

produce the sunspot magnetism, namely, in the upper layers of the sun's atmosphere. It can scarcely be doubted that extremely high electric space charges can develop there by the effect of variations in light pressure and ionization. It is known that the small *external* magnetic field of the earth is due to currents in the higher parts of the earth's atmosphere.<sup>2</sup> It seems probable, therefore, that the proportion of the external and internal field is the reverse for sunspots as it is for the earth and that the sunspot field is mainly of an external, atmospheric origin. This being

stated, we may expect the *primary* causes of the field to be the same as assumed above, namely, differences in temperature and state of aggregation exhibiting a typical asymmetry due to the action of the Coriolis force. These differences need not be located only in the higher atmospheric layers where the electric currents flow, but may arise in the lower and denser layers of the photosphere. They may then be transmitted to a higher level by local variations in intensity and composition of the radiation which flows upwards from these layers.

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## Thermal Vibrations and Atomic Structure of Zinc Crystals at 100° and 298°K

G. E. M. JAUNCEY AND E. M. McNATT

*Wayman Crow Hall of Physics, Washington University, St. Louis, Missouri*

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The diffuse scattering of x-rays from single crystals of zinc has been investigated over a greater range of angles of scattering than in previous experiments. As a result of this more extended study, the root-mean-square displacements of the crystal atoms at  $T=100^\circ\text{K}$  are 0.098 and 0.050Å for orientation angles of  $0^\circ$  and  $90^\circ$ , respectively. At the melting point of zinc it is likely that the root-mean-square displacement in a given direction is not more than 8 percent of the axial distance in the same direction. The atomic structure factors at a scattering angle of  $90^\circ$  are somewhat larger than would be expected on quantum-theoretical grounds. At large scattering angles the curves of the intensity of the scattered rays approach each other irrespective of temperature or orientation, the intensity approaching that of x-rays scattered from a gas of zinc atoms.

### 1. INTRODUCTION

THE senior author and others<sup>1</sup> have investigated the atomic structure and vibrations in single zinc crystals at various temperatures and angles of orientation by means of diffuse scattering. However, although temperatures of  $100^\circ$ ,  $200^\circ$ ,  $298^\circ$ ,  $370^\circ$  and  $550^\circ\text{K}$  and orientations of  $3^\circ$  and  $90^\circ$  were used, only one angle of scattering—namely,  $30^\circ$ —was used for temperatures other than  $298^\circ\text{K}$ . Further, even at room temperature the scattering angles were restricted to the range  $15^\circ$  to  $40^\circ$ . Hence, it was believed desirable to extend the investigation to a greater range of scattering angles.

In order to confine the present investigation to

reasonable limits, the scattering angles used were in the range  $15^\circ$  to  $90^\circ$ , the temperatures were  $100^\circ\text{K}$  and  $298^\circ\text{K}$ , and the crystal orientations were nearly  $0^\circ$  and nearly  $90^\circ$ . With the exception of a cryostat for the low temperature work which has been described elsewhere,<sup>2</sup> the apparatus and method used were essentially those of the previous papers. The spectral distribution was that shown in Fig. 2 of Jauncey and Bruce's paper.<sup>1</sup>

### 2. EXPERIMENTAL RESULTS

We make use of the experimental quantity<sup>3</sup>

$$(S\rho/\mu)_{\text{exp}} = \frac{4R^2 W m^2 c^4}{ANZe^4(1 + \cos^2 \phi)} \frac{D(\phi, \theta)}{D_0 \mu(\phi, \theta)}, \quad (1)$$

<sup>2</sup> E. M. McNatt, Rev. Sci. Inst. **10**, 42 (1939).

<sup>1</sup> G. E. M. Jauncey and W. A. Bruce, Phys. Rev. **51**, 1067 (1937); W. A. Bruce and E. M. McNatt, Phys. Rev. **51**, 1065 (1937). See these papers for earlier references.

<sup>3</sup> G. E. M. Jauncey and W. A. Bruce, Phys. Rev. **50**, 413 (1935); G. E. M. Jauncey and W. D. Claus, Phys. Rev. **46**, 941 (1934).

