to increase during morning twilight before ground sunrise but no continuous accurate measurements have been made of the effect. A series of data was obtained at Washington by the United States National Bureau of Standards for the purpose of comparison with the present theory. Their data are given in Fig. 5 for three mornings, May 11 and 18 and June 1, 1938, that appeared to be relatively free from sporadic *E* phenomena and magnetic disturbance. In Fig. 5 y_m is plotted against ζ and a scale of t is appended that was approximately the same for all three mornings. The monthly average hourly values of y_m for May,[10] assumed to apply to May 15, 1938, were used. These are plotted in Fig. 4 and the (y_m, t) curve for $\alpha = 2\times 10^{-8}$ was passed through them. This determined all the constants of (16) and underlying equations. Since the (y, z) curve at 30 minutes before ground sunrise was not known, a (y, z) curve was assumed with $y_m = 0.4\times 10^4$ in agreement with the observed values of Fig. 5. The curve is drawn in Fig. 6; but what follows does not depend at all critically on the assumed form of the curve. The (y, z) curves at succeeding epochs up to sunrise were calculated from (16); they are given in Fig. 6. From them the (y_m, t) curve was determined and is plotted in the curve of Fig. 5 for $\alpha = 2\times 10^{-8}$. It does not agree with the observations. The entire calculation was repeated with $\alpha = 4\times 10^{-8}$, the resulting curve is given in Fig. 5 and again is discordant with

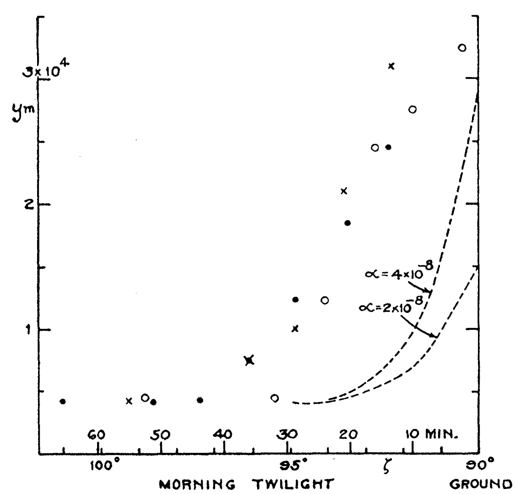

FIG. 5. Pre-dawn *E* data at Washington for May 11 and 18 and June 1, 1938, shown by dots, crosses and circles, respectively. Theoretical curves from (16).

[16] S. K. Mitra, Science and Culture **3**, 496 (1938); J. E. Best, F. T. Farmer and J. A. Ratcliffe, Proc. Roy. Soc. **A164**, 96 (1938).

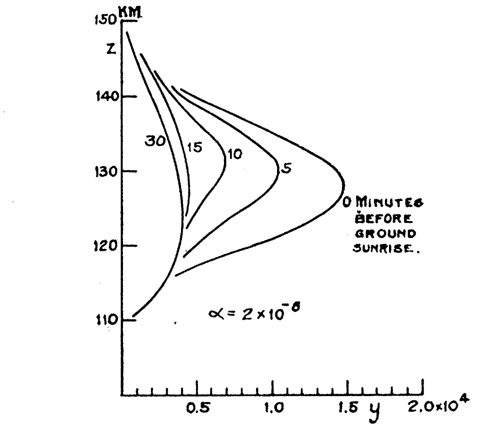

FIG. 6. Theoretical (y, z) curves from (17) during morning twilight at Washington, May 15, 1938.

observation. There is also a difficulty with virtual heights; these were observed to be about the same, within 10 km, during morning twilight as they were during the day, whereas the theoretical virtual heights from the curves of Fig. 6 are 20 km or more above the day values.

