The Widths of Nuclear Energy Levels†

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Neutron absorption measurements with Rh both as absorber and detector have been performed, in order to obtain information about the levels of the compound nucleus responsible for the 44″ period of Rh. The absorption curves have been analyzed by assuming a single sharp resonance and taking into consideration the angular distribution of the neutrons emerging from the source and the absorption of the neutrons and the electrons in the detector. This analysis yields the values: (a) 6100×10^{-24} cm² for the capture cross section at resonance; (b) 0.17 ev for the total width; (c) 1.6×10^{-3} ev for the neutron width.

THE selective capture of slow neutrons has been explained by a resonance process in which the neutron is captured in a virtual state of the compound system formed by the capturing nucleus and the incident neutron. According to this explanation, the cross section for the radiative capture of a neutron with kinetic energy E is

$$\sigma(E)=\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{\sigma_0(\Gamma/2)^2}{(E-E_0)^2+(\Gamma/2)^2}, \qquad (1)$$

provided only one level is effective for capture. Here Γ is the total (radiation) width of the virtual state of the compound nucleus, and σ_0 is the capture cross section for neutrons having the resonance energy E_0. The cross section at resonance may be expressed in terms of Γ_n, the partial (neutron) width of the virtual state arising from neutron emission, by the relation

$$\sigma_0=\pi\frac{2J+1}{2I+1}\frac{\hbar^2}{ME_0}\frac{\Gamma_n}{\Gamma}, \qquad (2)$$

where I is the angular momentum of the capturing nucleus, and J is the angular momentum of the virtual state. Since there appears to be no method of determining this latter quantity, we shall incorporate the unknown factor $(2J+1)/(2I+1)$ into the neutron width.

The neutron and radiation widths deduced from slow neutron data are of great importance for nuclear theory since they provide the only available information concerning the probabilities of emission of various kinds of particles from the compound nucleus. The ratio of the neutron to the radiation width may be obtained directly from the cross section at resonance, if the energy of resonance is known. If the total width is then determined by some method, such as the thermal and resonance activation method of Amaldi and Fermi, the neutron width may be calculated.

A direct method of obtaining the cross section at resonance is an absorption measurement in which the substance under investigation is used both as an absorber and as a detector of the resonance neutrons. The characteristic feature of this process is the "self-reversal" of the resonance line in the case of narrow resonances ($\Gamma/E_0\ll1$). Neutrons in the immediate neighborhood of the resonance are almost completely removed by a small thickness of absorber. Additional thicknesses are then much less effective in reducing the intensity of the beam since only the more penetrating neutrons in the "wings" of the resonance line remain. As a result, the apparent absorption coefficient will be a function of absorber thickness, which decreases rapidly with increasing thickness of absorber.

It is the purpose of this paper to obtain the neutron and radiation widths of the level responsible for the activation of the rhodium 44-sec. period by a theoretical analysis of such "self-reversal" absorption curves.

Experimental Procedure and Results

The irradiations were carried out on the top surface (30×22 cm) of a paraffin block 15 cm high. A radon-beryllium source of about 600 mC initial strength was symmetrically located 3 cm

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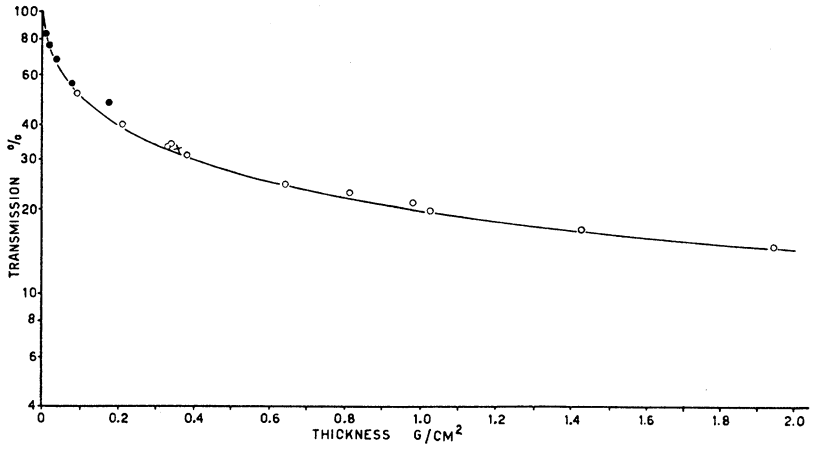

FIG. 1. Self-absorption in Rh as indicated by a 0.35 g/cm² detector. ● = Data of Preiswerk and v. Halban, × = Amaldi-Fermi, o = Present data, — = Calculated curve for $n\sigma_0/\rho = 36$ cm²/g.

