found in the liquid. However, the data do show that the filtration effect for gaseous hydrogen is not much larger than that found for the liquid.

In general, the results check the liquid work and show no evidence of serious interference of liquid forces in the scattering phenomena. The difference between the scattering cross sections for ortho- and parahydrogen appear to be large enough to establish definitely a spin dependence of the neutron proton force.

The authors are grateful to Professor Gilbert N. Lewis for the suggestion that a gaseous scatterer be used and for helpful criticism in general. They also are indebted to Professor J. R. Oppenheimer, Dr. Willis E. Lamb, and Dr. L. I. Schiff of the Physics Department and to both Professor Felix Bloch of Stanford University and Professor Enrico Fermi of the University of Rome for advice concerning theoretical aspects of the problem.

FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

Atomic Electron Velocities in Nitrogen and Methane

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(Received December 30, 1938)

When sufficiently fast electrons are scattered by gas atoms it can be shown that the *distribution of energies* among the inelastically scattered electrons has exactly the same shape as the *distribution of component velocities* among the atomic electrons. An experimental determination of the distribution of energies among the electrons scattered inelastically by nitrogen and methane has been made. From these results the distributions of component velocities among the atomic electrons in nitrogen and methane have been computed. Half of the electrons have component velocities less than 1.80×10^7 cm/sec. in nitrogen, and less than 1.55×10^7 cm/sec. in methane. The experimental results refer to the L electrons only, as the apparatus would not permit the use of electrons of high enough energy to give the same information about the K electrons. The experimentally determined distributions of component velocities among the atomic electrons for nitrogen and methane are decidedly flatter than the theoretical calculated distributions for atomic nitrogen and atomic carbon. No theoretical calculations are available for molecular nitrogen and methane, hence comparisons were made with the calculations for the atoms.

INTRODUCTION

MANY investigations, both theoretical and experimental, have been carried out during the last twenty years on the "atomic structure factor." The significance of the factor is that it gives us the probability of finding an electron at each point in the atom. A knowledge of the distribution of momenta among the electrons is complementary to this and is essential to a complete description of the state of the electrons in an atom. Whereas an immense amount of work has been done on the first distribution, that relating to the positions of the electrons, comparatively little has been accomplished in describing the momenta of the electrons. Since the mass of an electron does not depend appreciably on its velocity in the range with which we are concerned, we can use the term "atomic electron velocity" as equivalent to momentum insofar as we are interested in variations of these quantities. Information as to the distribution of atomic electron velocities has been obtained from a study of the profile of the modified band in the Compton effect.[1] A different method was devised later in which a study of the distribution of energies among the electrons scattered by a gas was used to give information about the velocities of atomic electrons.[2] When electrons of sufficient energy are scattered through an appreciable angle by a gas at low pressure it is possible to regard the process as resulting from

[1] P. A. Ross, Proc. Nat. Acad. Sci. **9**, 246 (1923); G. E. M. Jauncey, Phys. Rev. **25**, 314, 723 (1925); J. W. M. DuMond, Phys. Rev. **33**, 643 (1929); J. W. M. DuMond and H. A. Kirkpatrick, Phys. Rev. **37**, 136 (1931); **38**, 1094 (1931); J. W. M. DuMond and A. Hoyt, Phys. Rev. **37**, 1443 (1931); J. W. M. DuMond, Rev. Mod. Phys. **5**, 1 (1933); G. E. M. Jauncey, Phys. Rev. **46**, 667 (1934); J. W. M. DuMond and H. A. Kirkpatrick, Phys. Rev. **52**, 419 (1937); H. A. Kirkpatrick and J. W. M. DuMond, Phys. Rev. **54**, 802 (1938).