It is concluded that the simple theory does not yield exact agreement with the observed morning twilight augmentation of y_m of E. It is not safe to generalize too much from data of only three mornings at one station; further data, accurate and closely spaced, of all ionospheric regions would be desirable. If the discrepancy is found to persist, three effects, among those omitted from simple theory, come to mind toward its elucidation. They are winds, atmospheric refraction and scattering of the solar ionizing rays. To be effective winds must be greater than the peripheral velocity of rotation of the earth, or a very high index of refraction must be assumed. Scattering appears to call for a less extreme set of assumptions than do winds and refraction. However, any hypothesis for the purpose of accounting for the pre-dawn effect must necessarily be uncertain as long as explanations are doubtful of night-time and sporadic E echo phenomena.

Seasonal Variation of Noon y_m of E

Since the variation of y_m of E during daylight is expressed fairly closely by (16) with a high value of α, we may compare the simple Eq. (22), $y_m = y_0(\cos\zeta)^{\frac{1}{2}}$, with average monthly noon values of y_m observed at various stations scattered over the earth. Since E ionization varies with the sunspot cycle it is convenient to make the comparison for each year separately. Results for 1936, 1937 and 1938 are given in Fig. 7, in which the smooth curves for each station are from (22) and the observed points are shown by the initial letter of each station. In more detail, the curve of (22) was passed through the Washington observations, which determined y_0 for the year; with this value of y_0 and with ζ from the latitude of the station the curve of (23) for the station was drawn. The data of Fig. 7 were from Washington,[15] Huancayo,[13] Tromsö[17] and Northeast Land.[18] Measurements were made at Wuchang,[19] China, lat. 30° 34′ N for a few months in 1937 and 1938. Comparisons similar to those of Fig. 7 were prepared for the years

[17] L. Harang, Terr. Mag. **43**, 41 (1938).

[18] A. B. Whitman and R. A. Hamilton, Proc. Phys. Soc. London **50**, 217 (1938).

[19] P. L. Sung and C. T. Kwei, Terr. Mag. **43**, 453 (1938).

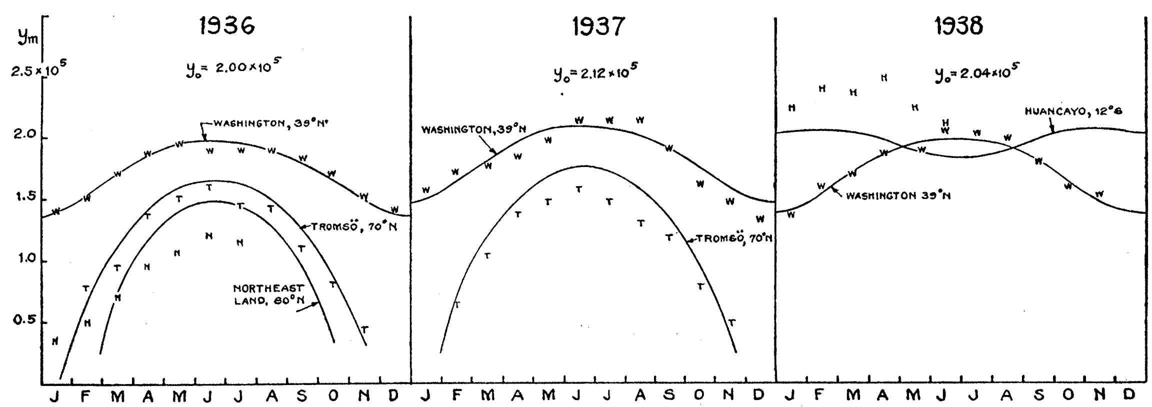

FIG. 7. Observed average monthly noon values of y_m of E shown by the initial letter of the station. Theoretical curves from (22).