below this surface. Cadmium sheet 0.45 mm thick covered the entire surface with the exception of a 9×9 cm opening at the center. For all measurements with D neutrons this square was covered with a plane sheet of cadmium 0.1×10×10 cm. The absorbers were placed on this cadmium sheet. The detectors were in contact with the upper surface of the absorber in order to provide a calculable geometry. Activation of the detector by stray thermal neutrons was prevented by a cadmium cover. Since the detector height was a function of the absorber thickness, it was necessary to correct for the variation of neutron intensity above the surface. This variation was measured and found to be closely linear with height in the range used. This correction, applied to all measurements, amounts to a maximum of 12 percent at 6 mm.

For the absorption curves we have used rhodium detectors of two different thicknesses. One, 0.35 g/cm² by 4×5 cm, was made of Rh powder in a paper packet which was slightly paraffined to prevent distortion. Since the paraffin might possibly alter the energy distribution of the neutrons, another type was constructed in which the Rh powder was bound to a thin nickel sheet by the minimum amount of lacquer. This type was 0.11 g/cm² by 5×5 cm.

The thick Rh absorbers consisted of powdered metal placed in containers of four-mil nickel sheet with the aid of a template to secure uniform thickness. Thinner absorbers were made to a convenient thickness by diluting with talc. The thinnest absorber was obtained by electroplating Rh on nickel. All absorbers were approximately 6×7 cm.

The activities of the detectors were measured with an ionization chamber similar to that described by Amaldi and Fermi.[1] This chamber was connected to an FP54 amplifier, and the current measured by the rate-of-drift method. The apparatus was very linear in the range used, and, at frequent intervals, care was taken to measure the chamber sensitivity with a uranium standard and the amplifier sensitivity by applying a known potential to the grid.

For the activity measurements a standard procedure of one-minute irradiation and one-half-minute interval followed by a one-minute ionization measurement was used. The results, therefore, while applying to the 44-sec. period, are independent of exact knowledge of the period, and the contribution of the longer period is sufficiently small to be neglected. Any data used for calculation of transmission values were the average of at least nine such activity measurements. From the individual measurements and their deviations it follows that no transmission value has a probable error greater than 3 percent.

The transmission of various thicknesses of Rh as determined by the "thick" Rh detector is given in Fig. 1 (open circles). Fig. 2 shows the

[1] Amaldi and Fermi, Phys. Rev. **50**, 899 (1936).

results for the "thin" detector. The calculated curves are shown in each case, and the "thick" detector calculated curve is shown in Fig. 2 to illustrate the effect of detector thickness which has so often been overlooked in previous work.

Analysis of Absorption Curves

The fundamental assumption which we shall make is that the resonance absorption arises from a single, narrow resonance level of the compound nucleus. By virtue of the assumption that only one resonance level is effective for capture, the simple one-level formula (1) is applicable. Furthermore, the assumption of a narrow resonance ($\Gamma/E_0 \ll 1$) enables us to neglect the variation of the term $(E_0/E)^{\frac{1}{2}}$ over the resonance region. We may thus write the capture cross section in the more simple form

$$\sigma(E)=\sigma_0/\left[1+\left(\frac{E-E_0}{\Gamma/2}\right)^2\right]. \tag{3}$$

The observed transmission curves are essentially measurements of the activity of a detector (defined as the number of β-rays emitted per second immediately after an infinite time of irradiation) with various thicknesses of absorber interposed between the paraffin and the detector. The essential elements involved in the determination of the detector activity are the distribution in angle and energy of the neutrons emerging from the paraffin, the reduction of neutron intensity in the absorber, the neutron absorption in the detector, and the absorption in the detector of the β-rays created by the capture process. Fermi has shown that the number of neutrons emitted per unit solid angle at an angle ϑ with the normal to a paraffin surface is approximately proportional to $\cos\vartheta+3^{\frac{1}{2}}\cos^2\vartheta$. For resonance neutrons, however, the assumption of spherically symmetric scattering used in deriving this distribution is no longer valid, and one would expect an approach to a cosine distribution. For this reason and for simplicity[2] we shall assume that the angular distribution of the resonance neutrons is proportional to $\cos\vartheta$, that is, we assume that the number of neutrons emitted per second per unit area of the paraffin with energies in the range between E and $E+dE$, and with angles to the normal of the surface between ϑ and $\vartheta+d\vartheta$, is equal to

$$2N(E)dE\cos\vartheta\sin\vartheta d\vartheta. \tag{4}$$

A detailed knowledge of the energy distribution function $N(E)$ is unnecessary since we may neglect the variation of the neutron intensity over the narrow resonance region.