[2] A. L. Hughes and M. M. Mann, Jr., Phys. Rev. **53**, 50 (1938). A. L. Hughes and M. A. Starr, Phys. Rev. **54**, 189 (1938).

the interaction of the impinging electron with either the nucleus or an atomic electron. If the "collision parameter" corresponding to the deflection under consideration is much smaller than the average distance between a nucleus and any atomic electron, then we can safely assume that the observed scattering is due to one, and only one, center, i.e., either to a nucleus or to an atomic electron. When this condition obtains we expect to find two distinct groups of scattered electrons, those scattered by the nuclei without loss of energy, and those scattered by the atomic electrons with loss of energy. If the atomic electrons are assumed to be at rest before collision, simple considerations based on the conservation of energy and momentum show that the electrons scattered by them through an angle θ would retain an amount of energy $V_0 \cos^2 \theta$, where V_0 is the energy of the impinging electron. If, however, the atomic electrons are in random motion, the electrons scattered through θ have a distribution of energies symmetrically arranged about $V_0 \cos^2 \theta$, the most probable value. The observed distribution of energies among the scattered electrons can therefore be used to give information as to the way in which the atomic electrons are moving. A particularly simple relationship exists between $f(u)$ the distribution of *component velocities* among the atomic electrons and $F(V'')$ the distribution of *energies* among the scattered electrons. The functions $f(u)$ and $F(V'')$ have precisely the same shape.[3] V'' is the excess energy (positive or negative) which the scattered electron acquires as a result of a collision with an atomic electron which has a component velocity u in a specific direction. (The particular component of velocity which determines V'' is the one which the atomic electron had, before collision, in a direction at right angles to that of the scattered electron after collision. As we may consider the distribution of atomic electron velocities to be isotropic, the determination of any particular component velocity automatically determines the other two, and also the resultant.) Thus a measurement of the distribution of *energies* among the scattered electrons gives at once the distribution of *component velocities*, which can then be compared with theoretical distributions. It can be shown that $f(u)$ is also identical in shape with $f(\lambda'')$ the distribution of intensity across the modified band in the Compton effect. Detailed discussion of the relationships will be found in an earlier paper.[4] These ideas have been applied to the determination of atomic electron velocities in helium and hydrogen.[2] In the present paper we shall discuss an exactly similar investigation to determine the atomic electron velocities in nitrogen and methane. It is un-

[3] G. E. M. Jauncey, Phys. Rev. **50**, 326 (1936).
[4] A. L. Hughes and M. M. Mann, Jr., Phys. Rev. **53**, 50 (1938).

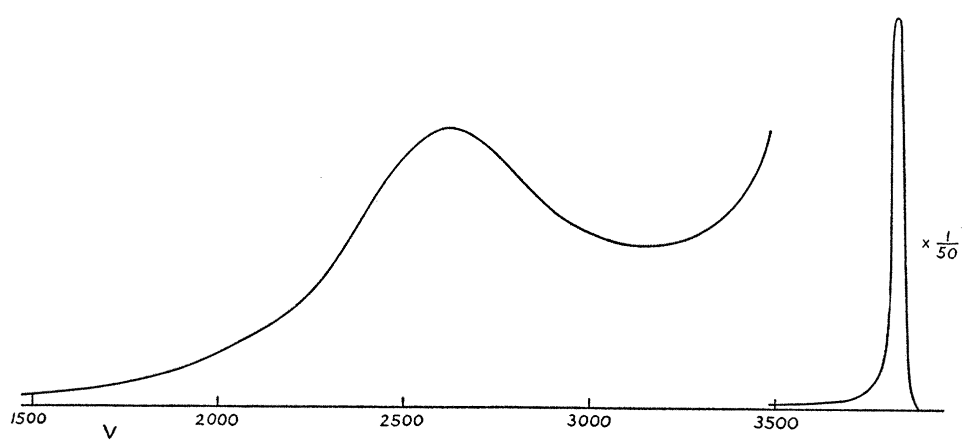

FIG. 1. Distribution in energy of 3825-volt electrons scattered through 34.2° by nitrogen. The narrow peak at 3825 volts (on a 1/50 scale) represents the electrons scattered elastically by the nucleus. The "foot" is due to the modifying effect of the K electrons. Collisions with the L electrons involve a loss of energy $V_0 \sin^2 34.2°$, plus or minus an amount depending on the component velocity of the L electron perpendicular to the direction of observation. The distribution of such component velocities is reflected in the distribution of energies centered at 2640 volts. The asymmetry of this distribution is due to the effect of the "foot" extending from the elastic peak.

