

### Electro-Optical Effects in Colloids

Recently the writer has repeated a few of Krishnan's experiments on the scattering of light in colloids. Most of his interesting results could be confirmed, but for the case of graphite sols our conclusions differ in one point. Contrary to his statement<sup>1</sup> the increase of the depolarization factor  $\rho_h$  in a magnetic field parallel to the direction of the incident light is due to a decrease of the component  $H_h$  while  $V_h$  remains constant. With this correction comparison with the theoretical predictions for the scattering in a strained glass<sup>2</sup> is somewhat improved. Changes of the depolarization factors which are similar to those for a graphite sol in a magnetic field can be produced also by electric fields. This effect is particularly large for a colloid of lead carbonate.

Since these effects depend on the anisometric form of the micelles it was expected that they should occur also in the sols of bentonite. However, neither a magnetic nor an electric field produces any noticeable changes of the scattering in these colloids if the incident light vibrates parallel or normal to the applied field. But a peculiar change of the Tyndall effect is observed when the incident light is polarized at 45° to the electric field. Observations normal to the plane of vibration of the incident light reveal a series of equi-distant dark regions along the path of the light beam. For observation normal to the plane of polarization these regions are light. The stragglings are not visible in observations parallel or normal to the electric field. The distance between the regions decreases rapidly with increasing field strength and approaches a minimum value of about 2 cm for a field of 70 volt/cm a.c. in a sol with small particles (10  $\mu\mu$ ).

The origin of this effect is due not to a change of the scattering properties of the sol but to a change of the incident light. It is caused by an unusually large electro-optical effect in the bentonite sols. The scattering phenomenon is analogous to that used in the well-known demonstration experiment in which polarized light is passed through a sugar solution which contains some scattering matter, but in our experiment the effects are due to ordinary and not to circular birefringence.

Investigations of the transmitted light verify this conclusion. For fields up to 40 volt/cm a.c. this "Kerr" effect follows the law  $\Delta n = n_0 K E^2$ . The "Kerr" constant  $K$  is extremely large. For a concentrated sol we found  $K = 1.2 \cdot 10^{-3}$ , i.e., the effect is  $10^6$  times larger than in nitrobenzene, 100 times larger than in Rochelle salt and 10 times larger than in the colloidal solutions of vanadium pentoxide.<sup>3</sup> For larger fields the birefringence approaches a saturation value which is so large that the two light components have a phase shift of 90° per cm. For d.c. voltage  $K$  is about twice as large as for 60-cycle a.c. An effect of the same order of magnitude exists also for high frequency fields ( $10^6$  cycles).

It should be pointed out that the electro-optical effect in these colloids differs from the ordinary Kerr effect insofar as the birefringence does not vary with the period of the field but remains constant. When the field is removed the birefringence disappears gradually with a

relaxation time of about 0.1 to 1 sec., depending on the concentration. In very viscous solutions it persists for hours. The relaxation time is of the same order of magnitude as in the experiments on streaming birefringence.<sup>4</sup> It appears therefore most probable that both effects have the same origin.

I am indebted to Professor Hauser and Dr. Le Beau for supplying the bentonite colloids.

HANS MUELLER

Massachusetts Institute of Technology,  
Cambridge, Massachusetts,  
February 13, 1939.

<sup>1</sup> R. S. Krishnan, Proc. Ind. Acad. Sci. 7, 91 (1938).

<sup>2</sup> H. Mueller, Proc. Ind. Acad. Sci. 8, 267 (1938).

<sup>3</sup> J. Errera, W. Overbeck and H. Sack, J. chim. Phys. 32, 681 (1935).

<sup>4</sup> I. Langmuir, J. Chem. Phys. 6, 873 (1938).

### Note on Reduction for the Rotation Group

The reduction of a product representation of the rotation group into irreducible representations is important in many physical problems. If  $U_m, V_{m'}$  form the basis vectors of irreducible representations of rank  $2j+1, 2j'+1$ , respectively ( $m$  running from  $j$  to  $-j, m'$  from  $j'$  to  $-j'$ ), then the products  $U_m V_{m'}$  can be combined to form basis vectors  $W_{M^J}$  of irreducible representations of rank  $2J+1$ .  $J$  has values decreasing from  $j+j'$  by integers to  $|j-j'|$  and  $M = m+m'$ .

A simple method of carrying out this reduction is given in van der Waerden's *Die Gruppentheoretische Methode in der Quantenmechanik*.<sup>1</sup> The connection between the vectors  $U, V, W$  is there given by

$$W_{M^J} = \rho_J \sum_m C_{m, m'} U_m V_{m'}, \quad (1)$$

$$C_{m, m'}^J = \sum_{\nu} (-)^{\nu} \frac{[(j+m)!(j-m)!(j'+m')!(j'-m')!]}{(J+M)!(J-M)!^{\frac{1}{2}}} \frac{(j-m-\nu)!(j+m-\lambda+\nu)!(j'+m'-\nu)!}{\times (j'-m'-\lambda+\nu)! \nu!(\lambda-\nu)!} \quad (2)$$

$$m' = M - m \quad \lambda = j + j' - J.$$

The coefficients  $\rho_J$  are not determined by van der Waerden's argument. But it is often useful to have the matrix connecting the  $W$ 's with the  $UV$ 's unitary. Suitable values of  $\rho_J$  can be found as follows:

A necessary condition that the transformation be unitary is

$$\sum_m (\rho_J C_{m, M-m}^J)^2 = 1. \quad (3)$$

We evaluate this for the particular case  $M=J$ . In the series (2), the only term that contributes for this case is  $\nu=j-m$  and we find

$$C_{m, J-m}^J = (-)^{j-m} \frac{[(2J)!]^{\frac{1}{2}}}{(2j-\lambda)!(2j'-\lambda)!} \times \left[ \frac{(j+m)!(j'+m')!}{(j-m)!(j'-m')!} \right]^{\frac{1}{2}} \quad (2a)$$

$m$  can take on values from  $j-\lambda$  up to  $j$ . With  $\sigma=j-m$  as summation index (3) becomes