It is essential that the absorption of the β-rays in the detector be included in the evaluation of the detector activity, for most measurements are performed with detectors which are thick with respect to electronic absorption. We shall make the questionable, though convenient assumption that the β-rays are absorbed exponentially, with an absorption coefficient μ_e.

From these assumptions and approximations, it immediately follows that the activity of a detector of thickness δ_2, with an absorber of thickness δ_1 interposed between it and the paraffin, is proportional to

$$\begin{aligned}A_{\delta_1,\delta_2}&=2N(E_0)\int_0^{\delta_2}dx\int_0^{\pi/2}\sin\vartheta d\vartheta\int_{-\infty}^{\infty}dE n\sigma(E)\\&\quad\times e^{-n\sigma(E)(\delta_1+x)/\cos\vartheta}e^{-\mu_e x}\\&=\Gamma N(E_0)\int_0^{\delta_2}dx\int_0^{\pi/2}\sin\vartheta d\vartheta\int_{-\infty}^{\infty}d\xi\frac{n\sigma_0}{1+\xi^2}\\&\quad\times e^{-n\sigma_0/(1+\xi^2)(\delta_1+x)/\cos\vartheta}e^{-\mu_e x},\end{aligned} \tag{5}$$

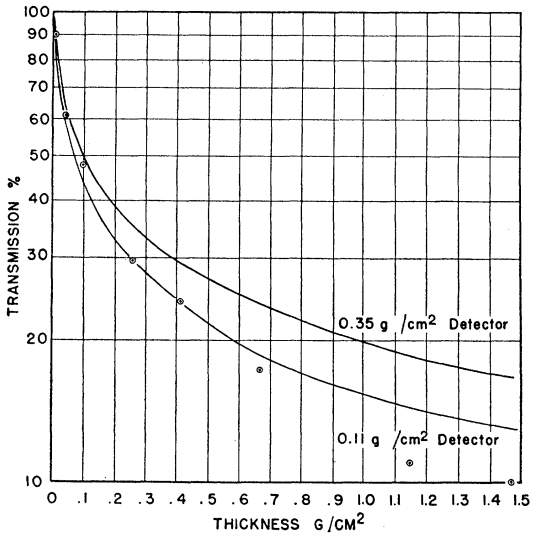

FIG. 2. Self-absorption in Rh as indicated by a 0.11-g/cm² detector (⊙). Solid curves are calculated for the detector thicknesses assuming $n\sigma_0/\rho=36$ cm²/g.

[2] Additional support for this simplification is provided by a calculation performed by Mr. M. Hamermesh which shows that the transmission at an absorber thickness of 0.1 g/cm² is inappreciably affected by replacing the cosine distribution with the Fermi distribution (cosine −49.9 percent, Fermi −49.6 percent).

where n represents the number of nuclei per cm^3 which are effective for capture, and $\xi=(E-E_0)/\frac{1}{2}\Gamma$. The integration with respect to ξ may be performed with the aid of the formula

$$\int_{-\infty}^{\infty}\frac{d\xi}{1+\xi^2}e^{-\zeta/(1+\xi^2)}=\pi e^{-\zeta/2}I_0\left(\frac{\zeta}{2}\right). \tag{6}$$

Hence,

$$A_{\delta_1,\,\delta_2}=\pi\Gamma n\sigma_0 N(E_0)\int_0^{\delta_2}dx\int_0^1 dz \times e^{-(n\sigma_0/2)(\delta_1+x)/z}I_0\left(\frac{n\sigma_0}{2}\frac{\delta_1+x}{z}\right)e^{-\mu_e x}, \tag{7}$$

in which $\cos\delta$ is designated by z.

For the purpose of numerical calculation, it is convenient to introduce a new quantity, which we shall term the "activity," defined by

$$\text{“}A\text{”}_{\delta_1,\,\delta_2}=\tfrac{2}{3}(\mu_e n\sigma_0)^{\frac{1}{2}}\int_0^{\delta_2}dx\int_0^1 dz \times e^{-(n\sigma_0)/2(\delta_2+x)/z}I_0\left(\frac{n\sigma_0}{2}\frac{\delta_1+x}{z}\right)e^{-\mu_e x}. \tag{8}$$