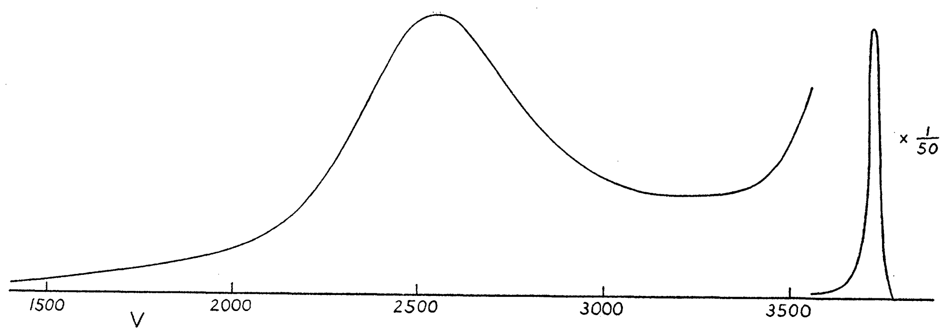

FIG. 2. Distribution in energy of 3725-volt electrons scattered through 34.2° by methane. Conditions for "single center scattering" are better fulfilled than in the case of nitrogen, because of the smaller perturbing effect of the "foot" of the elastic peak on the inelastic band.

necessary to describe the apparatus and method of using it as they are precisely the same as in our experiments on hydrogen and helium. The reader is referred to the published accounts of these experiments.[2]

Nitrogen was obtained from a commercial tank. It was said to be 99.5 percent pure, the chief impurity probably being oxygen. It was passed over hot copper and through charcoal cooled by "dry-ice" into the gas reservoir in our apparatus. Methane was obtained from a tank of the gas at high pressure. The manufacturers stated that the gas contained 86 percent methane, 12 percent ethane and 1 percent each of oxygen and nitrogen. We purified the gas by fractional distillation after condensation by liquid air and rejecting the last tenth of the liquid. This was repeated three times. Unlike the nitrogen, it was not passed through cooled charcoal.

RESULTS

In view of the fact that, in our earlier work on hydrogen and helium,[2] we had obtained the same values for $f(u)$ with incident electrons of various energies, it was not considered necessary in the work on nitrogen and methane to use any electron energy other than the highest we could conveniently use in the apparatus. This was 3800 volts. The angle of scattering was 34.2°, as before. The results are shown in the form of curves (Figs. 1 and 2) similar to those published in the papers on helium and hydrogen. Each curve in Figs. 1 and 2 is a mean curve constructed from nine separate curves representing individual runs. The total number of observations from which each of the curves shown here was drawn, exceeded 250. While the elastic peak is shown in each case in order to follow the scheme adopted in the previous papers, our efforts were concentrated on determining the shape of that part of the curve giving the distribution of inelastically scattered electrons. It will be noticed that the band representing the inelastically scattered electrons is not symmetrical about the maximum. The discussion given later in this paper will show that this lack of symmetry is to be attributed to the fact that we have not completely satisfied the requirements for perfect "single center scattering." To satisfy this requirement for nitrogen and methane calls for much higher electron energies than we can use in our present apparatus.

The ultimate purpose of this research is to determine the value of $f(u)$ for the atomic electrons in nitrogen and methane. Whereas it was possible to use both halves of the inelastic band for hydrogen and helium, since it was symmetrical, to compute $f(u)$, here we choose to use only the left half, or the low energy half. The reason we reject the high energy half is that we believe it to be distorted somewhat by the lack of perfect "single center scattering." A discussion of the underlying reasons for this action will be given later in the paper.

