The Energy Distribution in Field Emission*

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The energy distribution for field-current electrons from tungsten has been obtained. The distribution was obtained experimentally by the method of retarding potentials. It was possible to apply this method even at the high potentials necessary for field-current emission, since in this experiment the accelerating potential which liberated the electrons also served to decelerate them. This distribution exhibits three major features: (1) Only a minute portion of the electrons reach a copper collector unless the potential of the collector is positive by at least 4.5 volts with respect to the emitter; (2) the greatest number of electrons have energies very close to the maximum energy obtained; (3) the range of energies is at least 10 volts.

INTRODUCTION

THE emission of electrons from metals subjected to high surface fields was first discovered in a study by R. W. Wood of the radiation produced by discharges across small gaps.[1] The conclusion that the carriers of charge actually came from the metal itself was strengthened by later work on short spark discharges.[2–6] Some investigators supported the theory that field emission originated in the residual gas molecules between the electrodes.[7] It was not until the development of high vacuum technique that the laws governing field currents were formulated and its emission nature established.

Experiments by the General Electric Company of London[8] and by Millikan and his co-workers[9–11] established the first empirical field-current equation,

$$i = Ce^{-b/F}, \qquad (1)$$

where i is the current, F the strength of the surface field computed geometrically, and C and b constants for the particular surface under investigation. The values of C and b are subject to change by any process which might change the surface; such as heating, positive ion bombardment, etc. The observed variations in b led to the conclusion that the emission was from small localized areas or "points" where the field computed from the geometry of the apparatus is greatly magnified.

Classical methods alone have produced no satisfactory theory of field-current emission, but wave-mechanical methods give a justification for the empirical formula based on assumptions concerning the changes of the surface potential barrier with an accelerating field.[12–17] In this work of Oppenheimer, Fowler and Nordheim, Houston, and others the electrons within the metal are assumed to be incident upon a potential barrier at the surface of the metal. With zero accelerating field the barrier is infinitely thick; the electrons cannot penetrate the barrier and escape but must have enough energy to surmount the barrier. With an accelerating field the barrier becomes thinner and there is a finite probability of an electron escaping by tunneling the barrier. Field emission occurs when an appreciable

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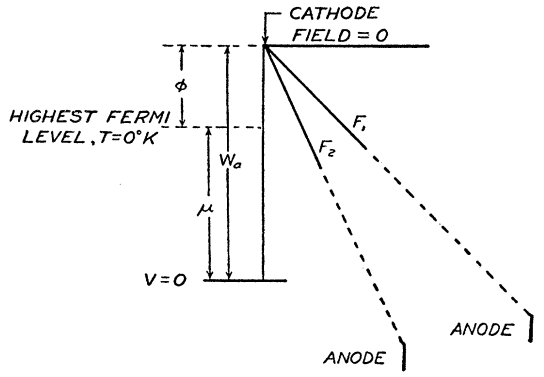

FIG. 1. Schematic potential barrier at the surface of a metal for field = 0; F_1; F_2; $(F_2 > F_1)$.

number escape and requires an accelerating field at the surface of the emitter which computed from the geometry of the electrodes is of the order of magnitude of 10^6 volts/cm. Because of surface irregularities the actual field where the emission occurs may be many times this value, possibly between 10^7 and 10^8 volts/cm.

Figure 1 shows the type of barrier postulated and the change in the nature of the barrier with accelerating field. The computation of the transmission coefficient for such a barrier with changing accelerating field gives rise to an equation for field currents which has the same form as the empirical Eq. (1) above. The evaluation of the actual current density necessitates the assumption of the number of electrons incident upon the barrier per second and their distribution in energy. Fowler and Nordheim, assuming a Fermi-Dirac energy distribution arrive at the following equation for the current density:

$$I = 6.2\times10^{-6}\mu^{\frac{1}{2}}F^2(\phi+\mu)^{-1}\phi^{-\frac{1}{2}} \times \exp\left[-6.9\times10^7\phi^{\frac{3}{2}}/F\right]. \quad (2)$$

Here μ is the usual parameter of electron distribution in the Fermi-Dirac statistics, ϕ the thermionic work function, and F the accelerating field in volts per cm. Although the equation is of the same algebraic form as that found experimentally its quantitative confirmation is difficult because of two facts. The actual value of F is indeterminable since the current is known to arise from small isolated regions where the field may be much higher than that given by geometrical considerations, and furthermore the areas involved in the emission are unknown.