The convenience of this definition lies in the fact that the "activity" of a detector, with zero thickness of absorber, is always less than one, and becomes equal to one only in the special case of an infinite cross section at resonance and an infinite detector thickness. It is evident from (7) and (8) that the relation between the activity $A_{\delta_1,\,\delta_2}$ and the "activity" "A"$_{\delta_1,\,\delta_2}$ is:

$$A_{\delta_1,\,\delta_2}=\tfrac{2}{3}\pi\Gamma(n\sigma_0/\mu_e)^{\frac{1}{2}}N(E_0)\text{“}A\text{”}_{\delta_1,\,\delta_2}. \tag{9}$$

A further simplification may be made in the expression for the "activity" by introducing the new variable η, defined by $x=-\delta_1+z\eta$, interchanging the order of the integrations and performing the elementary integration with respect to z. The resultant formula is a sum of two integrals with respect to η, *viz:*

$$\text{“}A\text{”}_{\delta_1,\,\delta_2}=\tfrac{2}{3}(\mu_e n\sigma_0)^{\frac{1}{2}}\left\{\int_{\delta_1}^{\delta_1+\delta_2}d\eta e^{-(n\sigma_0/2)\eta} \times I_0\left(\frac{n\sigma_0}{2}\eta\right)\frac{1+\mu_e\delta_1-(1+\mu_e\eta)e^{-\mu_e(\eta-\delta_1)}}{(\mu_e\eta)^2} + \int_{\delta_1+\delta_2}^{\infty}d\eta e^{-(n\sigma_0/2)\eta}I_0\left(\frac{n\sigma_0}{2}\eta\right) \times\frac{1+\mu_e\delta_1-(1+\mu_e\delta_1+\mu_e\delta_2)e^{-\mu_e\delta_2}}{(\mu_e\eta)^2}\right\}. \tag{10}$$

In general, this final integration must be carried out numerically. However, for large absorber thicknesses we may use the asymptotic form of $I_0(x)$, i.e.,

$$I_0(x)\sim\frac{e^x}{(2\pi x)^{\frac{1}{2}}} \tag{11}$$

which gives a result accurate to within 2 percent for values of x as small as 6. Inserting this expression for $I_0(x)$ into Eq. (8) we find that, for large absorber thicknesses ($n\sigma_0\delta_1\gtrsim 12$), "$A$"$_{\delta_1,\,\delta_2}$ is given to within ~ 1 percent by

$$\text{“}A\text{”}_{\delta_1,\,\delta_2}=\frac{1}{\pi^{\frac{1}{2}}}\int_0^{\mu_e\delta_2}\frac{e^{-x}}{(\mu_e\delta_1+x)^{\frac{1}{2}}} =e^{\mu_e\delta_1}\{\Phi(\mu_e(\delta_1+\delta_2)^{\frac{1}{2}})-\Phi((\mu_e\delta_1)^{\frac{1}{2}})\}, \tag{12}$$

where $\Phi(x)$ denotes the error integral, defined by

$$\Phi(x)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x e^{-t^2}dt. \tag{13}$$

The neutron transmission through an absorber of thickness δ_1 is defined to be the fraction by which the activity of the detector is reduced when the absorber is interposed between the detector and the paraffin. The transmission, therefore, can be written

$$P_{\delta_2}(\delta_1)=\frac{A_{\delta_1,\,\delta_2}}{A_{0,\,\delta_2}}=\frac{\text{“}A\text{”}_{\delta_1,\,\delta_2}}{\text{“}A\text{”}_{0,\,\delta_2}}, \tag{14}$$

which involves only the cross section at resonance and not the resonance energy and the total width. It is perhaps not entirely superfluous to point out that the transmission thus defined is not the fraction of the incident number of neutrons which penetrate the absorber. The transmission at large thicknesses assumes a particularly simple form in two special cases: (a) $n\sigma_0\delta_1\gg 1$, $n\sigma_0\delta_2\ll 1$, $\mu_e\delta_2\ll 1$; (b) $\delta_1\gg\delta_2$, $\mu_e\delta_2\gg 1$, $n\sigma_0\gg\mu_e$, which correspond, respectively, to a thin detector and to a thick detector whose electronic absorption coefficient is small compared with the cross section at resonance. It is easily shown that the transmissions in these two cases are $P=2/3(\pi n\sigma_0\delta_1)^{\frac{1}{2}}$ and $P=1/(\pi\mu_e\delta_1)^{\frac{1}{2}}$, respectively.

Since the transmission curves involve only a single adjustable parameter, the cross section at resonance, an attempt to bring the calculated curves into coincidence with the observed curves

by suitably choosing the resonance cross section is a severe test of the assumed dependence of the capture cross section on the energy of the neutron. We shall now describe calculations of this nature for rhodium.