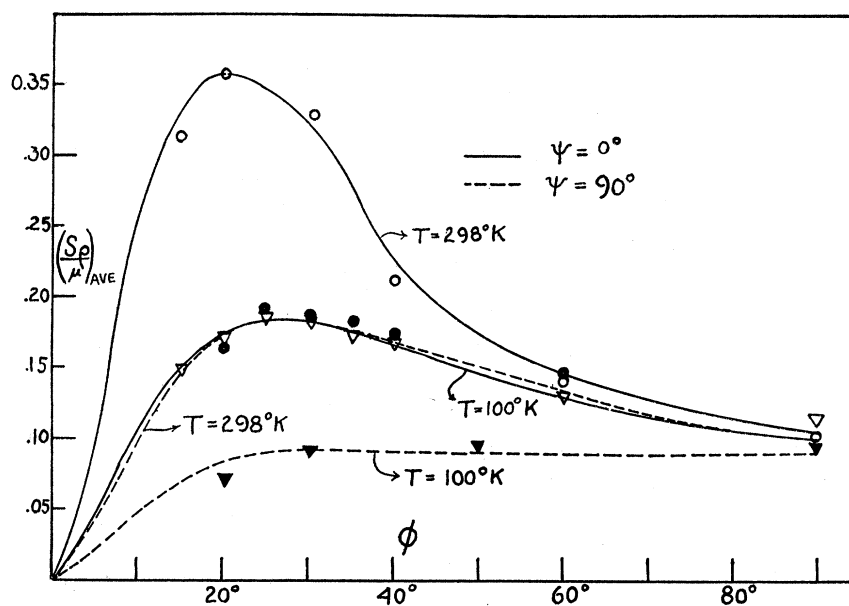


FIG. 1. Diffuse scattering of x-rays of average wave-length 0.44A from single zinc crystals.

where  $D(\phi, \theta)$  and  $D_0$  are, respectively, the ionization currents produced by the scattered and the primary rays, and where

$$u(\phi, \theta) = \frac{2 \sin(\phi - \theta)}{\sin(\phi - \theta) + \sin \theta}, \quad (2)$$

$\phi$  is the scattering angle,  $\theta$  is the angle between the primary rays and the surface of the crystal,  $R$  is the distance of the ionization chamber window from the axis of the spectrometer,  $A$  is the area of the chamber window,  $W$  the atomic weight of the crystal, and  $N$ ,  $Z$ ,  $e$ , and  $m$  have their usual meanings.

The experimental values of  $(S_p/\mu)_{\text{exp}}$  are shown as points in Fig. 1. The white circles represent values for the orientation angle  $\psi = 0^\circ$ ,  $T = 298^\circ\text{K}$ , the black circles represent values for  $\psi = 0^\circ$ ,  $T = 100^\circ\text{K}$ , the white triangles represent values for  $\psi = 90^\circ$ ,  $T = 298^\circ\text{K}$ , and the black triangles values for  $\psi = 90^\circ$ ,  $T = 100^\circ\text{K}$ .

The theoretical quantity  $(S_p/\mu)_{\text{th}}$  has previously been defined.<sup>4</sup> This quantity is a function of the true atomic structure factor and of the components of the atomic vibrations in a direction which bisects the angle between the direction

of the scattered rays and the backward direction of the primary rays. The angle between the direction of these components and the  $c$  axis of the zinc crystal is the orientation angle  $\psi$ . According to Zener<sup>5</sup> the mean square of the displacements due to these components is related to the orientation angle  $\psi$  by

$$\langle u_\psi^2 \rangle_{\text{AV}}^{\frac{1}{2}} = \{(a \cos^2 \psi + b \sin^2 \psi) / 8\pi^2\}^{\frac{1}{2}}, \quad (3)$$

where  $a/8\pi^2$  and  $b/8\pi^2$  are the mean square displacements due to the components of the vibrations in the directions  $\psi = 0^\circ$  and  $\psi = 90^\circ$ , respectively. Bruce<sup>6</sup> has described the method of analysis for determining the true atomic structure or  $f$  values from the measurements of the diffuse scattering. The  $f$  values shown in Table I for the range  $(\sin \frac{1}{2}\phi)/\lambda = 0.1$  to  $1.0 \text{ \AA}^{-1}$  are due to Bruce.<sup>6</sup> The  $f$  values for  $(\sin \frac{1}{2}\phi)/\lambda$  above  $1.0 \text{ \AA}^{-1}$  were obtained by us with the method of analysis described by Bruce.<sup>6</sup> It is also necessary to know

TABLE I. True atomic structure factors for zinc.

| $(\sin \frac{1}{2}\phi)/\lambda$ | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.8 | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 |
|----------------------------------|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|
| $f$ for $\psi = 0^\circ$         | 27.0 | 23.5 | 20.2 | 17.0 | 14.4 | 12.5 | 9.5 | 7.3 | 5.5 | 4.5 | 3.8 | 3.5 |
| $f$ for $\psi = 90^\circ$        | 28.4 | 25.9 | 21.5 | 18.2 | 14.9 |      |     |     |     |     |     |     |

<sup>4</sup>G. E. M. Jauncey and W. A. Bruce, Phys. Rev. **51**, 1062 (1937).

<sup>5</sup>C. Zener, Phys. Rev. **49**, 122 (1936).

