inquire as to its energy. The radiation, of course, may consist of several lines, but if it is embodied in a single line we see from the data of Fig. 5 that the energy is about 280 kv. This is obtained from the maximum of the K-photoelectron distribution by using the method explained in the case of Au^{198}. The relative intensity of the gamma-ray must be determined from the data obtained with all the positrons of the source stopped in its immediate neighborhood. The result is found that there is 0.21 quantum of 0.3-Mev radiation for every quantum of annihilation radiation. In other words, there is 0.4 quantum of 0.3-Mev radiation for every positron emitted. This intensity estimate is uncertain by a factor of two. The result given previously was in error.

It is interesting to observe that in the case of Cu^{64} there is indication of a high energy tail to the annihilation radiation, as was found previously in the case of a carbon radiator.[12] This may mean the presence of some gamma-radiation from this source.

[12] J. R. Richardson, Phys. Rev. **53**, 124 (1938).

Conclusion

It seems evident that for a complete investigation of a gamma-ray spectrum, it is necessary to employ radiators of various materials. A carbon radiator will give reliable information from 0.5 to 4 Mev, while an investigation of the photoelectrons ejected from a lead radiator will yield information about the region below 500 kv. If there are photoelectrons of energy 100 kv or less from the lead, then an experiment using some intermediate radiator such as cadmium is advisable. Only in this way can the complete spectrum be investigated properly.

The writer wishes to thank Professor J. M. Cork for his interest in this work. Thanks are due to Dr. R. L. Thornton and Dr. B. R. Curtis for their friendly cooperation. This research was supported by the Horace H. Rackham Trust Fund.

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The Penumbra at Geomagnetic Latitude 20° and the Energy Spectrum of Primary Cosmic Radiation

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The method of obtaining the penumbra presented in a previous paper is applied to the following energies: $r=0.385$, 0.400, 0.425, 0.450 and 0.500 Störmer at a geomagnetic latitude of 20°. Two graphs showing the variation of the penumbra with the energy are derived from the (γ_1, η) diagrams of these energies: one at a constant zenith angle of 60°, and the other along the east-west plane. If the number of primaries is assumed to vary inversely as the 2.8 power of their energy, the contribution of the penumbra to the directional intensity at a zenith angle of 60° is calculated, and is shown to be far from negligible. The calculated intensities are quite sensitive to the energy distribution used, and this suggests a possible method for determining the energy spectrum of primary cosmic rays.

The theory of the motion of charged primary cosmic particles developed by Lemaître and Vallarta leads to the conclusion that at every point of the earth there is a cone of many sheets within which all allowed directions are contained. In the terminology adopted by them,[1] the penumbra is the region of the allowed cone situated between the main cone and the Störmer cone, more precisely between the main cone and the simple shadow cone of Schremp.[2] In a previous paper[3] a description of methods for the determination of the penumbra was given, together with a brief summary of its structure. It was there pointed out that a complete analysis of this region, at least at one latitude, is imperative before definite theoretical conclusions, suit-

[1] G. Lemaître and M. S. Vallarta, Phys. Rev. **49**, 719 (1936).

[2] E. J. Schremp, Phys. Rev. **54**, 158 (1938).

[3] R. Albagli Hutner, Phys. Rev. **55**, 15 (1939).

able for an experimental verification, could be given. In this paper it is proposed to present such an analysis for the geomagnetic latitude 20°. Similar results may be presented later for other latitudes where the penumbra is still important.