General confirmation of the idea of transmission through potential barriers has been demonstrated by Henderson and Badgley[18] who showed that electrons emitted from platinum are not received by a copper collector unless the copper is higher in potential than the platinum by at least an amount equal to the work function of copper. The method used by Henderson and Badgley likewise permits the determination of the energy distribution of the emitted electrons. The present paper is concerned with this determination.

THEORY OF THE METHOD

The schematic diagram of the apparatus shown in Fig. 2 illustrates the method used. A fine wire F is stretched coaxially with the two hollow cylinders C_1 and C_2. Cylinder C_1 is perforated and functions as a grid. The potential V_1 produces field emission from the filament to C_1 where a portion of the electrons penetrate into the region between C_1 and C_2. C_2 is connected to the filament F through a small potential V_2, which tends to accelerate the electrons. Electrons passing through C_1 to C_2 are thus decelerated by a potential $V_1 - V_2$ where V_1 is the same potential that liberated them. They will reach C_2 and be collected provided V_2 is at least equal to the height of the potential barrier at the surface of the collector as the electron must pass over, but not through, this barrier to enter the copper (see Fig. 11). It was

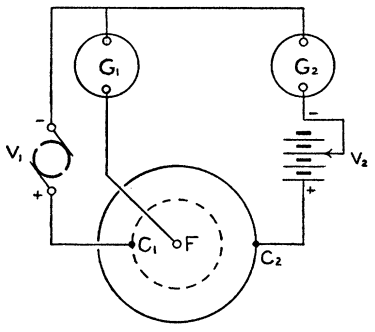

FIG. 2. Schematic diagram of apparatus. Electrons are emitted from the fine wire F by the action of the intense electrical field produced by the potential V_1 between F and C_1. Electrons passing through the grid C_1 only reach C_2 provided V_2 is at least equal to the work function of the copper collector C_2.

[18] R. E. Badgley, Thesis, University of Washington (1931); J. E. Henderson and R. E. Badgley, Phys. Rev. **38**, 590(A) (1931).

the minimum value of V_2 which permitted electrons to arrive at C_2 that was measured by Henderson and Badgley and found to agree approximately with the thermionic and photoelectric work functions of the metal used as collector.

If a distribution in energy exists for the emitted electrons, the current to C_2 will vary when V_2 is changed since different groups of electrons which have penetrated the potential barrier of the emitter will then be collected. Analysis of the current to C_2 as a function of V_2 will yield this distribution. By plotting the current against V_2 a curve is obtained, the slope of which represents the relative number of the emitted electrons which had been associated with the various energy levels of the emitter. This slope plotted against corresponding values of V_2 gives the distribution in energy among the emitted electrons.

The essential feature which makes the method experimentally feasible is the use of the large accelerating potential V_1 as a part of the decelerating potential $V_1 - V_2$. Consequently the resultant potential between the filament and collector is independent of V_1 and the distribution in energy of the emitted electrons is obtained by varying only the small potentials V_2 which can be accurately measured. In practice C_2 is made up of two cylinders usually connected together. Disconnected, however, there are two grids between the filament and the plate; and the second grid may be used in determining the effects of secondary electron emission.

Apparatus and Procedure

Figure 3 shows a cross section of the apparatus. The cylinders are mounted coaxially with the fine wire from which the electrons are emitted. They are made of one-mm copper and are supported by two 50-mil tungsten rods which also serve as electrical connections. The two inner cylinders, of inside diameters 1.2 and 2.4 cm, respectively, are seven cm long. The central four centimeters of this length are perforated by 60 holes of one-mm diameter per cm^2. The outer cylinder is eight cm long and has a diameter of 3.6 cm. It is unperforated; the central two cm is two mm thick and the rest one mm, to insure good heat conduction during outgassing. The

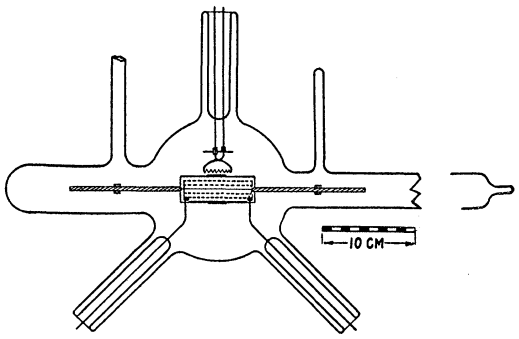

FIG. 3. Sectional diagram of the apparatus.

apparatus was outgassed by electron bombardment from an auxiliary filament mounted about three mm from the thick portion of the outer cylinder. The inner cylinders were heated by radiation from the outer one. The copper disk mounted directly behind the auxiliary filament protects the lead-in wires from ion bombardment.