From the low energy half of each of the inelastic bands in Figs. 1 and 2, we obtain a pair of curves giving, by definition, $F(V'')$. As shown in a previous paper,[2] to get $f(u)$ we merely change the abscissas in V'' into abscissas in u (or β) by the substitution

$$\beta(=u/c)\times 10^3=\lambda''=V''\div(0.5782\,V_0^{\frac{1}{2}}), \quad (1)$$

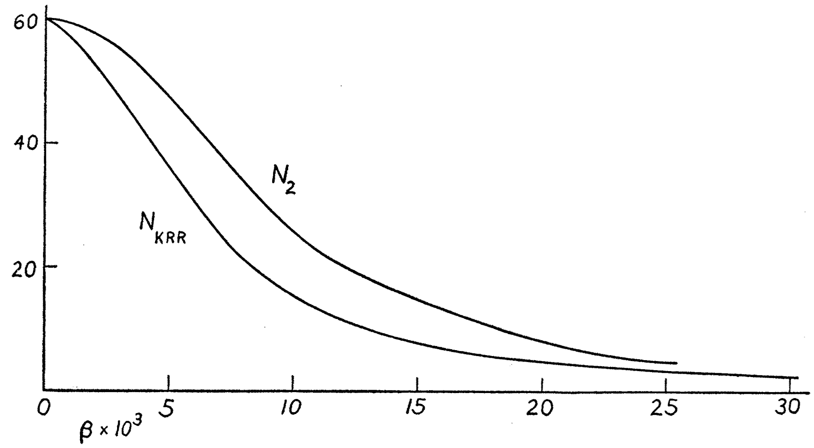

FIG. 3. Distribution of component velocities among the electrons in molecular nitrogen as obtained from the experimental curve shown in Fig. 1, and in atomic nitrogen as computed by Kirkpatrick, Ross and Ritland. (The latter curve includes the K electrons as well as the L electrons.)

where V'' is measured from the center of the inelastic band, V_0 is the energy of the impinging electrons, and c the velocity of light. Should we want to know the shape of the Compton modified band for nitrogen and methane for a primary wave-length of $\lambda = 695$ x.u. and $\theta = 90°$ we take the $F(V'')$ curve and relabel the abscissas in terms of λ'' as computed by the above formula. The experimentally computed $f(u)$ curves for nitrogen and methane are shown in Figs. 3 and 4. As we believe that the results are considerably more accurate than can be indicated in a small sized graph, we give the values of the ordinates of $f(u)$ for different values of u in Table I.

Discussion

The accuracy of the calculated values for the atomic electron velocities depends on the accuracy to which the basic assumptions of the method are satisfied in the experimental measurement. As was shown in the first paper[4] of this series, it is necessary that the collision parameter, p, for a finite deflection of an electron by a scattering center, which may be a nucleus or an atomic electron, be small compared with the average distance between any two scattering centers in order to meet the requirement that the observed scattering must be due to one, and only one, center. If the observed scattering is due to the cooperation of two or more scattering centers, we cannot use our method to compute the atomic electron velocities from the experimental results. In Fig. 5 we have shown the relation between the collision parameters for electrons of 4000 volts energy scattered through 34° by a helium atom, and the mean distance between the nucleus and an atomic electron. This mean distance is taken to be 0.31A which is obtained by combining the mean distance between the nucleus and the electron in a hydrogen atom, *viz.* 0.53A, and $Z' = 1\frac{11}{16}$ as the effective

TABLE I. *Theoretical velocity distributions for atomic electrons in* N *and* C *and experimental velocity distributions for atomic electrons in* N_2 *and* CH_4, *and the profile of the associated Compton modified band for* $\lambda = 695$ *x.u. and* $\theta = 90°$.*

$\beta \times 10^3$ and λ'' (x.u.)	u (cm/sec.)	$f(u)$ N (K.R.R.)	N_2 Experimental	C (K.R.R.)	CH_4 Experimental
0	0×10^7	60.0	60.0	60.0	60.0
1	3	57.0	59.3	56.4	59.5
2	6	52.5	57.7	49.8	57.6
3	9	46.9	55.1	42.0	53.5
4	12	41.6	51.7	34.4	48.4
5	15	36.0	47.5	27.4	42.9
6	18	30.6	43.0	22.2	37.5
8	24	21.4	33.9	14.9	27.2
10	30	15.3	25.8	10.8	19.5
12	36	11.4	20.2	8.3	14.5
14	42	8.65	16.6	6.5	11.0
16	48	6.84	13.5	5.2	8.8
18	54	5.5	10.5	4.3	7.5
20	60	4.55	8.0	3.68	6.5
25	75	3.2	4.7	—	4.0
30	90	—	—	1.7	—

* The last four columns could be labeled $f(\lambda'')$ thus giving the profile of the Compton modified band, (1) as calculated by Kirkpatrick, Ross and Ritland, and (2) as predicted from electron scattering experiments.

atomic number for calculating distances in the helium atom.