<sup>6</sup>W. A. Bruce, Phys. Rev. **53**, 802 (1938).

the  $\Sigma E_r^2$  values. The most satisfactory set of these values is shown in Table II. The values of Tables I and II were used in obtaining the curves of Fig. 1 from the formula for  $(S\rho/\mu_1)_{th}$ . In addition, the values of  $a$  and  $b$  shown in Table III were used in the calculation of ordinates of the curves in Fig. 1. It is to be noted that, although the values of  $a$  and  $b$  for room temperature (298°K) are the same as those given by Bruce,<sup>6</sup> the values of  $a$  and  $b$  for liquid-air temperature (100°K) differ slightly from those given by Jauncey and Bruce.<sup>1</sup> Since Jauncey and Bruce's value of  $a$  and  $b$  for 100°K were based on the diffuse scattering at one angle of scattering while our values are based on several angles of scattering as seen by the points shown in Fig. 1 we believe that our values of  $a$  and  $b$ —the values in Table III—are the more reliable.

### 3. DISPLACEMENTS DUE TO THERMAL VIBRATIONS

We feel that the experimental points shown in Fig. 1 fall reasonably well upon the theoretical curves shown in the same figure. Further, the experimental points fall pretty well on the theoretical curves out to a scattering angle of 90°. This causes us to feel that the values given in Tables I, II, and III are reasonably reliable. We should point out, however, that in calculating

TABLE II. Values of  $\Sigma E_r^2$  for zinc.

| $(\sin \frac{1}{2}\phi)/\lambda$ | $\Sigma E_r^2$ | $(\sin \frac{1}{2}\phi)/\lambda$ | $\Sigma E_r^2$ |
|----------------------------------|----------------|----------------------------------|----------------|
| 0.0                              | 30.0           | 0.8                              | 6.3            |
| 0.1                              | 24.8           | 0.9                              | 6.2            |
| 0.2                              | 19.0           | 1.0                              | 5.2            |
| 0.3                              | 14.7           | 1.3                              | 3.2            |
| 0.4                              | 12.5           | 1.6                              | 2.5            |
| 0.5                              | 10.4           | 1.9                              | 2.2            |
| 0.6                              | 9.4            | 2.2                              | 2.0            |
| 0.7                              | 7.3            |                                  |                |

the curves of  $(S\rho/\mu_1)_{th}$  for Fig. 1 we have not made correction for the Compton effect at large scattering angles such as 90°. Such a correction would lower each curve by about 0.008 in the units shown on the scale of ordinates of Fig. 1. However, the intensity of the diffuse scattering at  $\phi=90^\circ$  is small so that the error in the experimental points at this angle is somewhat large and of the same order as the correction for the Compton effect.

From the values of  $a$  and  $b$  can be calculated the root-mean-square displacements of the atoms in the zinc crystal. These displacements are shown in the fourth and fifth columns of Table III. Comparing the displacements with the axial distances,<sup>7</sup>  $a_0=2.657\text{A}$  and  $c_0=4.948\text{A}$ , we note that at  $T=298^\circ\text{K}$ , the thermal displacements in the  $a_0$  and  $c_0$  directions are each 3.5 percent of the corresponding axial distances. At  $T=100^\circ\text{K}$ , the ratio is reduced to 1.88 percent for the  $a_0$

TABLE III. Thermal displacements of zinc atoms.

| $T$   | $a$                | $b$                | $\psi=0^\circ$ $\langle u_\psi^2 \rangle_{Av}^{\frac{1}{2}}$ | $\psi=90^\circ$ |
|-------|--------------------|--------------------|--|-----------------|
| 100°K | 0.75A <sup>2</sup> | 0.20A <sup>2</sup> | 0.098A   | 0.050A          |
| 298   | 2.34               | 0.68               | 0.172  | 0.093           |

direction ( $\psi=90^\circ$ ) and 1.98 percent for the  $c_0$  direction ( $\psi=0^\circ$ ). At  $T=550^\circ\text{K}$ , Jauncey and Bruce<sup>1</sup> obtained  $a=6.20\text{A}$  from which the root-mean-square displacement is 0.280A or 5.65 percent of the  $c_0$  distance ( $\psi=0^\circ$ ). At the melting point (692°K) it does not seem likely that the ratio of displacement to axial distance will exceed 8 percent.