The filament is mounted between two copper rods centered with respect to the cylinders. In mounting the filament the assembly is first fastened firmly to a slender steel rod which is then inserted in the tube. The steel rod is removed after the copper rods have been fastened securely to the filament supports. The filament itself is of tungsten 0.0013 cm in diameter and 7.5 cm long.

The apparatus was mounted within a Pyrex glass envelope constructed from a two-liter flask. The whole was evacuated by a two-stage mercury diffusion pump used with liquid air. The vacuum was measured by an ionization gauge calibrated against mercury vapor at room temperature. During operation the tube is evacuated to a pressure of approximately 2×10^{-8} mm Hg. This order of vacuum is essential to work of this character if reproducible results are to be obtained. It was only attained after repeated bakings of the apparatus as a whole at about 500°C. After each baking the copper parts were maintained at a bright cherry red for several hours by electron bombardment. Under these vacuum conditions it is possible to maintain the emission current constant to within one percent for several hours at a time. Inconstancy of emission at higher pressures is probably due to bombardment of the surface by positive ions. In particular this would be true if the emission

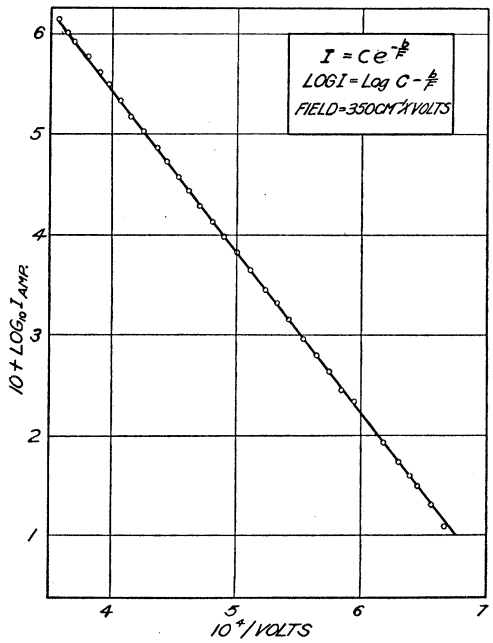

FIG. 4. Showing the straight line relationship between logarithm of the currents and $1/F$ in field current emission. F is the field as computed from the geometry of the apparatus.

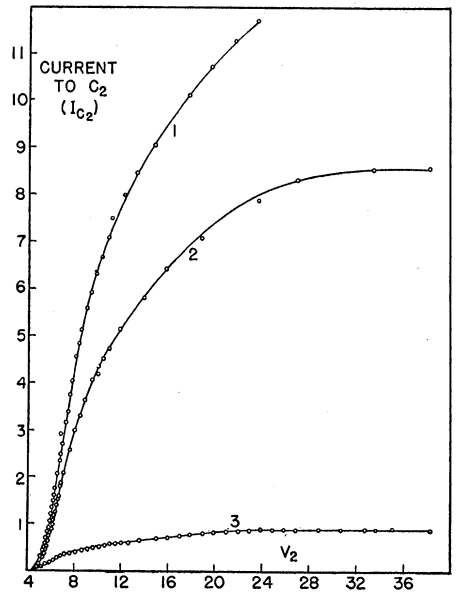

FIG. 5. The current to C_2 as a function of V_2. V_2 is the difference in potential between C_2 and the filament. Curves 1, 2, 3 are for field emission I_1, respectively equal to 9.6×10^{-6} ampere, 7.2×10^{-6} ampere and 7.2×10^{-7} ampere.

occurs from small regions so that the damage caused by one positive ion would represent a large portion of the emitting area.

The dimensions of the apparatus are such that there is a potential gradient of approximately $350 V_1$ volts per cm at the surface of the filament. This field is given by the equation

$$F = V/b \log (b/a),$$

where V is the potential difference between the cylindrical filament of radius b and the surrounding cylinder of radius a. The actual field at the small areas from which the emission comes is doubtless much larger than that computed from the geometry of the apparatus.