The collision parameters are given by

$$p_n=(Ze^2/mv^2)\cot\tfrac{1}{2}\theta \tag{2}$$

for a deflection by a nucleus, and

$$p_e=(2e^2/mv^2)\cot\theta \tag{3}$$

for a deflection by an atomic electron. Here Z is the atomic number, e the electronic charge, m the mass of the electron, v the velocity of the electron, and θ the angle of deflection. These equations give $p_n=0.0117\text{A}$ and $p_e=0.00525\text{A}$ for 4000-volt electrons scattered through 34° by helium atoms. One may conclude that when a helium atom scatters 4000-volt electrons through 34°, the scattering is due either to the nucleus or to an atomic electron but not to both.[5] The electrons scattered by a nucleus are distinguished from those scattered by an atomic electron by the fact that the former have the same energy as they had before collision, V_0, and the latter retain only a part, *viz.*, $V_0\cos^2\theta$.

We shall now apply these considerations to the nitrogen atom. Using Eqs. (2) and (3), we find

[5] To be sure, there is a possibility that on rare occasions the nucleus and an atomic electron are so orientated that they both contribute deflections of the same order to the observed deflection. If, in Fig. 5, we allow the atomic electron to take all possible positions on a sphere of radius 0.31A whose center is at the nucleus, it is easily seen that the chance of a deflection arising from a considerable contribution by both scattering centers, with the parameters appropriate to 4000-volt electrons, is small enough to be neglected.

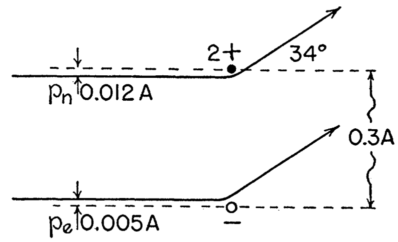

FIG. 5. Relation between collision parameters for 4000-volt electrons scattered through 34° by helium and the most probable nucleus to atomic electron distance in helium.

$p_n=0.041\text{A}$ and $p_e=0.00525\text{A}$ for 4000-volt electrons deflected through 34°. The mean distance between the nucleus and a K electron is approximately 0.08A, while the mean distance between the nucleus and an L electron is approximately 0.7A. These are shown in Fig. 6. It is evident that we may regard the scattering of 4000-volt electrons through 34° by the L electrons to be as unaffected by the nucleus as in the case of the helium atom. (Indeed by stretching the diagram for helium until it fits over the diagram for nitrogen, and allowing for the partial shielding of the nucleus by the K electrons, it can be seen that the conditions are at least as satisfactory in the case of nitrogen as in the case of helium for the application of our method to the L electrons.) On the other hand, since the distance between a K electron and the nucleus is of the same order as the nuclear collision parameter, p_n, it is impossible to use

