Electrodisintegration of Beryllium

We have succeeded in disintegrating beryllium with electrons having energies in excess of the photoelectric threshold. It is assumed (cf. preceding letter), as is the case with the photodisintegration of beryllium, that the products of disintegration are Be^8 and a neutron. The effect was detected by means of the activity which these neutrons produced in silver or rhodium. The activity was observed with a G-M counter system.

A sheet of beryllium 0.04 cm in thickness (in the vacuum system) was surrounded with a silver or rhodium foil and the whole encased in paraffin. When bombarded with 10 μa of 1.72-Mev electrons from an electrostatic generator for one minute the silver or rhodium gave about 90 net counts in the first minute. Both silver and rhodium decayed with the appropriate periods, 22 sec. and 44 sec., respectively. Assuming that, with the geometry used, 900 neutrons per second produce one count per minute from a silver detector (determined by using a known γ-ray source of radon and a cross section for photodisintegration of 3×10^{-28} cm^2) the cross section for the electrodisintegration process at this voltage is about 2×10^{-31} cm^2 in good agreement with the theoretical value predicted by Guth (cf. preceding letter). The threshold for this effect was established at about 1.65 Mev.

The possibility that the silver and rhodium activity was produced by photoneutrons resulting from stray x-rays, or x-rays produced in the beryllium, was eliminated; (1) by introducing sheets of graphite in front of the beryllium which reduced the activity essentially to zero; (2) by increasing the thickness of the beryllium target. Since the electrodisintegration should not increase for thicknesses of beryllium greater than 0.04 cm, while the photodisintegration is proportional to the thickness, it was possible to show that the activity produced in the thin target was essentially all due to electrons.

We wish to acknowledge our indebtedness to Dr. E. Guth who suggested this problem and who has provided many helpful suggestions.

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The Advantages of Scaling Circuits in Recording Random Counts

There are two distinct functions performed by a scaling circuit when used in electrical counting systems for nuclear investigations. The first function is that of reducing the input rate to the mechanical recorder. The second is that of changing the random distribution of pulses into one in which the time intervals between pulses are more nearly equal, before feeding these pulses to the mechanical recorder. The great importance of this second function for the reduction of the counting losses *in the mechanical recorder* does not yet seem to be generally recognized, although the matter has already been treated by others.[1]

The counting losses in a counting system, such as a Geiger-Müller system, arise from two causes; the finite resolving time of the components of the system and the random distribution of counts encountered with radioactive sources. The counting losses are determined by a function which is extremely sensitive to the distribution of counts and much less sensitive to the resolving times. The losses are rather surprisingly large in nuclear work mainly because one is dealing with random distributions of counts. For this reason, a method for the reduction of counting losses which depends on removing the random element in the distribution will be much more effective than a method which depends on reducing the resolving time of the mechanical recorder. It is this property which makes the scaling circuit method so powerful and elegant. A scale-of-n circuit changes a random distribution more and more nearly into a periodic one as the scaling ratio, n, increases. The marked periodicity of the counts is easily noted experimentally. If the scaling ratio is great enough, the recorder can be used at counting rates almost equal to its maximum counting rate for equally spaced pulses without the occurrence of appreciable counting losses. Experiment shows that a vacuum tube scale-of-eight is sufficient for counting rates up to about 20,000 counts a minute or more if the recorder is a Cenco counter.[2] Also it is found that the maximum counting rate which may be recorded with inappreciable counting losses in the mechanical recorder (say, one percent) increases by a factor of three hundred or more when changing from a scale-of-one to a scale-of-eight.[2] This is in general agreement with the theory of Alaoglu and Smith.[1]

A recent proposal[3] for the reduction of counting losses, in which the method of attack is that of decreasing the resolving time of the mechanical recorder in a scale-of-one circuit, is thus seen to be incapable of yielding an appreciable increase in the counting rate which may be recorded with negligible counting losses. Such high speed mechanical recorders would, of course, be very useful *in connection with* a scaling circuit, but cannot *replace* such circuits.

Some numerical results for a typical case, obtained by means of the theory of Alaoglu and Smith,[1] will give a quantitative idea of the magnitude of the scaling effect. This theory has been accurately verified experimentally for the case of a scale-of-one[2] and additional results in general agreement with the theory obtained by comparing a scale-of-eight against a scale-of-sixteen.[2]

Let the resolving time of the G–M tube and quenching circuit be $\sigma=5\times10^{-4}$ sec. Let us also consider two mechanical recorders, a fast one with resolving time $\tau_1=10^{-3}$ sec. and a slower one with resolving time $\tau_2=10^{-2}$ sec. These values are typical ones. Calculation shows that a vacuum tube *scale-of-two* circuit using the slower recorder will be just as accurate as a scale-of-one circuit using the faster recorder (faster by a factor ten) up to input rates to the system of over 1000 counts a minute. At 2000 counts a minute the reading of the scale-of-two will only be about two and one-half percent less than that of the scale-of-one. For a scale-of-eight feeding the slower recorder, there will be inappreciable losses *in the recorder* up to 30,000 counts a minute. The corresponding figures for scales-of-one are 60 and 1200 counts a minute for the slow and the fast recorders, respectively.

The writers wish to take this opportunity to correct a misprint in reference (2) below. In that reference, Eq. (10) reads $1/n_{1\,\max}$. This should be corrected to read $1/(n_{1\,\max}\times e)$.

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[1] L. Alaoglu and N. M. Smith, Phys. Rev. **53**, 832 (1938).
[2] H. Lifschutz and O. S. Duffendack, Phys. Rev. **54**, 714 (1938).
[3] H. V. Neher, Rev. Sci. Inst. **10**, 29 (1939).