I. Further Development of the Theory

In the part of the previous paper[3] concerning Fig. 8, it was pointed out that all orbits eventually going to (or coming from) infinity had to bounce between the two vertical envelopes F_0' and F_1' (Lemaître and Vallarta's F_0 and F_1, Fig. 1).[1] This leads to the fact first pointed out by Schremp,[4] that there is a value[5] of x, and therefore r, above which the penumbra is completely dark. The data as obtained directly from Bush's Differential Analyzer[6] are plotted in Fig. 4 (previous paper[3]), upper diagram. However when the work was continued in the summer and fall of 1938 it was found that a lower limit than that of the F_0 cut-off can be assigned for certain values of γ_1. In the method for generating re-entrant orbits[4] (third program[3]), starting points were taken along the equator at definite

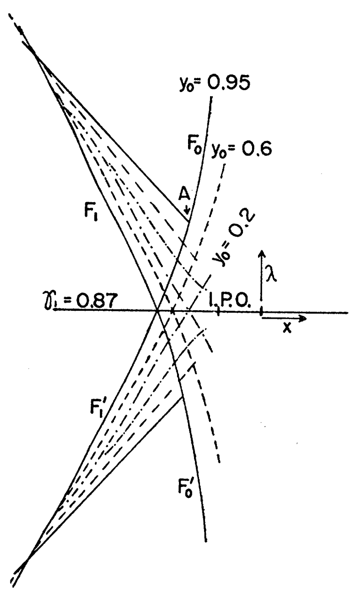


FIG. 1. The loci $dx/d\sigma=0$ of the re-entrant orbits through three values of y_0 for $\gamma_1=0.87$. The solid lines are, within the error of the Differential Analyzer, those for the asymptotic orbits. I. P. O. is the point of intersection of the inner periodic orbit and the equator.

[4] E. J. Schremp, Phys. Rev. **54**, 153 (1938).
[5] For the necessary definitions see reference 1.
[6] V. Bush, J. Frank. Inst. **212**, 447 (1931).

TABLE I. *Positions of A corresponding to each* γ_1.

γ_1	x	r(Störmers)
0.82	−0.070	0.568
0.85	−0.095	0.535
0.87	−0.118	0.511
0.89	−0.125	0.496
0.93	−0.143	0.466
0.97	−0.170	0.435

intervals between the outer and inner periodic orbits for each γ_1. The set of curves corresponding to each initial point is indicated by a value[3] of y_0; Fig. 7 of the previous paper[3] shows an example for $y_0=0.1$ and $\gamma_1=0.97$. The dots marked on these orbits, at the turning points on the left (not at the bottom), indicate the points at which $\dot{x}=0$ ($\dot{x}=dx/d\sigma$). A line connecting the successive points where $\dot{x}=0$ would intersect the two vertical dashed lines (the envelopes for the asymptotic orbits). Those orbits touching the line $\dot{x}=0$ in the part contained between the envelopes of the asymptotic orbits will go directly to infinity, while all other orbits will have at least one more re-entrant section.

The superposition of the different sets of curves for a given γ_1 but successive y_0's shows an orderly arrangement for the corresponding $\dot{x}=0$ curves, as indicated in Fig. 1 for $\gamma_1=0.87$ and for just three y_0's (to avoid confusion). The symmetry of the solutions about the equator permits the same set of lines to be drawn on each side of the $\lambda=0$ line, as shown. A study of Fig. 1 yields the fact that all re-entrant orbits must cross the envelopes of the asymptotic orbits (solid lines) to the left of the point A. Therefore, although the F_0 envelope reaches a value of x (and hence r) greater than that of A, it is the latter which determines the energy[7] above which the penumbra is dark for a given γ_1. Likewise all greater values of γ_1 are dark in the penumbra for the value of r at the point A. The quantitative data from the Differential Analyzer for the values of A corresponding to each γ_1 are listed in Table I, where the position of A is given in both Störmer units (r) and units of x. Fig. 2 shows the deviation between the results (solid line) published previously (referred to as the F_0 cut-off) and those (dashed line)

[7] For a table of equivalences between Störmer units and energy in ev see G. Lemaître and M. S. Vallarta, Phys. Rev. **43**, 90 (1933).

contained in Table I. For γ_1 equal to about 0.93 and above, the two curves coincide. An example of the use of the dashed curve appears in Fig. 5, line AB.