Rhodium

Experimental transmission curves have been obtained for the two detector thicknesses $\rho\delta_2=0.11$ g/cm² and $\rho\delta_2=0.35$ g/cm². The cross section at resonance is most conveniently calculated from the data obtained with the thicker detector. Anticipating that $n\sigma_0\delta_2>12$ for $\rho\delta_2=0.35$ g/cm², we may calculate the "activity" $"A"_{0,\delta_2}$ from Eq. (8) by integrating with respect to x from zero to infinity, subtracting the integral from δ_2 to infinity and utilizing the asymptotic expansion of $I_0(x)$ in the latter integral. Thus,

$$"A"_{0,\delta_2}=\tfrac{3}{2}(\mu_e n\sigma_0)^{\frac{1}{2}}\int_0^\infty dx\int_0^1 dz e^{-(\mu_e+n\sigma_0/2z)x}$$

$$I_0\left(\frac{n\sigma_0}{2}\frac{x}{z}\right)-\tfrac{3}{2}\left(\frac{\mu_e}{\pi}\right)^{\frac{1}{2}}\int_{\delta_2}^\infty dx\frac{e^{-\mu_e x}}{x^{\frac{1}{2}}}\int_0^1 z^{\frac{1}{2}}dz$$

$$=\tfrac{3}{2}\int_0^1 dz\left(\frac{z}{1+(\mu_e/n\sigma_0)z}\right)^{\frac{1}{2}}-(1-\Phi(\mu_e\delta_2)^{\frac{1}{2}}), \quad (15)$$

since $$\int_0^\infty e^{-ax}I_0(bx)dx=\frac{1}{(a^2-b^2)^{\frac{1}{2}}}, \quad a>b. \quad (16)$$

Performing the integration with respect to z we obtain, finally:

$$"A"_{0,\delta_2}=\tfrac{3}{2}\frac{n\sigma_0}{\mu_e}\left(1+\frac{\mu_e}{n\sigma_0}\right)^{\frac{1}{2}}+\tfrac{3}{2}\left(\frac{n\sigma_0}{\mu_e}\right)^{\frac{3}{2}}$$

$$\times\log\left(\left(1+\frac{\mu_e}{n\sigma_0}\right)^{\frac{1}{2}}-\left(\frac{\mu_e}{n\sigma_0}\right)^{\frac{1}{2}}\right)-(1-\Phi(\mu_e\delta_2)^{\frac{1}{2}}). \quad (17)$$

Using the asymptotic expansion of $I_0(x)$ in the second integral of the expression (10) for the "activity" $"A"_{\delta_1,\delta_2}$, we obtain

$$"A"_{\delta_1,\delta_2}=\tfrac{3}{2}(\mu_e n\sigma_0)^{\frac{1}{2}}\int_{\delta_1}^{\delta_1+\delta_2}d\eta e^{-(n\sigma_0/2)\eta}$$

$$\times I_0\left(\frac{n\sigma_0}{2}\eta\right)\frac{1+\mu_e\delta_1-(1+\mu_e\eta)e^{-\mu_e(\eta-\delta_1)}}{(\mu_e\eta)^2}$$

$$+\frac{1}{\pi^{\frac{1}{2}}}\frac{1+\mu_e\delta_1-(1+\mu_e\delta_1+\mu_e\delta_2)e^{-\mu_e\delta_2}}{(\mu_e\delta_1+\mu_e\delta_2)^{\frac{3}{2}}}. \quad (18)$$

We may now calculate the resonance cross section by fitting the experimental transmission at some value of the absorber thickness. Taking $\rho\delta_1=0.10$ g/cm², and using the experimental value of the electronic absorption coefficient, $\mu_e/\rho=7.3$ cm²/g,[3] we find that the observed transmission of 50 percent is obtained with $n\sigma_0/\rho=36$ cm²/g.[4] This value of the absorption coefficient, which corresponds to a cross section at resonance of $6{,}100\times10^{-24}$ cm²,[5] justifies the assumption that $n\sigma_0\delta_2>12$. With the value of the resonance cross section thus obtained, we may now calculate the complete absorption curve for comparison with experiment. The results of this calculation, which are in good agreement with the experimental points, are given as the solid line of Fig. 1.

Despite the good agreement, it seemed advisable to check the results with a thinner detector (0.11 g/cm²). The calculated curve ($n\sigma_0/\rho=36$) and the experimental points are shown in Fig. 2. The agreement is good for transmissions greater than 20 percent, but for lower values the observed transmissions are definitely less than those calculated. Agreement is hardly to be expected at large absorber thicknesses in view of the neglect of the factor $(E_0/E)^{\frac{1}{2}}$ which may well be of considerable importance in this region where only the wings of the line contribute to the activity of the detector. For the thick detector, the agreement at large thicknesses of absorber must be attributed to the effect of the paper container in increasing the number of neutrons in the center of the line where the capture probability is large. This effect increases the transmission. Such a process was demonstrated by a simple experiment.

