theoretical physicists provide us with $f(u)$ values computed for molecules. It is unfortunate that the gases which are relatively easy to study experimentally are very difficult to investigate theoretically.

We take this opportunity of thanking Professor Kirkpatrick for sending us the numerical values of the $f(\lambda'')$ curves for atomic carbon and nitrogen. The senior author takes pleasure in acknowledging the assistance afforded by a grant from the Penrose Fund of the American Philosophical Society.

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Scattering of Fast Electrons in Gases

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When sufficiently fast electrons are scattered elastically by atoms, it is possible to regard the effect as being due to the nuclei alone, the atomic electrons playing only a negligible part. The criterion which will allow such a simplification to be made is that the collision parameter for a deflection of an incoming electron by the nucleus must be small compared with the distance between the nucleus and the nearest atomic electron. When this is the case, the observed scattering should be that predicted by the Rutherford scattering formula. Kuper has recently made an experimental investigation of the scattering of electrons of energy above 49,000 volts by helium, neon and argon. His results for helium can be accounted for on the viewpoint proposed, *viz.*, that the nucleus is almost entirely responsible for the observed scattering and that, to a first approximation, the scattering is simple Rutherford scattering. The wave-mechanical formula, which takes into account the effect of the atom as a whole does not give as good agreement with Kuper's results for helium as the Rutherford scattering formula.

THE purpose of this note is to call attention to a simple way of interpreting certain experimental results in the scattering of fast electrons by gases, and in particular to apply it to an important experimental investigation recently published by Kuper.[1] Since an atom is made up of various centers (the nucleus and the atomic electrons), each individually acting on an electron passing through it with a force varying inversely as the square of the distance between the center and the electron, it is natural to regard the scattering of electrons by a single center of force as a first step in discussing the scattering of electrons by an atom. An electron of mass m, charge e and velocity v, on approaching a nucleus whose charge is Ze will be deflected by it. If p_n is the collision parameter, the angle of deflection θ will be given by

$$p_n=(Ze^2/mv^2)\cot(\theta/2). \quad (1)$$

If the scattering center is an electron at rest, the angle of deflection θ is related to the collision parameter by

$$p_e=(2e^2/mv^2)\cot\theta. \quad (2)$$

[1] J. B. Horner Kuper, Phys. Rev. **53**, 993 (1938).

The scattering of an electron passing through the atom is ordinarily the result of the interaction of that electron with several, possibly all, of the scattering centers in the atom. We could visualize the scattering of electrons by an atom as the result of each scattering center in the atom acting on the incoming electrons with an inverse square force. The mathematical difficulties encountered in attempting to solve such a problem are so formidable that no attempt has been made along these lines. The problem has been attacked successfully along very different lines. The electron stream impinging on the atom is replaced by an electron wave of the proper wave-length, and the atom is replaced by a region of suitably varying refractive index. Methods carried over from physical optics lead to scattering formulas. In many cases the results are in excellent quantitative accord with experiment and it may be concluded that, in principle, this wave-mechanical approach is quite satisfactory. In those cases in which quantitative agreement is wanting, it is generally due to technical mathematical difficulties in securing exact solutions of the equations.

A simpler viewpoint can be adopted with quantitative accuracy provided a certain condition is satisfied. The condition is as follows. If the collision parameter is much smaller than the distance between any two scattering centers in the atom, then any scattering that takes place can safely be attributed to one, and only one, scattering center. When this condition holds, we can apply the scattering formulas for the interaction of an electron with either a nucleus or another electron. This simplified viewpoint has been used successfully by the author and his collaborators.[2] It is of interest to show that it can be applied with fair success to Kuper's results on the scattering of fast electrons by helium atoms.

Kuper measured the elastic scattering of electrons with energies from 49,500 to 87,700 electron volts, through angles between about 0.5° and 2.5°, by helium, neon and argon. He compared his experimental results with the wave-mechanical calculations of Mott and Massey. While satisfactory agreement was obtained in the case of argon and neon, no agreement at all was found in the case of helium. We shall show, first of all, that the relation of the collision parameters to the distance between the nucleus and an atomic electron in the helium atom, for the scattering of 50,000-volt electrons through 1°, is such as to allow us to regard the scattering as a good approximation to a single center scattering problem. The relation is shown to scale in Fig. 1. It is evident that the electrons, passing close

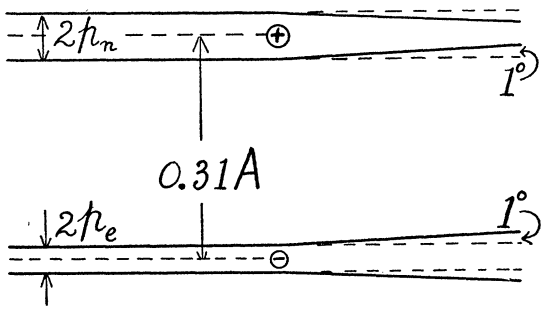

FIG. 1. Comparison of the collision parameters for the scattering of 50,000-volt electrons through 1° by a helium atom with the most probable distance between the nucleus and an atomic electron. The incoming electrons move from left to right.

[2] A. L. Hughes and S. S. West, Phys. Rev. **50**, 320 (1936); **52**, 43 (1937). A. L. Hughes and Marvin M. Mann, Jr., Phys. Rev. **53**, 50 (1938). A. L. Hughes and Merle A. Starr, Phys. Rev. **54**, 189 (1938). A. L. Hughes and Merle A. Starr (preceding paper).