II. Analysis of Further Results Obtained from the Differential Analyzer

A description of the third program used on the Differential Analyzer, as well as a table designating all the curves obtained, appeared in a previous paper.[3] These data were used as the bases of the attempt to complete the diagrams of the penumbra for all pertinent energies at one latitude, i.e., for the range $0.376 < r < 0.700$ störmer at geomagnetic latitude 20°. In practice, energies above $r = 0.500$ Störmer contribute very little so that the final selection of values of r were: 0.385, 0.400, 0.425, 0.450 and 0.500 Störmer.

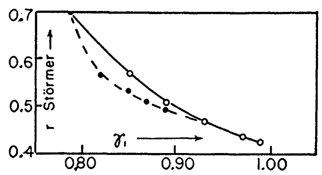


FIG. 2. All values of (γ_1, r) above the dashed line have the penumbra completely dark. The solid line was determined from the F_0 cut-offs; the dashed line is a result of all the diagrams like that of Fig. 1.

Description of the fourth program

As the η diagrams for these values were drawn it became evident that more data were needed, and the Differential Analyzer was resorted to once again in the spring of 1938. Since a more definite and limited goal was then in view, a new approach (the fourth program) was decided upon, in which runs were started at the particular (r, λ) desired and for such values of η as were needed to complete the η diagrams derived from the trajectories obtained in the third program. As a check on the results deduced from the third program, the runs of the fourth program were made to overlap those of the third program, with a resulting discrepancy that proved larger than was expected. It had always been known that the ends of long trajectories were not to be trusted, and one of the excellent features of the third program was the fact that all runs were cut approximately in half by starting at the equator. But even with this advantage some of the results deduced from the third program were shown to be definitely wrong. This depended on the fact that the points (r, λ) through which the orbits were desired came at the ends of the runs of the third program and very often at curved portions of the trajectories, where the slope (and hence the angle η) changes very rapidly. (See Fig. 10 of previous paper.[3]) Thus, because of the cumulative error, orbits were made to reach points in the immediate vicinity of the bounce which they would not have attained had the path been traced correctly. On the other hand the runs of the fourth program were made to start at the desired point and slope, and hence were known to go through that point, but, at the same time, the total trajectories were usually longer.

A study of the two sets of curves and several test runs led the writer to believe that most of the error is introduced whenever there is a long drive in any one direction for either x or λ and considerably amplified when this is followed by a sharp turn. In this case the error would not depend merely on the total arc length traversed but rather on the particular twists of each orbit. It was thought that a rough calibration of some sort between the two programs could be obtained by running a complete set of curves, by the method of the fourth program, for the η diagram of $\gamma_1 = 0.91$, $r = 0.425$, $\lambda = 20°$, and comparing it with the corresponding η diagram as derived from the curves of the third program. This and other direct comparisons between trajectories of the third and fourth programs were the bases for the successive approximations which brought the curves of the two programs into agreement and led to the final results presented here. In the case of high values of η (positive or negative) the curves of the fourth program were relied on entirely, which was perfectly valid, for the majority of such cases were short runs. For the entire η diagrams of $r = 0.385$ and 0.400, the third program orbits were not to be trusted since the orbits were in the immediate vicinity of a bounce when passing through these points; the results were based wholly on the fourth program.

Results

Figures 3 to 5, inclusive, give the final appearance of the penumbra for the energies designated, in the form of (γ_1, η) diagrams rather than in the cone projection of Lemaître and Vallarta.* The reason for the use of the (γ_1, η) diagram is that the cone projection distorts the results to the disadvantage of the regions close to the horizon, whereas the former gives the same weight to all angles. Areas of "light" are shaded by lines NW to SE, closely spaced; while those of "darkness" are NE to SW, widely spaced. Blank spaces between the strips have bands of narrow to infinitesimal width; any long blank columns indicate lack of sufficient data. The exact limits of the regions bordered by dashed lines are uncertain, but there is evidence for the shape of each such region. The point marked "zenith" is the horizontal projection of the zenith direction. The type of orbit in the conformal, meridian plane (x, λ) giving rise to each patch is shown. The curve marked $z=60°$ is the projection of the locus of points at an angle of 60° with the zenith direction at the given point (r, λ); this zenith angle, rather than the one for $z=45°$, is chosen because the con-