### 4. STRUCTURE FACTORS AT LARGE ANGLES

In an element like zinc ( $Z=30$ ) we believe that at  $(\sin \frac{1}{2}\phi)/\lambda=2.2 \text{A}^{-1}$  the contributions to the atomic structure factor come only from the  $K$  and  $L$  electrons. At this value of  $(\sin \frac{1}{2}\phi)/\lambda$ ,  $f=3.5$ , and  $\Sigma E_r^2=2.0$ . If as an approximation we assume that the contribution of each  $K$  electron to  $f$  is  $E_K$  and that of each  $L$  electron is  $E_L$ , we then have the two equations

$$2E_K+8E_L=3.5, \quad (4)$$

$$2E_K^2+8E_L^2=2.0. \quad (5)$$

Solving these we obtain  $E_K=0.905$  and  $E_L=0.211$ . For a hydrogen-like atom

$$E_K=1/(1+4\pi^2\gamma_K^2x^2)^2, \quad (6)$$

where  $x=(\sin \frac{1}{2}\phi)/\lambda$  and  $\gamma_K$  is a parameter. Putting  $E_K=0.905$  and  $x=2.2$  in (6), we obtain  $\gamma=0.0162\text{A}$ . The quantity  $\gamma_K$  corresponds to the radius of the  $K$  orbit in the Bohr model of the atom. For  $Z=30$ , the radius of the Bohr model is  $0.53/30=0.0177\text{A}$ .

<sup>7</sup> See R. W. G. Wyckoff, *The Structure of Crystals* (Chemical Catalog Co., 1931), second edition, p. 207.

A similar expression to (6) can be worked out for the  $L$  electrons. However, there are two types of  $L$  electrons, the  $2s$  and the  $2p$  electrons. The wave functions for the  $L$  electrons are given by quantum mechanics.<sup>8</sup> From these the  $E_r$  value for each kind of  $L$  electron may be calculated from the formula

$$E_r = \int_0^\infty u(r) \frac{\sin kr}{kr} dr, \quad (7)$$

where  $k = (4\pi \sin \frac{1}{2}\phi)/\lambda$  and  $u(r)dr$  is the probability of the electron being between spheres of radii  $r$  and  $r+dr$ . The formulas for the  $E_r$ 's so obtained contain a parameter  $\gamma$ . If it is assumed that  $\gamma$  is the same for each type of electron, the formulas may be added so as to give

$$f_L = 2E_{2,0} + 6E_{2,1} = 8(1-y)(1-y/2)(1+y)^{-4}, \quad (8)$$

where

$$y = (16\pi^2 \gamma_L^2 \sin^2 \frac{1}{2}\phi)/\lambda^2. \quad (9)$$

Now at  $(\sin \frac{1}{2}\phi)/\lambda = 2.2$ ,  $f_L = 8 \times 0.211 = 1.688$ . Solving (8) for  $y$ , we obtain  $y = 0.30$ . From (9) this gives  $\gamma_L = 0.020A$ . The radius of the  $L$  orbit in the Bohr model of a hydrogen-like atom

<sup>8</sup> See L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Co., 1935), pp. 134, 135.

( $Z=30$ ) is  $(4 \times 0.53)/30 = 0.071A$ . With a screening constant of about 6 this radius would be  $(4 \times 0.53)/(30-6) = 0.088A$ . The discrepancy between  $\gamma_L = 0.020A$  and 0.071 or 0.088A is too great. Putting it another way, if we make  $\gamma_L = 0.088A$  in (9) and then calculate from (8), we obtain  $f_L = 0.232$  so that  $f = f_K + f_L = 2.140$ . Our experimental value  $f = 3.5$  is thus considerably larger than what would seem to be a reasonable value of  $f$  on theoretical grounds. We have carefully repeated the diffuse scattering experiment at  $\phi = 90^\circ$  several times but we consistently obtain values of  $(S\rho/\mu)_{\text{exp}}$  which necessitate high values of  $f$ . Our results require that  $f$  decrease more slowly at high values of  $(\sin \frac{1}{2}\phi)/\lambda$  than would otherwise be expected. Either the  $K$  or the  $L$  electrons or both are on the average concentrated more closely to the nucleus than on theoretical grounds we had expected.

In Fig. 1 it is interesting to note that all curves and points approach each other closely at  $\phi = 90^\circ$ . At large angles the effect of the atomic vibrations becomes negligible and the zinc crystal scatters x-rays in the same way as gaseous zinc atoms would scatter the rays.

In conclusion we wish to thank Mr. J. E. Nafe for assistance in the calculation of the curves in Fig. 1.

### An Attempt to Observe the Absorption of Neutrinos

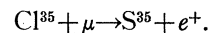
H. R. CRANE

*University of Michigan, Ann Arbor, Michigan*

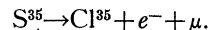
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IT HAS been quite conclusively demonstrated<sup>1</sup> that the presence of neutrinos cannot be detected by an ionization effect, of the kind which results from the passage of charged particles, neutrons or gamma-rays through matter. At least one possibility of detecting them remains, however, and that is by a process which is the reverse of the  $K$ -electron capture process. An

example of this is



The product  $\text{S}^{35}$  is a radioactive isotope (as would be true of the product in general), and decays back to  $\text{Cl}^{35}$  with the emission of a negative electron and a neutrino:



Adding the equations we see that the energy of

<sup>1</sup> M. E. Nahmias, Proc. Cambridge Phil. Soc. 31, 99 (1935).