The high potential represented by V_1 in Fig. 2 was supplied by a 3000-volt d.c. generator driven by a d.c. motor operating on storage batteries. This furnished a very steady source of potential as soon as the generator was in thermal equilibrium. During the later stages of the work an especially constructed 9000-volt generator similarly driven was used. Voltages up to 3000 v were measured with a Model 1 Weston voltmeter with multiplier. Above this voltage a rotary voltmeter[19] was used, which was found to be a very simple and trustworthy instrument for these measurements. The emission current, which ranged from 10^{-10} to 10^{-4} ampere, was measured by a Leeds & Northrup type R galvanometer, G_1. The variable potential V_2, between C_2 and the filament, is supplied by a battery of storage cells connected across a potentiometer. A voltmeter which could be read to 0.01 volt was used to measure this voltage. The current I_2 between C_1 and C_2 was measured by a type H. S., Leeds & Northrup galvanometer, G_2, of sensitivity of 3.0×10^{-11} amp./mm.

RESULTS

In Fig. 4 is shown a typical curve illustrating the characteristics of the emission current I_1 between the filament and C_1. By plotting the logarithm of the current against the reciprocal of the field a straight line is obtained, which verifies the experimental relationship obtained earlier by Millikan and Lauritsen.[11] This linear relationship can be regarded as the test for field currents. Curves exhibiting breaks may be ob-

[19] P. Kirkpatrick and I. Miyake, Rev. Sci. Inst. **3**, 1 (1932).

tained when the filament conditions are subject to change. In the following work only those filaments which exhibited unchanging characteristics are considered.

The field currents obtained were steadier than reported by previous investigators. This is probably due to two causes. The first is the potential source. As the emission varies exponentially with the field this is extremely important. The d.c. motor generator operated on storage batteries was much superior to any rectified a.c. sources of potential that were tried. The second is the degree of evacuation of the tube which is equally as important as the potential source. The variations in current were less than one percent over a period of several hours when the vacuum was approximately 2×10^{-8} mm Hg. With vacuum of 10^{-7} mm Hg it was found impossible to obtain currents free from variations as great as 10 percent. Even with the best vacuum obtainable the existing currents are not always steady. The initial emission from a new filament generally exists at abnormally low values of the field (2 to 3×10^{5} volts/cm). These currents are ordinarily unsteady and only become steady after large emissions have been drawn from the filament. In the process of increasing the field to increase the emission the current sometimes decreases suddenly by a factor as great as 10. The smaller currents are then generally quite steady. If the emission is from a localized area this sudden decrease can be interpreted as a breakdown of such a point resulting in decreased field and smaller and more stable currents.

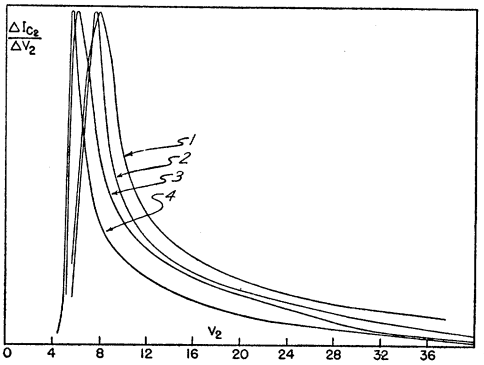

FIG. 6. Curves obtained from those in Fig. 5 by plotting the slope against V_2. Curve 4 is for a value of I_1 too small to be shown satisfactorily in Fig. 5. The slopes are arbitrarily adjusted to a common maximum. They are distribution curves representing the relative number of the emitted electrons which had been associated with the various energy levels of the emitter.

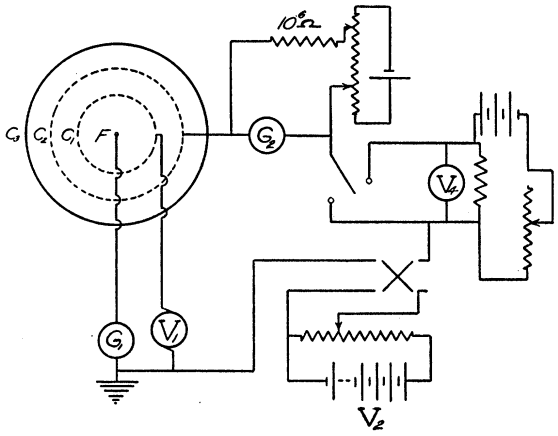

FIG. 7. Diagram of apparatus which permits the distribution curves to be obtained directly.