* See reference 3 for examples of each for the same (r, λ).

tribution of the penumbra is greater. The values of θ corresponding to the values of some of the γ_1's are marked at the bottom of each diagram. Note that the diagram for $r=0.425$ Störmer published previously[3] has been not only completed but also improved. All diagrams in this paper are for positive particles in the Northern Hemisphere.

Deductions

From the relation:

$$\cos z = \cos \theta \cos \eta,$$

where z is the zenith angle, the values of θ and η corresponding to a constant zenith angle can be calculated; the γ_1 belonging to each θ for a given (r, λ) can be determined; and finally the curves of constant z drawn on the (γ_1, η) diagrams, as shown on Figs. 3 to 5 for $z=60°$. In order to see the variation of the penumbra with the energy, r, the changes along the curve of constant zenith angle may be plotted as in Fig. 6 for $z=60°$. There is the choice of plotting either η or θ against r. Both were tried and the (η, r) graph found to be more convenient; however

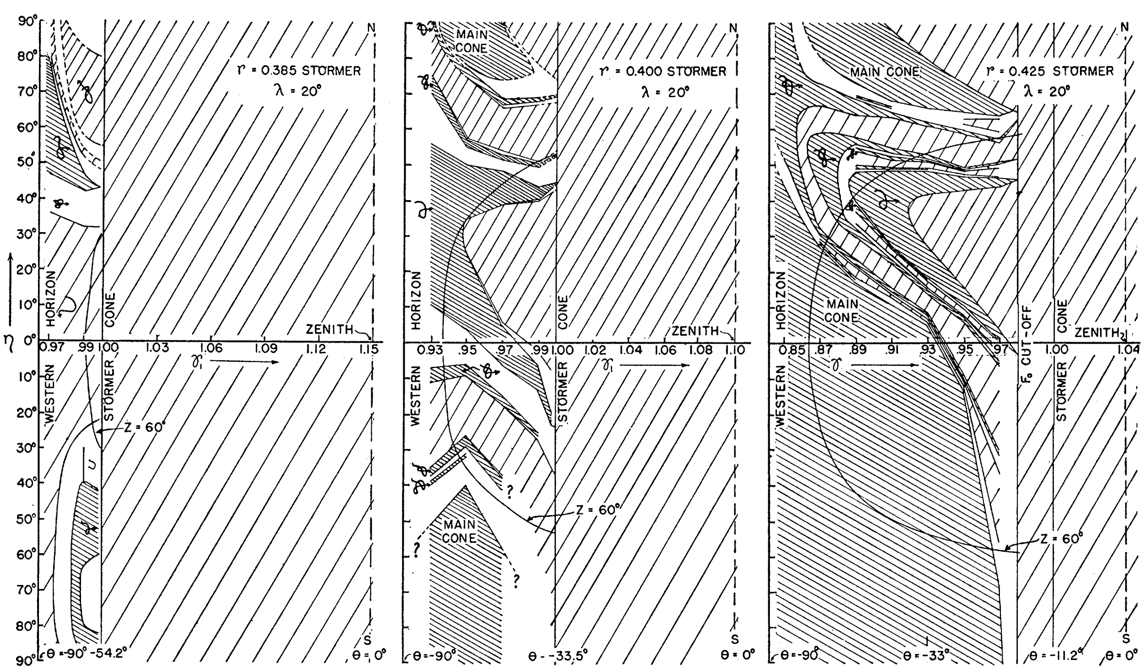
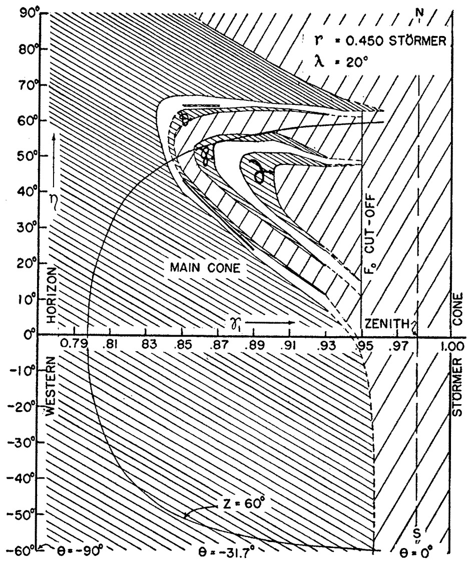


