The Masses of Li^6, Li^7, Be^8, Be^9, B^{10} and B^{11}

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The energy values of the two modes of disintegration of beryllium under proton bombardment, determined by electrostatic analysis, are used in connection with other selected energy release values to set up masses for some of the lightest atoms. It is shown that introduction of the revised electronic charge makes negligible changes (less than 0.07 percent) in the accepted range-energy curve for alpha-particles. It is assumed that the masses of H, D and He^4 are 1.00813 ± 0.00002, 2.01473 ± 0.00002 and 4.00386 ± 0.00006. The deduced masses are:

$Li^6=6.01670\pm0.00012$ $\quad$ $Be^9=9.01474\pm0.00014$
$Li^7=7.01799\pm0.00011$ $\quad$ $B^{10}=10.01579\pm0.00022$
$Be^8=8.00753\pm0.00013$ $\quad$ $B^{11}=11.01244\pm0.00019.$

These masses are compared with several observed nuclear reactions and mass-spectrographic doublets.

Introduction

The precise determinations of the energy released in the two modes of disintegration of beryllium under proton bombardment reported from this laboratory[1, 2] are used in the present paper, in connection with other selected energy data, to construct a revised set of masses for six of the lightest atoms. Masses computed mainly from disintegration data have been previously presented by other authors,[3–6] and it is well known that the masses proposed by Oliphant, Kempton and Rutherford disclosed a rather large error in the mass of helium in use at that time, based on results with the mass spectrograph.

The Masses of H, D and He^4

In setting up the revised masses, we shall use the values of the masses of hydrogen, deuterium and helium obtained by Bainbridge, and by Bainbridge and Jordan with the mass spectrograph.[7] Our confidence in the mass-spectrographic values for these light atoms seems justified by the excellent agreement between the results of Aston[8] and Bainbridge. The fundamental brackets used for the calculation of H, D and He^4 are shown in Table I. It is instructive to note that the agreement on H, D and He^4 persists in spite of the lack of agreement on the methane-oxygen bracket. This is largely due to the fact that only fractions of the methane-oxygen bracket enter into the computed atomic weights of these lightest atoms. The equations used are given in Table II. In the calculation of the atomic weights of atomic species nearer oxygen, the discrepancy between the two methane-oxygen brackets will appear more and more prominently, and is a large factor in the difference between the weights of C^{12} and B^{10} computed by the two observers.

Re-investigation of the Alpha-Particle Range Curve

In order to set up a mass scale, we need to supplement our present results, obtained from the electrostatic analyzer, with other data

Table I. *Brackets used in the determination of* H, D *and* He^4 *with the mass spectrograph. Numerical values in thousandths of a mass unit.*

Bracket	Bainbridge	Aston
$[H_2-D]$	1.53 ± 0.04	1.52 ± 0.04
$[D_3-\frac{1}{2}C^{12}]$	42.19 ± 0.05	42.36 ± 0.18
$[C^{12}H_4-O^{16}]$	36.49 ± 0.08	36.01 ± 0.24
$[D_2-He^4]$	25.61 ± 0.04	25.51 ± 0.08

[1] S. K. Allison, Lester S. Skaggs and Nicholas M. Smith, Jr., Phys. Rev. **54**, 171 (1938).
[2] S. K. Allison, E. R. Graves, L. S. Skaggs and N. M. Smith, Jr., Phys. Rev. **55**, 107 (1939).
[3] M. L. E. Oliphant, A. E. Kempton and Lord Rutherford, Proc. Roy. Soc. **150**, 241 (1935).
[4] H. A. Bethe, Phys. Rev. **47**, 633 (1935).
[5] T. W. Bonner and W. M. Brubaker, Phys. Rev. **50**, 308 (1936).
[6] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).
[7] This work is summarized by M. S. Livingston and H. A. Bethe, reference 6.
[8] F. W. Aston, Proc. Roy. Soc. London **A163**, 391 (1937); and Nature **138**, 1094 (1936); **139**, 922 (1937).