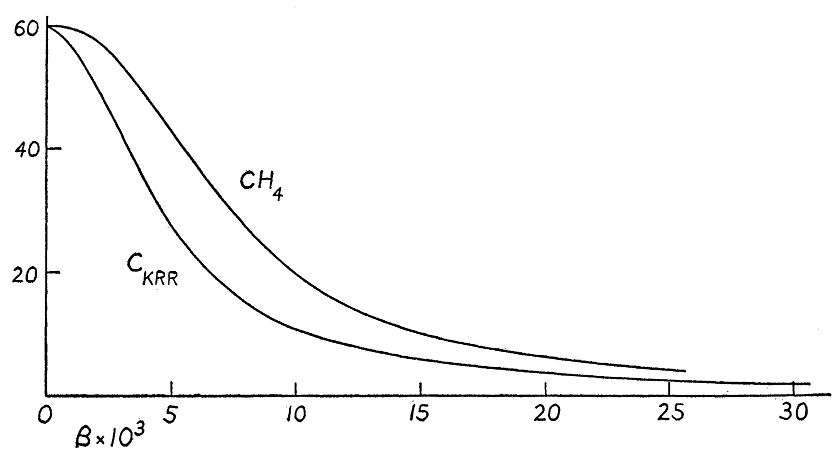

FIG. 4. Distribution of component velocities among the electrons in methane as obtained from the experimental curve shown in Fig. 2, and in atomic carbon as computed by Kirkpatrick, Ross and Ritland. (The latter curve includes the K electrons as well as the L electrons.)

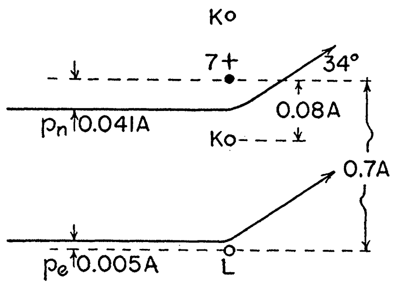

FIG. 6. Relation between the collision parameters for 4000-volt electrons scattered through 34° by nitrogen and the most probable nucleus to atomic electron distances in nitrogen.

our method to determine the velocities of the K electrons by means of 4000-volt electrons. To get the same degree of separation between the effect of a K electron and the effect of the nucleus in the nitrogen atom as we get in the helium atom, we must increase the electron energy by $7/2$ to compensate for the change in nuclear charge (see Eq. (2)), and increase it again by a factor of $0.31/0.08$ to compensate for the smaller distance between the scattering centers with which we are concerned in nitrogen as compared with the corresponding distance in helium. The final result is a factor of 14. Thus 56,000-volt electrons in nitrogen would be necessary to allow us to calculate the velocities of *all* its atomic electrons with the same degree of accuracy as we obtain in helium at 4000 volts. However, if we are content with information about the velocities of the L electrons, then it is quite unnecessary to use electrons of energy 56,000 volts.

In view of the foregoing considerations it may be asked why experiments were attempted with 4000-volt electrons in nitrogen and methane when it was known in advance that the measurements would give no information about the velocities of the K electrons. The answer is that we had available the apparatus with which very dependable results had been obtained with hydrogen and helium and that it seemed well worth getting all the information it could give us about some other gases, even though we might have to forego information about the K electrons. The apparatus was not designed to operate at more than about 4000 volts. However within its operating range it gave excellent results with helium and hydrogen as judged by the remarkable agreement between our results and those obtained by a totally different method using the scattering of x-rays,[6] and as judged by the agreement in the case of helium with Hicks' theoretical computations.[7] These considerations lead us to infer that the apparatus when used for helium and hydrogen yielded unusually dependable values for $f(u)$ and that unrecognized sources of error in the apparatus and method do not exist to any considerable extent. It seemed highly desirable therefore to use the same apparatus, without modification, for the study of other gases. The results so obtained could then be regarded just as dependable as those obtained for hydrogen and helium, even though they are limited to electrons outside the K shell. The results would also have value for comparison with the results obtained with a future apparatus of very different design in which we hope to make use of electrons of much higher energies to secure information about the K electrons.

The investigation on nitrogen and methane cannot be regarded as fully completed until we have as much information about the velocities of the K electrons as we have about the L electrons. All that is necessary—in principle at any rate—is to use electrons with energies of the order of 50,000 volts. However the change from our present apparatus to one suitable for use with 50,000-volt electrons introduces a number of difficult technical problems. In the first place a different kind of electron gun will have to be developed. Difficulties due to high voltage will be much accentuated, and in particular it is doubtful whether our present type of electrostatic analyzer will stand the necessary 16,000 volts. However, a magnetic analyzer would remove this particular difficulty but it has the disadvantage that it does not give directly a distribution in energy. The greatest difficulty of all is in connection with the measurement of the scattered electrons. The number of scattered electrons varies inversely as the square of the energy of the impinging electrons. Moreover we cannot offset this by making the defining slits wider, for, since the inelastic band will be relatively narrower at higher energies, we need at least as good resolution as we have at present. In view of the fact that, with our present apparatus, the number of scattered electrons is so small as to tax our FP-54 tube severely, it seems that we shall be compelled to measure the scattered electrons by a G-M counter tube when we go to much higher energies.