FIG. 3. General appearance of the penumbra for various values of r at $\lambda=20°$. Positive particles; Northern Hemisphere. For $r=0.385$ Störmer (0.881×10^{10} ev for electrons) the eastern horizon appears at $\gamma_1=1.33$; for $r=0.400$ Störmer (0.954×10^{10} ev) it begins at $\gamma_1=1.29$; and for $r=0.425$ (1.07×10^{10} ev) it begins at $\gamma_1=1.24$. In all cases the region from the Störmer cone line to the eastern horizon are "dark."

FIG. 4.



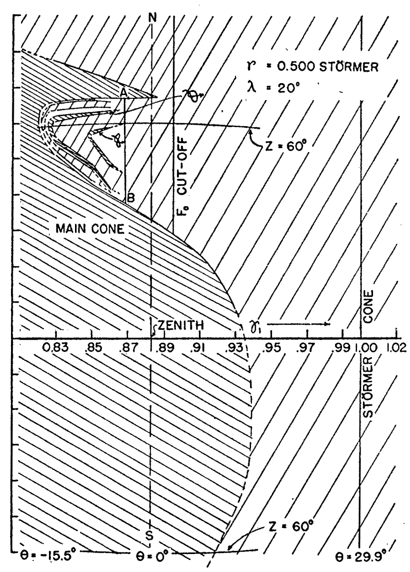


FIG. 5.

FIG. 4. General appearance of the penumbra for $r=0.450$ Störmer (1.20×10^{10} ev for electrons). On the right, the eastern horizon appears at $\gamma_1=1.19$, all of the region from the Störmer cone line to the eastern horizon being dark. The diagram continues in essentially the same way from $-60°$ to $-90°$ in η. Positive particles; Northern Hemisphere.

FIG. 5. General appearance of the penumbra for $r=0.500$ Störmer (1.49×10^{10} ev for electrons). On the left, the western horizon appears at $\gamma_1=0.648$, with complete light except for Schremp's simple shadow cone; on the right, the eastern horizon appears at $\gamma_1=1.12$. The line AB is determined by the dashed line in Fig. 2. The diagram continues in essentially the same way to $\eta=-90°$. Positive particles; Northern Hemisphere.