With each different point, or possibly distribution of points, the characteristics of curves of the type shown in Fig. 4 vary. This takes the form of a different slope of the curve. The variation may be interpreted as due to a change in b in the Eq. (1), or to a different proportionality between the applied potential and F or to a combination of these.

Figure 5 shows a typical group of curves obtained by plotting the current I_2, between C_1 and C_2, against the small potential V_2. It will be noted that independent of the magnitude of the current I_1, emitted by the filament, no electrons arrived at C_2 until V_2 reached a value of about 4.5 volts. This is very close to the accepted work function of copper. As V_2 was increased beyond 4.5 volts the current approached saturation and indicated that the probability of an electron penetrating the potential barrier decreases rapidly for electrons in the lower energy levels of the emitter.

The slope of these curves plotted against V_2 gives the energy distribution of the emitted electrons. The curves 1, 2 and 3, shown in Fig. 6, were obtained from the corresponding curves in Fig. 5 by mechanically measuring the slopes. They were arbitrarily adjusted to a common maximum. Curve 4 in Fig. 6 corresponds to a value for I_1 of 4.8×10^{-8} ampere, which is too small to be shown on Fig. 5. This mechanical differentiation proved to be a laborious and somewhat inaccurate method. A better and more

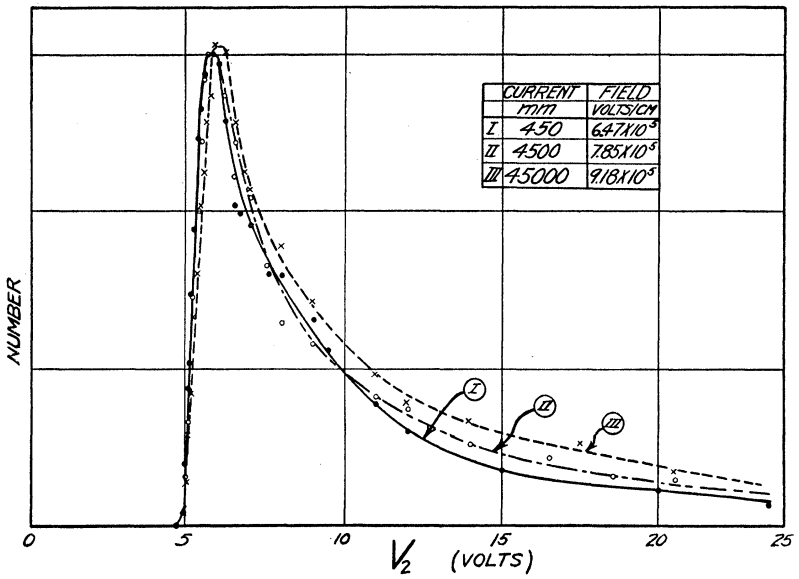

FIG. 8. Typical group of distribution curves obtained from the same filament for different values of the field. The curves are all adjusted to the same maximum.

accurate method of obtaining these distribution curves directly is illustrated in Fig. 7. Field currents were produced as before. The deflection of the galvanometer G_2 was then reduced to zero by adjustment of the auxiliary circuit connected to G_2. When the circuit is just balanced for a particular value of I_2 any change in I_2 will produce a corresponding deflection in G_2. The introduction into the circuit of V_4 which was kept constant and equal to 0.1 volt effectively increases V_2 by 0.1 volt and produces an accompanying increase in I_2. By plotting the deflection of G_2 against V_2 one obtains the distribution curves directly since this deflection represents the average change in I_2 over a 0.1-volt interval at V_2.

Figure 8 shows a typical set of distribution curves taken by this method for different magnitudes of the field. The curves rise sharply at 4.5 volts and rapidly reach a maximum after which they decrease more slowly. In this region the curves for larger emission currents are the higher. The range of this distribution which extends, in the curves shown, approximately 20 volts is of interest. The exact range is indeterminate since the curves in the region of large values of V_2 are not particularly significant. This is due to two causes to be discussed later, namely: (1) the existence of secondary electrons emitted at the last cylinder C_2; and (2) a distortion of the true distribution due to the geometry of the apparatus. Since tungsten was used as the emitter and copper as the collector the contact potential existing between them would be expected to be very small. Thermionic measurements showed it to be less than 0.2 volt.