Another check of the value of $n\sigma_0/\rho$ can be obtained very simply by measuring the activity of both faces of a rhodium detector. The ratio of the activities is given by

$$\frac{\int_0^\delta dx\int_0^1 dz e^{-(n\sigma_0x)/2z}I_0\left(\frac{n\sigma_0x}{2z}\right)e^{-\mu_e x}}{\int_0^\delta dx\int_0^1 dz e^{-(n\sigma_0x)/2z}I_0\left(\frac{n\sigma_0x}{2z}\right)e^{-\mu_e(\delta-x)}}. \quad (19)$$

[3] Amaldi, Hafstad and Tuve, Phys. Rev. **51**, 896 (1937).

[4] An estimate of the sensitivity of this method may be obtained from the following: (a) An assumed absorption coefficient of 52 cm²/g yields a transmission of 45 percent. (b) 29 cm²/g gives a transmission of 55 percent.

[5] Bethe (Rev. Mod. Phys. **9**, 146 (1937)) has mentioned our reported result (Phys. Rev. **51**, 1022 (1937)), but has incorrectly given the absorption coefficient at resonance as 19 cm²/g instead of 36 cm²/g, although the cross section given in his Table XXVII, p. 151, is correct.

For a detector thickness $\rho\delta=0.36$ g/cm^2 and $n\sigma_0/\rho=36$ Eq. (19) gives the value 2.1. Amaldi and Fermi have observed a ratio of two for this thickness of detector. We have measured the ratio for a Rh sheet 0.18 g/cm^2 (kindly loaned us by Professor E. O. Salant) and found it to be 1.43 ± 0.01. The calculated value is identical.

The total width of the rhodium resonance level may be obtained from Amaldi and Fermi's measurements of the resonance and thermal activity of a detector, averaged over all positions and orientations inside a large block of paraffin. The average over-all directions may be replaced by an average with respect to two opposite orientations of the detector, whence it may be shown that the mean resonance activity of the detector is proportional to

$$\int A_{\text{res}}d\tau=\tfrac{1}{2}\pi n\sigma_0 Q\lambda_0\frac{\Gamma}{E_0}\int_0^\delta dx\int_0^1 dz \times\exp\left[-\frac{n\sigma_0 x}{2z}\right]I_0\left(\frac{n\sigma_0}{2z}x\right)\cdot\tfrac{1}{2}(e^{-\mu_e x}+e^{-\mu_e(\delta-x)}), \quad (20)$$

where Q is the number of neutrons emitted per second from the source, λ_0 is the mean free path in paraffin of the resonance neutrons, and δ denotes the thickness of the detector. The average thermal activity, which is most conveniently measured by subtracting the activity obtained with a Cd filter on both sides of the detector from the activity measured with the detector screened by Cd on only one side, is similarly proportional to

$$\int A_{\text{therm}}d\tau=\frac{2}{2+(3)^{\frac{1}{2}}}n\sigma_c Q\lambda_c N^{\frac{1}{2}}\int_0^\delta dx\int_0^1 dz \times e^{-(n\sigma_c x)/z}(1+(3)^{\frac{1}{2}}z)e^{-\mu_e x}. \quad (21)$$

In deriving this expression, the dependence of the thermal cross section σ_c on energy has been neglected. Here λ_c represents the mean free path in paraffin of the thermal neutrons and N symbolizes the average number of mean free paths traversed by a thermal neutron before capture.

From these two expressions, Γ/E_0 may be calculated in terms of the experimentally determined ratio of the resonance and thermal activities, *viz;*

$$\frac{\Gamma}{E_0}=\frac{4}{\pi}\frac{\sigma_c}{\sigma_0}\frac{\lambda_c N^{\frac{1}{2}}}{\lambda_0}\frac{\int A_{\text{res}}d\tau}{\int A_{\text{therm}}d\tau}\cdot\frac{\frac{1}{1+\frac{1}{2}(3)^{\frac{1}{2}}}\int_0^\delta dx\int_0^1 dz e^{-(n\sigma_c/z)x}(1+(3)^{\frac{1}{2}}z)e^{-\mu_e x}}{\int_0^\delta dx\int_0^1 dz e^{-(n\sigma_0/2z)x}I_0\left(\frac{n\sigma_0}{2z}x\right)(e^{-\mu_e x}+e^{-\mu_e(\delta-x)})}. \quad (22)$$