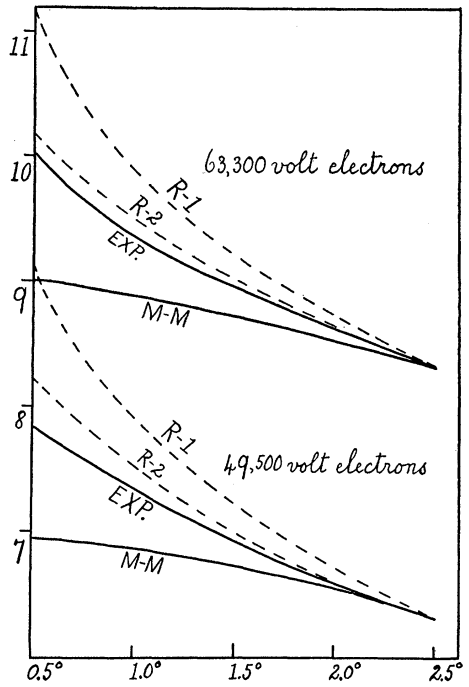

FIG. 2. Scattering of fast electrons through small angles by helium. EXP.—experimental values due to Kuper. M–M.—theoretical curve due to Mott and Massey. R–1.—Rutherford scattering. R–2.—Rutherford scattering on the assumption that the effective angle differs from the measured angle by 0.5°. The ordinates are the logarithms of the scattered current.

enough to the nucleus to be scattered through 1°, are practically unaffected by the presence of the atomic electrons. This justifies us in supposing that the elastic scattering is due practically entirely to the nucleus and that the well-known formula for scattering by a nucleus applies. According to this, the number of electrons scattered through unit solid angle at θ should be proportional to $\mathrm{cosec}^4(\theta/2)$, the well-known Rutherford scattering law. In Fig. 2 we have reproduced Kuper's experimental curves for 49,500- and 63,300-volt electrons, together with those calculated from Mott and Massey's theory. It will be seen that not only is there no numerical agreement but that the curvature of the experimental and theoretical curves are in opposite directions. The curves marked R–1 are calculated by means of the Rutherford scattering law. While the R–1 curves are steeper than the experimental curves, it is interesting to note that their curvatures are in the same direction. It is of some significance that the experimental curve

comes closer to the Rutherford curve as the electron energy is stepped up from 49,500 to 63,300 volts. This is just what we should expect to follow from a comparison of Fig. 1, which is drawn for 50,000-volt electrons, with a similar figure drawn for 63,000-volt electrons. It is concluded that, when electrons of energy 50,000 volts and over are scattered through angles of 1° or more by helium atoms, the elastic scattering may be regarded as the result of an inverse square force between the incoming electrons and the nucleus, and that the effect of the atomic electrons is negligibly small.

It is natural to ask if the simple interpretation proposed can be made to give quantitative agreement with the experimental results. If we make the assumption that the effective angles of scattering are 0.5° greater than the scattering angles recorded, then we obtain the Rutherford scattering curves marked R–2 which are seen to be in almost quantitative agreement with Kuper's curves. That Kuper's measured angles should be in error by 0.5° is of course an *ad hoc* assumption which is not justified by any internal evidence in his paper. Indeed Kuper states that his angles were correct of 0.05°. It could be argued, however, that while the measured angles are correct to 0.05°, the effective angles may differ appreciably from the measured angles. In experiments of this kind the stream of electrons shot into the gas is not a strictly parallel beam. The electrons accepted by the analyzer are also to be found in a narrow cone. We have therefore the difficult problem of determining just what the effective angle is when we have a stream of electrons limited within a narrow cone from which electrons are scattered into another cone, which makes an angle of only 1° or so with the first, and when in addition, the scattering varies inversely as the fourth power of the angle. As in all problems of scattering of electrons in gas, it is necessary to correct for the change in scattering volume as the angle θ alters by introducing a factor of $\sin\theta$. This factor is very sensitive to errors in θ when it is small. In addition the factor $\sin\theta$ is strictly true only for intersecting cylinders and when the primary and scattered electrons are quite parallel to the axes of their respective cylinders. The replacement of intersecting cylinders by intersecting cones, especially at very small angles, may make $\sin\theta$ only a rough approximation as a scattering volume correction factor. These considerations indicate that the accurate determination of the effective angle in a small angle scattering experiment is a very difficult matter. The uncertainty in the effective angle may possibly account for some, or perhaps all, of the differences between the experimental curves and the curves computed by the Rutherford scattering formula.

It is not the purpose of the preceding paragraph to insist that we must add 0.5° to each of Kuper's angles, but merely to point out that one should be aware of the real difficulty in determining the effective scattering angle when it is very small. The main purpose of this paper is to point out that the elastic scattering of fast electrons by helium may be regarded as a simple Rutherford scattering by the nucleus and that it gives a better description of what happens than the more complicated Mott and Massey formula. In the case of argon and neon the Rutherford scattering is unapplicable because there are many more scattering centers in the atom. In particular the K electrons are so close to the nucleus that the distance between either of them and the nucleus is no longer much larger than the collision parameter p_n. Hence the considerations raised in this paper do not apply to neon and argon.

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Sodium-Like Spectra of Potassium, Calcium and Scandium

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Further identifications have been made in the sodium-like spectra of K, Ca and Sc. The constant second difference law has been used for some time in the analysis of spectra of isoelectronic ions. It is shown that this second difference can be calculated readily, and that to a first approximation it depends only on the total quantum numbers of the initial and final states. Correspondence between calculated and observed second differences is shown for several transitions. Application of the law to ionization potentials is indicated.

REFERENCES to previous work on the first six ions of the sodium sequence are listed in a paper on the Cl VII spectrum recently reported from this laboratory.[1] Edlén[2] has re-

[1] L. W. Phillips, Phys. Rev. **53**, 248 (1938).
[2] B. Edlén, Zeits. f. Physik **100**, 621 (1936).