Of more than 30 separate distribution curves obtained at room temperature all show a value of V_2 equal to 4.5 volts before an appreciable number of electrons are collected. There is no doubt but that the curves would extend below 4.5 volts as they approach the axis asymptotically. However, it appears that an extremely small portion of the electrons in a metal at room temperature are in the higher energy levels. The maxima in the distribution curves appear at somewhat different values of V_2. Twenty-four curves show the most probable energy to be between 5.5 and 6.0 volts. Seven showed a maximum between 6 and 6.5 volts, and on four the maximum occurred at 7.5 volts. The thirty-five curves studied represent fourteen different filament conditions. The appearance of the maximum at different values of V_2 is quite definite but the factors which determine its position are not. The variation in the position of the maximum is probably due to experimental conditions not controlled, though in general the

presence of the maximum at higher V_2 was associated with higher applied potentials.

Although the distribution curves indicate that field current electrons have a range of energy greater than thirty volts, secondary electron effects arising both at C_1 and C_2 distort the distribution in the region of lower energies (large V_2) and make that region unimportant except for purposes of comparison. The effect of the reflection at C_1 is negligible since electrons losing more than twenty volts energy would never reach C_2 with significant values of V_2. As twenty volts is less than two percent of the energy of the incident electrons, the number reflected with an energy loss smaller than this is insignificant.[20–22] Secondary electrons emitted at C_2 return to C_1 and cause an effective decrease in the electron current between C_1 and C_2. This slight decrease is shown on Fig. 5 for V_2 greater than 35 volts and it also produces the negative slope shown in Fig. 6.

If V_2 is fixed and C_2 used as a grid, the energy distribution may be determined by applying a retarding potential between C_2 and C_3 (Fig. 7) and analyzing this current as before except that it is a function of V_3, the potential between C_2 and C_3. Curve II of Fig. 9 is the result of such an analysis. In this case it is apparent from the figure that the secondary electrons emitted at C_2 go to C_3 and produce the increase for low energies or small V_3. The decrease from secondaries emitted at C_3 is also recognizable so there is no doubt that for the higher values of V_2 the curve is distorted.

A comparison of distribution curves obtained by the two different methods is shown in Fig. 10 where curve I was taken without C_3 being used. Since the secondary electron effects should be opposite for this curve the agreement shows that for V_2 less than ten volts secondary electron effects are negligible. Curve III was obtained with a new point and agrees with curve I only at higher energies. It is safe to conclude that these data are independent of secondary electron effects for V_2 less than 10 volts and that the energy of the field electrons extends over at least ten volts.

Curves I and II of Fig. 9 exhibit the effect of secondary electrons at different parts in the distribution curve. They show that the range of energies affected is only about six volts. This is further verification of the previous conclusion that the range of energies is at least 10 volts since that part of the curve is unaffected by secondary electrons and is therefore reliable.

Since the apparatus is cylindrical, if a general emission occurs electrons can escape through the ends of the cylinder and not be measured at all. Likewise velocity components along the axis of the cylinder would not be measured by the retarding potential gradient perpendicular to the filament. If the emission is from points

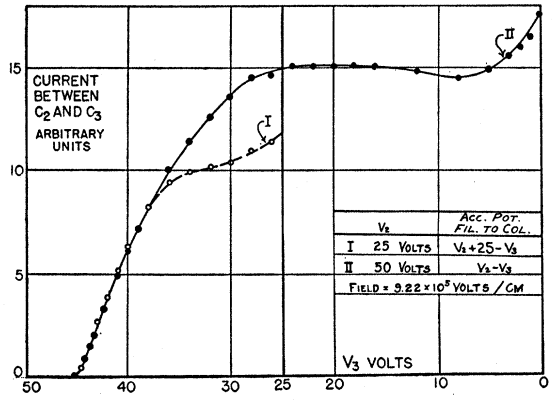

FIG. 9. Current voltage curves between C_2 and C_3 for constant values of V_2. V_3 is the retarding potential between C_2 and C_3. A range of energies extending over about 10 volts is free from secondary electron effects.

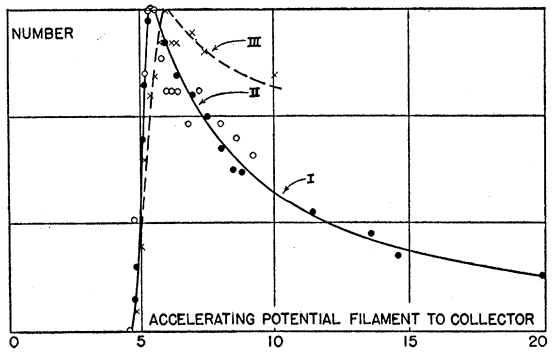

FIG. 10. Distribution curves I and II obtained by the two methods for the same "point." Note that they agree over the energy range investigated. Curve III was obtained with a new "point."

