Lifetime of the Yukawa Particle

Recent investigations by various authors[1] have made it very probable that the hard rays of the cosmic radiation (mesotrons), now identified with the particle of Yukawa[2] of mass $\mu \sim 200m$ (m = mass of the electron), are unstable and will decay spontaneously into electrons and neutrinos. The lifetime for a mesotron at rest has been estimated from experience to be of the order $2\text{–}4 \times 10^{-6}$ sec.

Yukawa himself calculated the lifetime on the basis of his ideas to be of the order 0.25×10^{-6} sec., a result not far from the observed value. However, the present author[3] obtained on the same assumptions a much smaller value. The importance of this question may justify a restatement of the theoretical result and an explanation of this difference.

The final formulae for the lifetime obtained by both authors is the same, apart from differences in notation. It can be written in the form

$$\tau = \frac{G^2}{\hbar c} 4\pi^2 \left(\frac{m}{\mu}\right)^4 \frac{\hbar}{\mu c^2} \frac{1}{G_F^2}. \tag{1}$$

In this formula $\hbar$, m, c have the usual meaning, and μ is the rest mass of the mesotron. G is the constant of dimension of a charge in the potential between nuclear particles $V(r) = (G^2/r)e^{-r\mu c/\hbar}$ following from Yukawa's theory. $G^2/\hbar c$ is of the order[3, 4] μ/M (M = mass of the proton) but probably somewhat larger than this quotient. The lifetime τ is therefore essentially proportional to μ^{-4}. G_F finally is the constant in Fermi's theory of β-decay, normalized to be a pure number. The form of interaction assumed for the coupling between proton, neutron and the electron neutrino field is

$$G_F mc^2(\hbar/mc)^3(\psi_N^* \beta \psi_P)(\varphi_\nu^* \beta \varphi_e) + \text{c.c.}$$

(ψ_N, ψ_P, φ_ν, φ_e being the wave functions of neutron, proton, neutrino and electron, respectively). This leads to the probability for emission of an electron of energy ϵ

$$w(\epsilon)d\epsilon = |M|^2 \frac{G_F^2}{(2\pi)^3} \frac{mc^2}{\hbar} \frac{(\epsilon_0 - \epsilon)^2(\epsilon^2 - mc^2)^{\frac{1}{2}} \epsilon d\epsilon}{(mc^2)^5},$$

where ϵ_0 is the maximum energy of the emitted electrons and M a matrix element from the motion of the heavy particles inside the nucleus.

The discrepancy in the calculated lifetimes comes from the different values used for the constant G_F. As discussed by Bethe and Bacher[5] and by Nordheim and Yost,[6] the experimental value of G_F depends quite appreciably on the group of elements which are taken for comparison, the difference being due in all probability to the matrix element M, which is smaller than unity for heavy elements but can be expected to be unity for light positron emitters. The value for G_F used by Yukawa (0.87×10^{-12} in our units) corresponds to the heavy natural radioactive elements, while the value deduced for the light positron emitters[6] is $G_F = 5.5 \times 10^{-12}$. It seems beyond doubt that this later value has to be taken for our purpose.

With the present most probable values $G^2/\hbar c = 0.3$; $\mu = 200m$; $G_F = 5.5 \times 10^{-12}$, we obtain from (1) $\tau = 1.6 \times 10^{-9}$ sec., i.e., a value about 10^{-3} times too small. A decrease in the assumed value for μ to $150m$ would increase τ only by a factor of order 3.

In view of this definite discrepancy the question arises whether any modifications of the theory could give a better result. It is to be noted firstly that the introduction of the Konopinski-Uhlenbeck form of the β-decay theory would only make matters worse as it would introduce roughly another factor $(m/\mu)^2$. A real improvement can only be expected by a complete reformulation of the theory. One possible suggestion would be to assume that the disintegration of a free mesotron is in first order approximation a forbidden transition, while in nuclei it is made allowed by the influence of the other nuclear particles.

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[6] L. W. Nordheim and F. Yost, Phys. Rev. **51**, 942 (1937). It has to be noted that the formula for τ_0 on p. 943 should be $\tau_0^{-1} = (G_F^2/(2\pi)^3) \times (mc^2/\hbar)$. The value of G_F is then determined from the empirical value $\tau_0^{-1} \cong 10^{-4}$.

The Scattering of Cosmic Rays by the Stars of a Galaxy

The problem dealt with in this note may be formulated in the following way: imagine a galaxy of N stars, each carrying a magnetic dipole of moment μ_n ($n = 1, 2, \cdots N$) and assume that the density, defined as the number of stars per unit volume, varies according to any given law, while the dipoles are oriented at random because of their very weak coupling. Under this condition the resultant field of the whole galaxy almost vanishes. Let there be an isotropic distribution of charged cosmic particles entering the galaxy from outside. Our problem is to find the intensity distribution in all directions around a point within the galaxy. Its importance arises from the fact that if the distribution should prove to be anisotropic a means would be available for determining whether cosmic rays come from beyond the galaxy, independent of the galactic rotation effect already considered by Compton and Getting.[1]

Suppose we consider a particle sent into an element of volume dV of scattering matter in a direction given by the vector R. Let the probability of emerging in the direction R' be given by a scattering function $f(R, R')$ per unit solid angle. Conversely a particle entering in the direction R' will have a probability $f(R', R)$ of emerging in the direction R. Let us assume that the scatterer (magnetic field of the star) has the reciprocal property so that $f(R, R') = f(R', R)$. In our case this property is satisfied provided the particle's sign is reversed at the same time as its direction of motion. That is, the probability of electron's going by any route is equal to the probability of positrons going by the reverse route. If it has the reciprocal property for each element of volume it will also have it for