The bands representing the distributions of inelastically scattered electrons, shown in Figs. 1

[6] H. A. Kirkpatrick and J. W. M. DuMond, Phys. Rev. **54**, 802 (1938); J. W. M. DuMond and H. A. Kirkpatrick, Phys. Rev. **52**, 419 (1937).
[7] B. Hicks, Phys. Rev. **52**, 436 (1937).

and 2, are unsymmetrical about the (inelastic) maximum. This is to be attributed to the disturbing effect of the "foot" which makes the elastic peak unsymmetrical and extends from it towards smaller energies. Because of the rapid way it falls off, it has a negligible distorting effect on the *low* energy side of the inelastic band, even though the effect on the *high* energy side is appreciable. The "foot" is probably due to the combined effect of the nucleus and the K electrons on the impinging electrons which pass close to them. Had only the nucleus been involved, the electrons passing close to it would have lost no energy and the elastic peak would have been as narrow as the resolution would allow. The presence of the K electrons in the vicinity of the nucleus modifies the situation. The scattering center is now made up of a nucleus together with two K electrons. Inasmuch as these electrons are of small mass, it is possible for them to be set in motion by the impinging electron thereby taking energy away from it. Thus we may see qualitatively how the complex center may result in an appreciable number of electrons being scattered with some loss of energy, in addition to those which are scattered elastically. The situation is, in fact, very similar to the scattering of low energy electrons (say below 300 volts) by helium atoms since the nucleus and the two K electrons constitute a system exactly like a helium atom. We may summarize the broader features of the experimental results with low energy electrons in helium by saying that most electrons are scattered without any loss of energy (giving the elastic peak) and that the number of electrons losing energy decreases very rapidly as the amount of that energy loss increases. Returning again to our present experiments, this is just what we observe to happen near the elastic peak and which is to be explained by similarity of the inner part of the nitrogen atom to the helium atom. Thus when the two sides of the inelastic band are unsymmetrical, it is clear that the *low* energy side is the one to use in computing the atomic electron velocities.

This use of the low energy side of the inelastic band may also be justified by the following argument. In the earlier work on helium and hydrogen[2] both sides of the inelastic bands obtained with 3000- and 4000-volt electrons were identical and led to the same value for $f(u)$ the distribution of component velocities among the atomic electrons. But with 1000-volt electrons the foot of the elastic peak was large enough to distort the high energy side of the inelastic band, yet the low energy side was unaffected and led to the same values for $f(u)$ as before. This result then gives us an empirical justification for using only the low energy side of an inelastic band when it is unsymmetrical.

In the experiments on hydrogen and helium it was possible to compare the experimental values of $f(u)$ with the values calculated by wave mechanics. Unfortunately no such calculations are available for comparison with our results on nitrogen and methane. We are forced therefore to attempt a comparison with what calculations are available. Kirkpatrick, Ross and Ritland have calculated $f(u)$ values for atomic carbon and atomic nitrogen.[8] We shall compare these with our experimental results for molecular nitrogen and methane. One might perhaps expect a considerable divergence between the experimental $f(u)$ curve for methane and the theoretical $f(u)$ curve for atomic carbon, because of the modifying effect of the four hydrogen atoms. However it is interesting and perhaps significant to find that the experimental $f(u)$ curves for nitrogen and methane obtained in this paper and for hydrogen obtained in an earlier paper are all wider, and by approximately the same amount, than the theoretical curves computed by Kirkpatrick, Ross and Ritland for atomic nitrogen, carbon and hydrogen.[9] The comparisons are shown graphically in Figs. 3 and 4 and in Table I. No further comparison between theory and experiment can be made in this field until

[8] P. Kirkpatrick, P. A. Ross, and H. O. Ritland, Phys. Rev. **50**, 928 (1936). These authors actually express their results in terms of $f(\lambda'')$ which gives the profile of the modified band in the Compton effect, for x-rays of wave-length 695 x.u. scattered through 90°. To convert their computed $f(\lambda'')$ curves into $f(u)$ curves, all that is necessary is to relabel the abscissas in terms of u in place of λ'' by means of Eq. (1).