According to Amaldi and Fermi, the experimental value of this activity ratio, obtained with a detector thickness of $\rho\delta=0.36$ g/cm^2, is

$$\frac{\int A_{\text{res}}d\tau}{\int A_{\text{therm}}d\tau}=\frac{4.58\cdot10^5}{1.5\cdot10^6}=0.30. \quad (23)$$

The other experimental quantities involved in Eq. (22) have also been determined. The thermal cross section of Rh has the value $\sigma_c=120\times10^{-24}$ cm^2, corresponding to an absorption coefficient of 0.7 cm^2/g. The mean free path in paraffin of the thermal neutrons is $\lambda_c=0.30$ cm. The mean free path of the resonance neutrons has not been measured accurately. It may, however, be obtained from the thermal mean free path, using the theoretically calculated effect of molecular binding. Assuming the protons in a paraffin molecule to be isotropically bound, Fermi has shown that $\lambda_0/\lambda_c=3.3$. Bethe, however, has lowered this value to $\lambda_0/\lambda_c=2.8$ by taking into consideration the anisotropic binding of the protons. The mean free path of the resonance neutrons is, therefore, equal to $\lambda_0=0.85$ cm. The last experimental quantity we shall require is $N^{\frac{1}{2}}$. Experiments on the "albedo" of thermal neutrons in paraffin yield the result that $N^{\frac{1}{2}}=14$.

Combining these experimental values with the calculated values of the integrals, we obtain $\Gamma/E_0=0.072$. This result for Γ/E_0 with a reso-

nance energy $E_0=2.0$,[6] leads to a total width of the Rh resonance of $\Gamma=0.14$ ev. The value of the width thus obtained from relative activity measurements may be compared with the width calculated from the thermal and resonance cross sections by assuming the one-level formula (1) to be applicable. Taking the average energy of the thermal neutrons as $kT=0.026$ ev, we obtain

$$\sigma_c=\left(\frac{E_0}{kT}\right)^{\frac{1}{2}}\left(\frac{\Gamma}{2E_0}\right)^2\sigma_0, \qquad (24)$$

since the resonance energy is large compared to both kT and the natural width. Inserting the values of σ_c and σ_0, namely $\sigma_c=120\times10^{-24}$ cm^2 and $\sigma_0=6{,}100\times10^{-24}$ cm^2, we find that $\Gamma=0.19$ ev, which is in substantial agreement with the value of the width deduced from activity measurements in view of the uncertainty in the experimental values of many of the quantities involved.[7] If the total width be taken as 0.17 ev, a weighted mean, the value of the neutron width is $\Gamma_n=1.6\times10^{-3}$ ev.

In their early experiments on resonance neutrons Amaldi and Fermi measured the self-absorption of rhodium for a single absorber thickness (0.36 g/cm^2). Their detector thickness and geometry were the same as in the present "thick" detector case, and their value which is included in Fig. 1 is in excellent agreement with our data for this absorber thickness. Unfortunately the importance of self-reversal was not realized at the time of their measurements, and their effective absorption coefficient $n\sigma_0/\rho=2.0$ at 0.36 g/cm^2 is almost twenty times smaller than the absorption coefficient at the resonance energy.

Preiswerk and von Halban[8] discovered the dependence of absorption coefficient on absorber thickness and correctly attributed it to self-reversal. Their data is also included in Fig. 1, although, since their geometry and detector thickness have not been stated, complete agreement cannot be expected.

Since this work was completed, Jaeckel[9] has applied a similar method to the case of rhodium obtaining an absorption coefficient at resonance of 25 cm^2/g. The use of an old value of the electronic absorption coefficient (6 instead of 7.3), however, should make his value too low.

Discussion

The experimental transmission curve decreases more rapidly with absorber thickness than $\delta_1^{-\frac{1}{2}}$, which is the shape of the absorption curve deduced from (12) when the absorber thickness is large compared with the detector thickness. This discrepancy between observed and calculated transmissions at large absorber thicknesses indicates that one or more of the approximations which have been made in the preceding analysis are invalid under these conditions. Approximations have been introduced concerning the neutron intensity variation with energy and angle, the energy dependence of the capture cross section, and the law of electronic absorption. The discrepancy is necessarily to be attributed to an inadequacy of the approximate formula (3) representing the cross section, and to the neglected variation of the neutron intensity with energy, for it may be shown that with our previous assumptions concerning these quantities the calculated transmission varies at large thicknesses as $\delta_1^{-\frac{1}{2}}$, quite independently of the laws governing electronic absorption and neutron angular distribution.