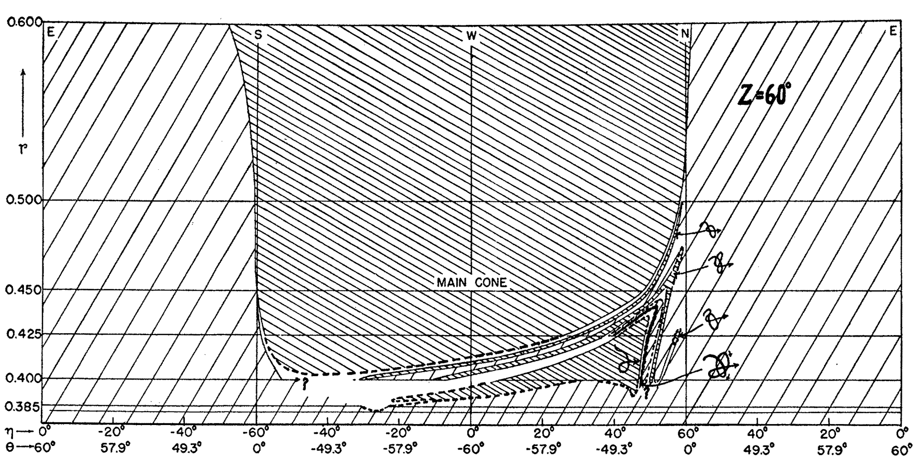


FIG. 6. The effect of r, in Störmers, on the areas of light and dark along the constant zenith angle of 60°. The heavy, black band above the right-hand question mark is a region of light, the type of orbit being indicated by the lower right-hand figure. The letters at the top of the diagram represent the East, North, West and South directions. Positive particles; Northern Hemisphere.

the corresponding values of θ are given in the bottom line of Fig. 6. The East, North, West and South directions are marked. The data from the (γ_1, η) diagrams (Figs. 3 to 5) are represented by horizontal, heavy lines at the marked values of r in Störmers. Sections of "light" are cross-hatched from NW to SE and closely spaced; those of "darkness" are from NE to SW and widely spaced. The narrow blank regions contain strips of small to infinitesimal widths; the blank space around each question mark is due to insufficient data. By studying the diagrams of the successive r's (Figs. 3 to 5) quite thoroughly, some conjectures have been hazarded as to how the strips of "light" and "dark" may be extrapolated, and in all cases have been bordered with dashed lines. The values of r for which these particular strips terminate are not at all certain but there is evidence that the general shapes of the regions are as indicated.

In Fig. 7 the effect of the energy on the "light" and "dark" regions along the East-West plane is shown by plotting r against the zenith angle along the East-West plane. The markings have the same significance as in the previous diagram.

To obtain cosmic-ray intensities, diagrams like those of Figs. 6 and 7 are especially convenient for determining the limits of integration for the distribution function depending on r.[8] Fig. 6 makes it clear that for any intensity distribution which emphasizes the lower energies, the penumbra will make important contributions.

To illustrate this, the number of rays per second, $n=\int F(E)dE$, was calculated for the three energy distributions:

$$F_1(E)=B/E^{2.8}, \quad F_2(E)=K/E, \quad F_3(E)=D/e^E.$$

The first is made plausible by Nordheim's[9] paper, where he discusses the results and suggestions of many other workers; the others were conveniently chosen to show how sensitive the resultant intensity may be to the energy distribution function used. In all cases n, in arbitrary units, (Fig. 8) is plotted against the azimuthal angle, α, along the curve $z=60°$ from a direction due North to one due West. The relation between the azimuthal angle, α, counted clockwise from the North, the zenith angle, z, and the angle θ, already introduced before is:

$$\sin\theta=\sin z\sin\alpha.$$

This zenith angle and range of azimuths were chosen because a study of Fig. 6 indicates that these directions are the most promising for striking contributions by the penumbra. All of the humps are due entirely to the irregularities of the penumbra; without it the curve in each case would rise very gradually from N to W, without any maxima or minima. Fig. 8 shows that the presence and size of these humps is indeed sensitive to the energy distribution function.

It should be pointed out that the possibility of the presence of humps in curves of this sort was qualitatively predicted by Schremp several years ago and first published[4] in 1938.

The curves of Fig. 8 are valid for positive particles in the Northern Hemisphere. For negative particles in the Northern Hemisphere each (γ_1, η) diagram is to be rotated about the N-S line, so that what appears on the West for posi-

[8] For electrons at high energies: $E=Mer^2/R^2=0.095r^2$; where M is the magnetic moment in e.m.u., e is the electronic charge in e.s.u., R is the radius of the earth in cm, E is the energy in ergs and r is in Störmers.

[9] L. W. Nordheim, Phys. Rev. **53**, 694 (1938).