[9] Some question might be raised as to why we do not make the comparison with the theoretical work of Hicks in the case of hydrogen. While admitting that Hicks' work is probably the more accurate as judged by the better agreement with our previous results on hydrogen, it seemed somewhat more consistent to make the comparison for all three cases with the theoretical curves of Kirkpatrick, Ross and Ritland since all their computations were made according to the same procedure.

theoretical physicists provide us with $f(u)$ values computed for molecules. It is unfortunate that the gases which are relatively easy to study experimentally are very difficult to investigate theoretically.

We take this opportunity of thanking Professor Kirkpatrick for sending us the numerical values of the $f(\lambda'')$ curves for atomic carbon and nitrogen. The senior author takes pleasure in acknowledging the assistance afforded by a grant from the Penrose Fund of the American Philosophical Society.

FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

Scattering of Fast Electrons in Gases

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(Received December 30, 1938)

When sufficiently fast electrons are scattered elastically by atoms, it is possible to regard the effect as being due to the nuclei alone, the atomic electrons playing only a negligible part. The criterion which will allow such a simplification to be made is that the collision parameter for a deflection of an incoming electron by the nucleus must be small compared with the distance between the nucleus and the nearest atomic electron. When this is the case, the observed scattering should be that predicted by the Rutherford scattering formula. Kuper has recently made an experimental investigation of the scattering of electrons of energy above 49,000 volts by helium, neon and argon. His results for helium can be accounted for on the viewpoint proposed, *viz.*, that the nucleus is almost entirely responsible for the observed scattering and that, to a first approximation, the scattering is simple Rutherford scattering. The wave-mechanical formula, which takes into account the effect of the atom as a whole does not give as good agreement with Kuper's results for helium as the Rutherford scattering formula.

THE purpose of this note is to call attention to a simple way of interpreting certain experimental results in the scattering of fast electrons by gases, and in particular to apply it to an important experimental investigation recently published by Kuper.[1] Since an atom is made up of various centers (the nucleus and the atomic electrons), each individually acting on an electron passing through it with a force varying inversely as the square of the distance between the center and the electron, it is natural to regard the scattering of electrons by a single center of force as a first step in discussing the scattering of electrons by an atom. An electron of mass m, charge e and velocity v, on approaching a nucleus whose charge is Ze will be deflected by it. If p_n is the collision parameter, the angle of deflection θ will be given by

$$p_n = (Ze^2/mv^2) \cot (\theta/2). \quad (1)$$

If the scattering center is an electron at rest, the angle of deflection θ is related to the collision parameter by

$$p_e = (2e^2/mv^2) \cot \theta. \quad (2)$$

[1] J. B. Horner Kuper, Phys. Rev. **53**, 993 (1938).

The scattering of an electron passing through the atom is ordinarily the result of the interaction of that electron with several, possibly all, of the scattering centers in the atom. We could visualize the scattering of electrons by an atom as the result of each scattering center in the atom acting on the incoming electrons with an inverse square force. The mathematical difficulties encountered in attempting to solve such a problem are so formidable that no attempt has been made along these lines. The problem has been attacked successfully along very different lines. The electron stream impinging on the atom is replaced by an electron wave of the proper wave-length, and the atom is replaced by a region of suitably varying refractive index. Methods carried over from physical optics lead to scattering formulas. In many cases the results are in excellent quantitative accord with experiment and it may be concluded that, in principle, this wave-mechanical approach is quite satisfactory. In those cases in which quantitative agreement is wanting, it is generally due to technical mathematical difficulties in securing exact solutions of the equations.