		Field $\frac{\text{volts}}{\text{cm}}$	I—mm	V_2 volts	V_3 volts	Acc. Pot. Fil. to Col.
I	●	$6.9 \cdot 10^5$	400	varied	0	V_2
II	○	$7.7 \cdot 10^5$	400	50.1	varied	$50 - V_3$
III	×	$9.2 \cdot 10^5$	4000	2.5	varied	$25 - V_3$

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[21] A. Becker, Ann. d. Physik **78**, 228 (1925).
[22] K. H. Stehberger, Ann. d. Physik **86**, 825 (1928).

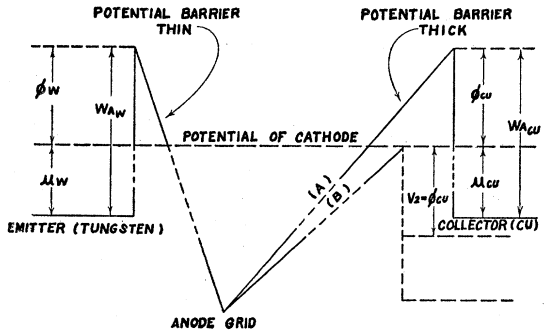

FIG. 11. Schematic representation of the surface potential barriers involved. The potential of the copper collector is lowered by an amount $V_2=\phi_{Cu}$ in order that electrons from the highest Fermi level ($T=0°$K) may surmount the barrier.

and in the form of definitely directed beams towards the grid, the energy distribution would be very close to that for total energies. The steep portions of the curve near small V_2 would be unaffected except for certain electrons with large velocity components along the filament. These higher energy electrons would be recorded as electrons of lower energy making the curve too high for large values of V_2. This is precisely the effect observed in some preliminary results on the total energy distribution reported by Dahlstrom, McKenzie and Henderson[23] where a point emitter at the center of a sphere was used. The results obtained from the cylindrical apparatus used in this experiment may be interpreted as measuring the total energy distribution of the emitted electrons only in the region of small V_2.

In view of these facts, the following are listed as the most important experimental conclusions of this investigation for field currents emitted from tungsten and collected on copper.

(1) Electrons are collected by the copper plate only when its potential is at least 4.5 volts higher than that of the emitter.

(2) Some electrons of lower energies penetrate the potential barrier of the emitter and give rise to an energy distribution for field current electrons which has the following characteristics: (a) A range of energies of at least 10 volts. (b) A most probable energy for the emitted electrons very close to the maximum observed energy. (c) A small variation of the position of this maximum depending upon filament conditions and upon the field strength.

DISCUSSION OF RESULTS

In Fig. 11 is shown schematically the potential barriers involved in this experiment. Because of the geometry of the apparatus the barrier at the surface of the tungsten emitter is thin compared to the barrier at the surface of the copper collector. Electrons which may tunnel through the barrier at the surface of the tungsten cannot enter the copper unless they pass over the barrier at its surface because of the thickness. It is immediately evident that the potential of the copper must be lowered by an amount equal to its work function in order that electrons may enter. This fact is demonstrated by the curves on Fig. 5 which shows clearly that no appreciable numbers of electrons reach the copper unless it is positive with respect to the tungsten by approximately 4.5 volts. This is close to the accepted value for the work functions of copper and tungsten. However, by using a platinum emitter and copper collector Henderson and Badgley[24] obtain, to within their experimental error, this same value which, because of the wide differences between the work functions of platinum and

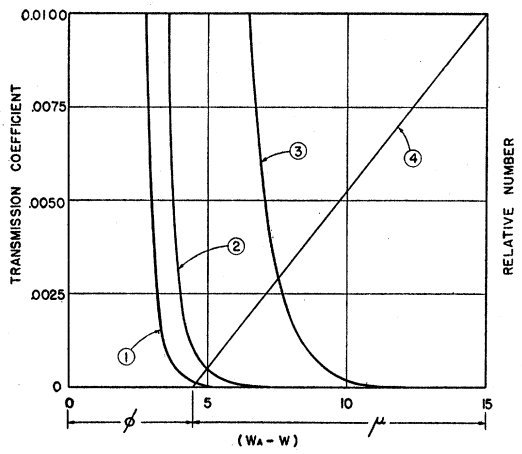

FIG. 12. Curve 4 represents the relative number of electrons with energies W normal to the surface in accord with the Fermi statistics. These are plotted as a function of (W_a-W). ϕ is assumed as 4.5 volts and μ, 10 volts. Curves 1, 2 and 3 are computed values of the transmission coefficient for fields of 7×10^7 volts/cm, 10^8 volts/cm, and 2.5×10^8 volts/cm, respectively.