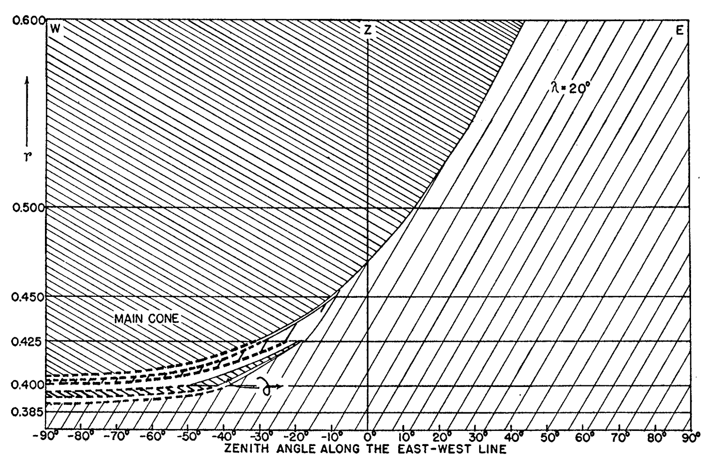


FIG. 7. The energies, r, are in Störmers. The W, Z and E at the top of the diagram indicate the West, zenith and East directions, respectively. Positive particles; Northern Hemisphere.

tive particles is now on the East for negative ones. The study of Fig. 6 shows that there is no contribution to the penumbra in the N to W section by the negative particles. The effect of their main cones is a rapidly decreasing contribution from N to W, which would just barely wipe out the first small hump of the $B/E^{2.8}$ distribution, if there were an equal number of positive and negative particles; but would serve to increase the difference between the large hump and the rest of the curve toward the West.

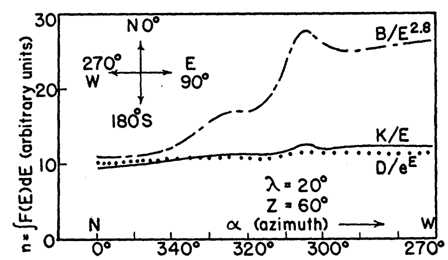


FIG. 8. The azimuthal effect resulting from the three energy distribution functions indicated, with the energy limits obtained directly from Fig. 6. The integrations were made over all energies, including those which are too high to be affected by the earth's magnetic field. Positive particles; Northern Hemisphere. For a mixture of positives and negatives see text.

For the values of the azimuth from 0° to 90°, i.e., from N to E, the curves of Fig. 8 hold for the negative particles, and the positive particles would contribute by their main cones only, in the way mentioned in the previous paragraph for negative particles. Thus if suitable counter experiments could be made, it would seem that the number *versus* azimuth curve between the N and E directions should be less irregular than that between the N and W (in the Northern Hemisphere) because of the supposed predominance of positive particles.

Of course the results shown here refer strictly to primaries which have not passed through the atmosphere. As for the secondaries, it is possible that the soft cosmic rays do not show up these irregularities, since, if they are produced to a large extent by cascade showers, any original directional effect might well be hidden by the time that the rays reach low altitudes. On the other hand the more penetrating secondaries would seem to be more promising, especially at altitudes higher than sea level, as already emphasized by Rossi.[10] He points out that the East-West asymmetry is greater if the particles of least energy are filtered out. At Asmara, Eritrea ($\lambda = 11.5°$, altitude 2730 m) a ratio of 1.32 ± 0.03 at $z = 45°$ was found with a lead screen of 8 cm between counters, while without the screen this dropped to 1.19 ± 0.02.

It should be pointed out that although the penumbra for this latitude has been fairly well outlined, no definite decisions on that for other latitudes can be reached. By referring to Fig. 5 of the previous paper,[3] it is seen that latitudes from 25° to 40° may well prove interesting because of the range of energies involved.

Acknowledgments

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[10] B. Rossi, International Conference on Physics 1, 237 (1934).