TABLE II. *Calculation of* H, D *and* He^4 *from the values of Table I.*

	Bainbridge*	Aston
$\frac{1}{16}O^{16}+\frac{3}{8}[H_2-D]+\frac{1}{8}[D_3-\frac{1}{2}C^{12}]+\frac{1}{16}[C^{12}H_4-O^{16}]=H$	$=1.00813\pm0.00002$	1.00812 ± 0.00004
$\frac{1}{8}O^{16}-\frac{1}{4}[H_2-D]+\frac{1}{4}[D_3-\frac{1}{2}C^{12}]+\frac{1}{8}[C^{12}H_4-O^{16}]=D$	$=2.01473\pm0.00006$	2.01471 ± 0.00007
$2\{D-\frac{1}{2}[D_2-He^4]\}=He^4$	$=4.00386\pm0.00006$	4.00391 ± 0.00016

* See also K. T. Bainbridge, Phys. Rev. **53**, 922(A) (1938).

obtained from the ranges of produced particles. It seems that the most reliable supplementary data to use is that obtained in cases in which the produced particle is an alpha-particle whose range lies within or close to the region in which the range curve for alpha-particles has been accurately calibrated by magnetic analysis. This region extends in energy from 5.3 to 10.5 Mev, and in range from 4 to 11.5 cm of air under standard conditions. We have recalculated the energies of the alpha-particles from Po, RaC′ and ThC′ (long), using recent values of the fundamental constants. The value of $H\rho$ for the RaC′ alpha-particle is taken as 3.99280×10^5, according to Rosenblum and Dupouy.[9] This value agrees with that of Briggs[10] and was accepted by Rutherford, Wynn-Williams, Lewis and Bowden,[11] who used 3.9930×10^5. The $H\rho$ values for Po and ThC′ (long) particles may then be obtained from the table on page 634 of the paper of Rutherford *et al.* by multiplying 3.99280×10^5 by the appropriate v/v_0 listed under "new magnet." Table III gives some of the fundamental and derived constants used in the computation of the velocity and energy of the alpha-particle. The following equations are used for the computation of the velocity and energy from the $H\rho$ value:

$$v=c\left[\frac{2e}{mc^2}H\rho-\frac{1}{2}\left(\frac{2e}{mc^2}H\rho\right)^3\cdots\right],$$

$$E=mc^2\left[\frac{1}{2}\left(\frac{v}{c}\right)^2+\frac{3}{8}\left(\frac{v}{c}\right)^4\cdots\right]\text{ in ergs.}$$

The results of the computation, and comparison with previously used values, are shown in Table IV. The mean and extrapolated ranges are from Duncanson[12] and the "previous values" of the velocities are likewise taken from his paper. The "previous values" of the energies are taken from Rutherford, Wynn-Williams, Lewis and Bowden.[11] Disregarding the slight difference of 0.06 percent between the new and previous value of the energy of the RaC′ alpha-particle, it is seen that no revision of the range-energy curve in the accurately known region is required, because of revision of the fundamental constants.

Energy Values Used in Setting up the Masses

We shall use without comment the two energy values found in this laboratory by electrostatic analysis,[1, 2] namely

$$_4Be^9+{_1H^1}={_3Li^6}+{_2He^4}+2.152\pm0.04\text{ Mev},$$
$$_4Be^9+{_1H^1}={_4Be^8}+{_1D^2}+0.557\pm0.006\text{ Mev}.$$

In addition we will use

$$_3Li^6+{_1D^2}={_2He^4}+{_2He^4}+22.08\pm0.07\text{ Mev}.$$

Oliphant, Kempton and Rutherford[13] compared the range of the alpha-particles from this reaction, when produced with 0.19-Mev deuterons, with the range of 8.62 (extrapolated range) ThC′ particles. They used an absorption tube with thin mica windows for the comparison and report that "the range of the alpha-particles emitted at

TABLE III. *Numerical values useful in computation of the range-energy relation of alpha-particles.*