[23] R. K. Dahlstrom, K. V. McKenzie and J. E. Henderson, Phys. Rev. **48**, 484(A) (1935).

[24] J. E. Henderson and R. E. Badgley, reference 18.

copper, demonstrates clearly that it is the work function of the collector rather than the emitter that determines the minimum value of V_2 that will permit the electrons to reach the collector. This verifies the general application of the idea of potential barriers to field emission.

As V_2 is increased beyond this minimum value, electrons which have tunneled through the barrier at energies below the highest Fermi level ($T=0°$) will be collected and will give rise to the distribution in energy observed. Eventually with further increase in V_2 a value will be obtained where either the lowest energy is reached or else electrons are unable to penetrate the barrier. The current to the collector will then attain a limiting or saturation value. That this is true is shown by Fig. 5. Increase of the applied field apparently increases the probability of transmission through the barrier as distribution curves taken at higher fields when no detectable change in the emitter has occurred are invariably above those taken at lower values in the region of large V_2.

A quantitative comparison of these results with the Fowler-Nordheim theory[12] is of interest. The transmission coefficient as obtained by them is:

$$D(W)=(4/W_a)W^{\frac{1}{2}} \times(W_a-W)^{\frac{1}{2}}\exp\left[-4K(W_a-W)^{\frac{3}{2}}/3F\right], \quad (W_a>W); \quad (3)$$

where W is the kinetic energy normal to the surface, W_a the total height of the barrier, F the applied field, and $K^2=8\pi^2m/h^2$ where h and m have their usual significance. Curve 4 in Fig. 12 represents the relative number of electrons with energies normal to the surface in accord with the distribution function at 0°K predicted by the Fermi statistics. The transmission coefficient for successively increasing values of the field are shown in curves 1, 2 and 3. These curves assume $\phi=4.5$ volts and $\mu=10$ volts, which values are reasonable in view of the nature of the emitter and the experimental results. The resultant curves for the distribution in normal energies of the emitted electrons are shown as curves 1, 2 and 3 in Fig. 13. For ease of comparison, these distributions are given with the ordinates adjusted to a common maximum. It is evident that these are of the same form as the typical experimental curves as given in Fig. 8. As has already been pointed out, the excess of slow electrons exhibited by the experimental curves can be accounted for by secondary emission, and because the cylindrical geometry of the apparatus does not permit the normal energy of the electrons to be measured directly. Nevertheless the general form of the curves is so nearly that predicted that the experiments give strong direct evidence for the Fowler-Nordheim-Sommerfeld theory of field emission. Certainly the experiments constitute further convincing evidence that the electrons leaving the metal come through and not over the surface barrier.

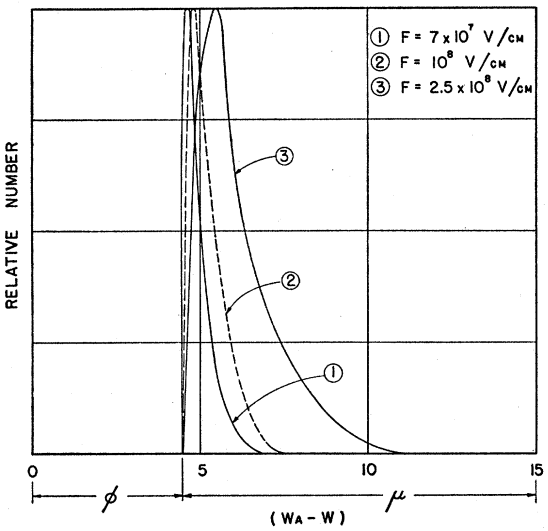

FIG. 13. Theoretical distribution of normal energies of electrons in field emission according to the Fowler-Nordheim-Sommerfeld theory. These are obtained from Fig. 12 by multiplying corresponding ordinates of curve 4 by those of curves 1, 2 and 3. The general form of these curves corresponds well with the typical experimental curves as given in Fig. 8. As in Fig. 8, the curves are adjusted to a common maximum ordinate.

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