1931 to 1935, inclusive. The Washington observations were continuous throughout these years. Data were available from Slough,[20] lat. 51° 30′ N, and Tromsö[20] for the year from August 1932 to August 1933; from Deal,[21] lat. 40° 15′ N for the year from March 1933 to March 1934; and for Slough[19] for 1934. These comprise practically all reported monthly average noon values of y_m of *E*. The conclusion was that at any station the variation of y_m through the year agreed closely with (22) but that between stations widely separated in latitude the agreement with (22) was far from close. The discrepancy in 1938 between Huancayo and Washington is particularly noticeable. These are accurate stations; it is to be remembered that Huancayo has been operating on its present schedule for only about a year. The discrepancy appears to be that noon y_m of *E* falls off more rapidly with increasing latitude than would be expected from (22); y_m is below the value from (22) by roughly 0, 10, 20 and 30 percent at latitudes 0°, 40°, 70° and 80°, respectively. If the discrepancy turns out to be real, additional theoretical hypotheses may be called for.

In this connection a suggestion, albeit vague and qualitative, may be in order. The assumption was made that the atmosphere of the *E* region was the same at all latitudes. It may be necessary to depart from the assumption to some extent and to assume that the character of the atmosphere changes from the equator to the poles. For example, sunlight may cause excitation or may dissociate molecular oxygen or nitrogen, or both, to the atomic condition, the amount of excitation or dissociation being a function of the latitude. Therefore the absorption and recombination coefficients, in short the entire photochemistry of the *E* region, may vary sufficiently with latitude to account for the departures from the simple photochemistry underlying the present calculations.

Variation of y_0 of *E* with Sunspots

The values of y_0 of *E* were determined from the Washington[14] observations for the years 1931 to 1938, which cover a portion of a sunspot cycle. In Fig. 8 y_0 is plotted with the yearly average relative sunspot number[22] s; it is seen that y_0 and s rise and fall together. A theoretical relation between y_0 and s may be derived as follows: From (23) $y_0 \sim i_0^{\frac{1}{4}}$, where i_0 is the intensity of the solar radiation which causes *E* ionization. Assume that $i_0 \sim (s+b)$, where b, a constant, is introduced in recognition of the fact that y_0 and i_0 have values other than zero when s is zero, i.e., when there are no sunspots. Then

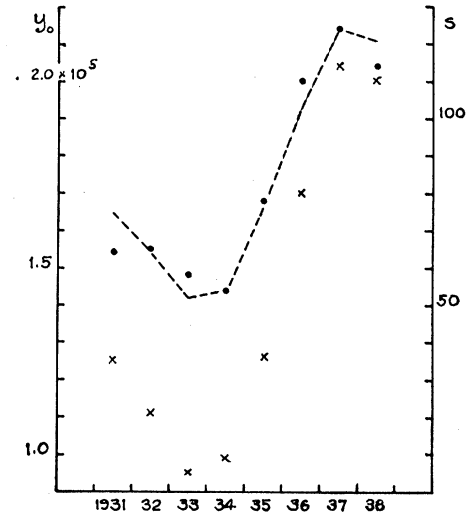

FIG. 8. Dots, y_0 of *E* from Washington observations (reference 15); crosses relative sunspot numbers (reference 22) s. Dotted curve from (25).

$$y_0 = a(s+b)^{\frac{1}{4}}. \qquad (25)$$

The dotted curve of Fig. 8 is plotted from (25) with $a = 0.154 \times 10^5$ and $b = 78.6$; it agrees fairly well with y_0. It may be noted that (25) with different constants was found[23] to agree with long distance short wave radio communication data. The difference in the constants was due to the facts that the radio data depended in a complex manner on the *F* regions as well as *E* region and that the variation of *F* with sunspots is not the same as the variation of *E*.

In conclusion, it is a pleasure to thank Dr. J. H. Dellinger and the Radio Section of the National Bureau of Standards for the unpublished data plotted in Fig. 5.

[20] E. V. Appleton, R. Naismith and L. T. Ingram, Phil. Trans. Roy. Soc. **236A**, 218 (1936–1938); E. V. Appleton and R. Naismith, Proc. Roy. Soc. **A150**, 685, (1935).

[21] J. R. Schafer and W. M. Goodall, Proc. Inst. Rad. Eng. **23**, 670 (1935).

[22] W. Brunner, each volume of Terr. Mag.

[23] L. C. Young and E. O. Hulburt, Phys. Rev. **50**, 45 (1936).