$e=4.803\times10^{-10}$ e.s.u.	$N=6.0221\times10^{23}$
$c=2.99774\times10^{10}$ cm sec.$^{-1}$	$e/m=1.7597\times10^7$ e.m.u./g
one unit of atomic weight	1.6606×10^{-24} g
mass of the electron	9.11×10^{-28} g
mass of helium atom	6.649×10^{-24} g
one electron volt	1.6022×10^{-12} erg
one unit of atomic weight	9.313×10^8 ev
mass of the alpha-particle	6.647×10^{-24} g
mc^2 for the alpha-particle	5.973×10^{-3} erg
$mc^2/2e$ for the alpha-particle	6.218×10^6 ergs/e.s.u.

[9] S. Rosenblum and G. Dupouy, Comptes rendus **194**, 1919 (1932).

[10] G. H. Briggs, Proc. Roy. Soc. London **A118**, 549 (1928).

[11] Lord Rutherford, C. E. Wynn-Williams, W. B. Lewis and B. V. Bowden, Proc. Roy. Soc. London **A139**, 617 (1933).

[12] W. E. Duncanson, Proc. Camb. Phil. Soc. **30**, 102 (1933).

[13] M. L. E. Oliphant, A. E. Kempton and Lord Rutherford, Proc. Roy. Soc. London **A149**, 406 (1935).

right angles to the bombarding beam is 12.70 ± 0.05 cm under standard conditions." The context leaves little doubt that extrapolated range is meant. From Table VI of Duncanson's paper, which applies to extrapolated range, it is found that

$$(v-v')/v_0 = -21.80\times 10^{-3},$$

where v is the true velocity, v_0 is the RaC′ alpha-particle velocity of Table IV, and v' is the velocity calculated from the v^3 law, that is,

$$v' = 1.9210\times 10^9(12.70/6.945)^{\frac{1}{3}}.$$

One obtains $v=2.3071\times 10^9$, and from the relativistic energy equation, 11.09 Mev. The energy release in the reaction can be calculated by

$$Q = 2(\text{energy of one of the alpha-particles}) - \tfrac{1}{2}(\text{energy of the deuterons}),$$

and one obtains 22.08 Mev, in agreement with the value 22.06 ± 0.07 given by the original authors. It is instructive to note that the slope of the range-energy curve in this region is such that

$$dE/dx = 0.050 \text{ Mev/mm},$$

so that the author's estimated error corresponds to only 25,000 volts. Furthermore if we are to make $_4Be^8$ unstable on the mass scale proposed here by assuming an error in this reaction, we must suppose that the authors made an error of two mm in the range. Livingston and Bethe have discussed corrections for obliquity and thick target effect in this reaction and give an energy release of 22.07 ± 0.07 Mev.

The reactions thus far listed lead to masses of $_3Li^6$, $_4Be^8$, and $_4Be^9$. To include the mass of $_3Li^7$ in our system, we use the reaction

$$_3Li^7 + {}_1H^1 = {}_2He^4 + {}_2He^4 + 17.13\pm 0.06 \text{ Mev}.$$

The range of the alpha-particles produced in this reaction lies within the calibrated portion of the range-energy relation, and has been investigated by Oliphant, Kempton and Rutherford.[13] The range is about two mm shorter than that of the well-known ThC′ alpha-particle of 8.764 Mev. Computing directly from their stated mean range of 8.29 ± 0.03 cm at 0.19-Mev protons, we obtain a reaction energy 17.08 ± 0.06 Mev. We adopt the value 17.13 ± 0.06 given by Livingston and Bethe after a consideration of thick target corrections, etc.