Equation (3) presupposes a single resonance which is sufficiently narrow to permit the neglect of the factor $(E_0/E)^{\frac{1}{2}}$ in the resonance region. Furthermore, we have implicitly neglected the effect of the thermal motion of the capturing nuclei, i.e., the Doppler effect. That the Doppler effect is of no great importance may be seen by comparing the Doppler width[10], [11]

[6] Data of Goldsmith-Rasetti corrected for boron scattering and electronic absorption in the detector.

[7] The agreement is somewhat improved by utilizing recent mean free path measurements. The thermal neutron mean free path deduced from the measurements of H. Carroll and J. R. Dunning is $\lambda_c=0.25$ cm. According to unpublished measurements of V. W. Cohen, H. H. Goldsmith, and J. Schwinger (see Letter to the Editor, this issue), the mean free path in paraffin of resonance neutrons is $\lambda_0=0.6$ cm. The experimental ratio λ_0/λ_c is therefore 2.4 which must replace the calculated ratio of 2.8. The value of the total width is altered to 0.17 ev which compares more favorably with the value of 0.19 ev deduced from the thermal cross section of Rh.

[8] Preiswerk and von Halban, Nature **138**, 163 (1936).

[9] Jaeckel, Zeits. f. Physik **107**, 669 (1937).

[10] Bethe, Rev. Mod. Phys. **9**, 140 (1937).

[11] The quoted formula for the Doppler width is deduced by neglecting the binding of the capturing nuclei. Recent calculations by W. E. Lamb, Jr. (in press) have shown that

$$\Delta = 2((mE_0kT)/M)^{\frac{1}{2}} = 0.045 \text{ ev}$$

with the total width $\Gamma = 0.17$ ev. In consequence of the small value of Δ/Γ, the Doppler effect alters the capture cross section only in the immediate vicinity of the resonance, a region which contributes but little to the activity at large absorber thicknesses. Therefore the Doppler effect exerts no influence on the shape of the theoretical transmission curve in the region where disagreement with experiment exists.

Under the conditions obtaining at large absorber thicknesses, appreciable absorption is not confined to the immediate neighborhood of the resonance energy, thus invalidating the neglect of the variation of the neutron intensity $N(E)$. In the absence of experimental knowledge concerning this variation, it is reasonable to assume that the neutron intensity varies approximately as $1/E$.

In order to determine whether the removal of the approximations made in connection with the single resonance formula is sufficient *per se* to explain the discrepancy, thus obviating the necessity of invoking additional resonances, we have calculated the shape of the transmission curve at large thicknesses, using the complete formula (1) for the cross section, and $N(E) = 1/E$. Although these refinements do not suffice to reconcile theory and experiment, a somewhat slower variation of neutron intensity, *viz:* $1/E^{\frac{1}{2}}$, was found quite sufficient to reproduce the shape of the experimental transmission curves. It would therefore appear that the activity arising from the resonance level at two volts is competent of accounting for the observed transmission curves.

It is of some interest to note that Rh 3.9 min. has a resonance level at approximately the same energy as the two-volt resonance of Rh 44 sec.[12] The possibility of an effect of the resonance associated with the longer period should be considered, for although we are measuring the activity of the 44-sec. period, the absorption process involves the excitation of both periods. However, the approximate equality of the measured resonance energies of the two periods, coupled with the fact that the concentration of the isotope Rh^{103} is about 1000 times that of the isotope Rh^{101} proves almost conclusively that both periods arise from the same resonance level of the radioactive isotope Rh^{104}; i.e., Rh 44 sec. and Rh 3.9 min. are isomeric forms of the nucleus Rh^{104}. Inasmuch as the branching-ratio for the two periods is presumably independent of the energy of the captured neutron, the cross section for the formation of Rh 44-sec. differs from the total cross section by a constant factor, which obviously changes all the activities by a constant factor, and thus leaves the calculated transmissions unaffected.

In conclusion, the authors wish to express appreciation to the American Platinum Company for the rhodium which it so kindly loaned to the Columbia Department of Physics; and to Mr. M. Hamermesh for aid in some of the calculations.

the effect of crystal binding is to replace kT by the average vibrational energy per degree of freedom in the case of large natural width. This would lead to an effective temperature about 20 percent higher than T, which has no material effect on our conclusions. We are indebted to Dr. Lamb for the privilege of quoting his results before publication.

[12] Goldsmith and Rasetti, Phys. Rev. **50**, 328 (1936).