On the Equilibrium of Massive Spheres

It is usually stated[1] that general relativity sets an upper limit to the mass and radius of a sphere of constant proper energy density ρ. This result is obtained by considering only those solutions of Einstein's field equations which give a finite central proper pressure p; the minimum mass and radius for which p first becomes infinite at the center are taken as the limiting values. In his original paper[1] Schwarzschild points out the existence of other solutions with infinite central p, but dismisses them as physically inadmissible because of this singularity without a further discussion of their properties. However, an examination of these solutions, which are described below, shows that they lead to arbitrarily large masses and radii.

On the other hand a cold neutron gas model[2] leads to an upper limit on the size of a static sphere. It is of interest to try to account for the difference in behavior of these two models. If for a material consisting of particles in motion (and which may exert forces on each other) the energy-momentum tensors for the particles, and for the force fields (apart from gravitation) associated with them, are additive and have non-negative traces, then for such a material $T=\rho-3p\geqq 0$ must always hold. The $\rho=$const. model for sufficiently high pressures has $T<0$. This not altogether consistent model corresponds to the case of perfectly incompressible particles packed tightly together and treated essentially nonrelativistically in that the contribution of the forces to ρ is not taken into account. A negative T near the center of the sphere, such as makes possible an arbitrarily large mass for the $\rho=$const. model, may be regarded as analogous to a negative (repulsive) mass which keeps the sphere from collapsing.

Both the original Schwarzschild solutions, and the new singular solutions leading to arbitrarily large spheres, may be obtained by a method considerably simpler than the one used by Schwarzschild. Einstein's equations for constant ρ and for the line-element

$$ds^2=-e^{\lambda(r)}dr^2-r^2d\theta^2-r^2\sin^2\theta d\phi^2+e^{\nu(r)}dt^2$$

reduce to:[2]

$$e^{\nu(r)}=\frac{(1-2u_b/r_b)\rho^2}{[p(r)+\rho]^2} \tag{1}$$

$$dU/dx=3x^2 \tag{2}$$

and

$$\frac{dP}{dx}=-\frac{P+3}{2x(x-U)}[Px^3+U], \tag{3}$$

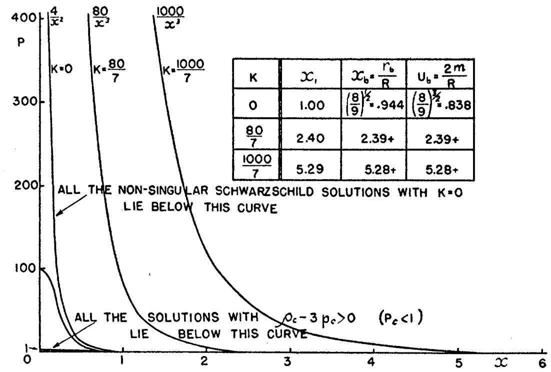

FIG. 1.

where x, U, P, are dimensionless quantities defined by $x=r/R$, $U=2u/R$, $P=8\pi R^2p$; $u\equiv\frac{1}{2}r(1-e^{-\lambda})$, R is a characteristic length determined in terms of the given constant ρ by $R^2=3/8\pi\rho$; and relativistic units, making $c=1$, $G=1$, are used throughout. The radius r_b of the sphere is determined by that value x_b of x where P first vanishes, and the gravitational mass of the sphere is given by $m=u_b$, the value of u at $x=x_b$. The integral of (2) above is $U=x^3-K$, where $K\geqq 0$ in order that $e^{-\lambda}=[1-r^2/R^2+KR/r]$ should nowhere change sign. The usual procedure[1] corresponds to making $K=0$, and then Eq. (3) may be at once explicitly integrated, and gives the well known Schwarzschild interior solution:

$$P=\frac{3(P_c+1)(1-x^2)^{\frac{1}{2}}-(P_c+3)}{(P_c+3)-(P_c+1)(1-x^2)^{\frac{1}{2}}},$$

where P_c is the value of P at $x=0$. If $P_c\rightarrow\infty$, which is commonly taken as the limiting solution, then as $x\rightarrow 0$, $P\sim 4/x^2$, and the corresponding radius and mass are fixed by $x_b=(8/9)^{\frac{1}{2}}$, and $U_b=(8/9)^{\frac{3}{2}}$. This solution makes $e^{\nu}\rightarrow 0$ and $e^{\lambda}\rightarrow 1$ as $r\rightarrow 0$.

The new singular solutions are obtained by making $K>0$. Eq. (3) now can not be explicitly integrated in terms of known functions, but it may be shown that solutions exist which near the origin behave like $P\sim 7K/x^3$. Numerical integration of (3) for several values of K shows that x_b is very closely given by the largest positive root x_1 of $x-U=x-x^3+K=0$. Thus $x_b\sim x_1$, $U_b=x_b^3-K\sim x_1^3-K=x_1$. For very large K, $x_1\sim K^{\frac{1}{3}}$, so that $x_b\sim K^{\frac{1}{3}}$, $U_b\sim K^{\frac{1}{3}}$, and it is seen that no upper limit on the size of the sphere exists, if the singular solutions for the pressure are not excluded. For these solutions both $e^{\lambda}\rightarrow 0$ and $e^{\nu}\rightarrow 0$ as $r\rightarrow 0$. Fig. 1 gives a general idea of the various solutions, including the Schwarzschild limiting solution and the last one for which $\rho_c-3p_c\geqq 0$ ($P_c\leqq 1$) holds.

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[1] K. Schwarzschild, Berl. Ber. (1916), p. 424; A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge, 1924), p. 168; R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford, 1934), pp. 246–247, and others.
[2] J. R. Oppenheimer and G. M. Volkoff, this issue, Phys. Rev. **55**, 374 (1939). Eqs. (1), (2) and (3) are Eqs. (7), (9) and (10) of that article rewritten for constant ρ.