To include the masses of the stable boron isotopes in our system, we use two reactions linking them with $_4Be^8$.

$$_5B^{10} + {}_1D^2 = {}_4Be^8 + {}_2He^4 + 17.80\pm 0.15 \text{ Mev},$$
$$_5B^{11} + {}_1H^1 = {}_4Be^8 + {}_2He^4 + 8.55\pm 0.11 \text{ Mev}.$$

The second of these reactions was investigated by Oliphant, Kempton and Rutherford[3] and the alpha-particles were found to have a mean range of 4.40 ± 0.06 cm with 0.19-Mev protons. Alpha-particles of RaC, having 4.043-cm mean range are known to have an energy of 5.508 Mev, and RaA alpha-particles of mean range 4.620 cm have 5.994-Mev energy. Interpolating between these two points it appears that 4.40-cm particles have an energy of 5.81 Mev. This leads to the reaction energy 8.55 Mev listed above. The original authors gave 8.49 as the reaction energy in Mev.

The first of the above boron reactions was investigated by Cockcroft and Lewis.[14] The high energy alpha-particles were absorbed mostly in mica, which diminishes the accuracy of the range measurement. The most accurate of the various trials they made was with 0.55-Mev deuterons, and the extrapolated range was 14.74 ± 0.05 cm. Again from Duncanson's Table VI, it is found that $(v-v')/v_0$ is -29.25×10^{-3}, and the alpha-particle energy is 12.20 Mev. This gives a reaction energy of 17.88 Mev, and allowing some weight to the other trials of Cockcroft and Lewis,

[14] J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. London **A154**, 246 (1936).

TABLE IV. *Effect of new values of e and e/m on the range-energy relationship of the alpha-particle.*

Source	Mean Range	Extrapolated Range	$H\rho\times 10^{-5}$	Velocity $\times 10^9$	Energy (Mev)	Previous Values: Energy	Previous Values: Velocity
Po	3.805	3.848	3.3189	**1.5978**	**5.305**	5.303	1.5976
RaC′	6.870	6.945	3.9928	**1.9210**	**7.678**	7.683	1.9220
ThC′	11.533	11.644	4.6736	**2.2469**	**10.543**	10.538	2.2495

TABLE V. *Reaction energy values used in computation of the masses of* Li^6, Li^7, Be^8, Be^9, B^{10}, B^{11}.

REACTION	ORIGINAL AUTHOR'S ESTIMATE	LIVINGSTON AND BETHE	ADOPTED VALUE MEV	EQUIVALENT MASS, MMU
${}_3Li^7(p, \alpha){}_2He^4$	17.06 ±0.06	17.13±0.06	**17.13 ±0.06**	18.40 ±0.07
${}_4Be^9(p, d){}_4Be^8$	0.557±0.006	——	**0.557±0.006**	0.598±0.006
${}_4Be^9(p, \alpha){}_3Li^6$	2.152±0.04	——	**2.152±0.04**	2.311±0.04
${}_3Li^6(d, \alpha){}_2He^4$	22.06 ±0.07	22.07±0.07	**22.08 ±0.07**	23.71 ±0.08
${}_5B^{10}(d, \alpha){}_4Be^8$	17.5 ±0.15	17.76±0.08	**17.80 ±0.15**	19.13 ±0.16
${}_5B^{11}(p, \alpha){}_4Be^8$	8.49	8.60±0.11	**8.55 ±0.11**	9.18 ±0.12

TABLE VI. *Masses computed from Table V.*

ATOM	ADOPTED MASS	BAINBRIDGE	ASTON	METHOD
${}_3Li^6$	**6.01670±0.00012**			$23.71 + 2{}_2He^4 - {}_1D^2$
${}_3Li^7$	**7.01799±0.00011**	7.01818±0.00012		$18.40 + 2{}_2He^4 - {}_1H^1$
${}_4Be^8$	**8.00753±0.00013**			$-0.598 + {}_4Be^9 + {}_1H^1 - {}_1D^2$
${}_4Be^9$	**9.01474±0.00014**	9.01516±0.00020		$2.311 + {}_3Li^6 + {}_2He^4 - {}_1H^1$
${}_5B^{10}$	**10.01579±0.00022**	10.01631±0.00020	10.0161±0.0003	$19.13 + {}_4Be^8 + {}_2He^4 - {}_1D^2$
${}_5B^{11}$	**11.01244±0.00019**	11.01292±0.00016		$9.18 + {}_4Be^8 + {}_2He^4 - {}_1H^1$

which gave slightly lower values, we arrive at 17.80±0.15. Cockcroft and Lewis gave 17.5 Mev, and Livingston and Bethe on recalculation of their results gave 17.76 Mev as the energy release.

The reactions used and the resultant masses are summarized in Tables V and VI. The uncertainties in the adopted masses are computed from those in the basic data by the formula

$$\epsilon = (\epsilon_1^2 + \epsilon_2^2 + \cdots)^{\frac{1}{2}}.$$

AGREEMENT WITH OTHER EXPERIMENTS

The masses of Table VI may be used to predict the energy release in nuclear reactions other than those of Table V, and the result compared with experiment. Such a comparison is given in Table VII. It is seen that the calculated and observed values of Q are in agreement within the specified limits.

The extent of the agreement of the adopted masses with certain doublets observed in the mass spectrograph is shown in Table VIII. In these doublets, then, we have agreement between the disintegration results and the mass spectrograph.

In Table VI it is evident that the adopted masses from Li^6 to B^{11} are lower than the mass spectrographic results of Bainbridge. This may be connected with the unexplained discrepancy between Bainbridge's methane-oxygen bracket and Aston's measurement of the same difference.

It is hoped that H^3 and He^3 can soon be added to the list of masses from experiments made with the electrostatic analyzer in our laboratory.

TABLE VII. *Calculated energy releases from Table VI compared with experiment.*

REACTION	CALCULATED Q	OBSERVED Q	REFERENCE
${}_4Be^9(d, \alpha){}_3Li^7$	7.09±0.17 Mev	7.03±0.12	1
${}_3Li^6(d, p){}_3Li^7$	4.94±0.11	5.02±0.12	2
${}_5B^{11}(d, \alpha){}_4Be^9$	7.98±0.23	8.13±0.12	3
${}_5B^{10}(d, p){}_5B^{11}$	9.26±0.20	9.14±0.06	3
${}_5B^{10}(n, \alpha){}_3Li^7$	2.71±0.24	2.90	4

[1] M. L. E. Oliphant, A. E. Kempton and Lord Rutherford, Proc. Roy. Soc. **150**, 241 (1935) give 7.21 Mev as the energy release here, which is higher than corresponds to their stated extrapolated range of 3.0 cm at 0.19-Mev deuterons. J. H. Williams, R. O. Haxby and W. G. Shepherd, Phys. Rev. **52**, 1031 (1937) found 2.92 cm mean range at zero deuteron energy, which gives 6.95 Mev. The 7.03 value used here is from a private communication of H. A. Bethe, who applied thick target corrections to the result of Williams, Haxby and Shepherd.

[2] J. D. Cockcroft and E. T. S. Walton, Proc. Roy. Soc. London **144**, 704 (1934) as revised in Livingston and Bethe, reference 6.

[3] J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. **154**, 246 (1936) as revised in Livingston and Bethe, reference 6.

[4] W. Maurer and J. B. Fiske, in publication. Made available to us by Dr. W. Gentner. A neutron mass of 1.00897±0.00006 is assumed in the calculated energy.

TABLE VIII. *Comparison of calculated doublet differences with those observed in the mass spectrograph.*

BRACKET	BAINBRIDGE	ADOPTED MASSES
$[Be^9H - B^{-0}]$	6.96±0.20	7.08±0.25
$[Be^9H - \frac{1}{2}Ne^{20}] - [B^{10} - \frac{1}{2}Ne^{20}]$	7.16±0.25	
$[B^{10}H - B^{11}]$	11.60±0.10	11.48±0.29
$[B^{10}H_2 - C^{12}] - [B^{11}H - C^{12}]$	11.61±0.22	
$[B^{10}H - \frac{1}{2}Ne^{22}] - [B^{11} - \frac{1}{2}Ne^{22}]$	11.50±0.52	