On Interpreting Related Magnetic Moments of Light Nuclei

D. R. INGLIS
Rowland Physical Laboratory, The Johns Hopkins University, Baltimore, Maryland
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Under the assumption that neutrons and protons are simple nuclear building stones, endowed with constant spin magnetic moments and bound together by simple short range interactions of an exchange (primarily space-exchange) nature, it is (a) not found possible to explain the order of magnitude of the observed difference between the magnetic moments of Li^6 and the deuteron, without a very forced theory of spin-orbit coupling; (b) not understandable that the N^{14} and Li^6 moments should differ considerably; (c) not possible to explain the difference between the magnetic moments of Li^7 and the proton in the central-field model (with perturbations) within 25 percent, and not possible to reduce the smaller discrepancy in the alpha-particle model to less than four percent, unless the electric polarizability of the triton is much greater than estimated by the central-field model; (d) understandable in the central-field model that the F^{19} nucleus is in a ^2S state with magnetic moment about five percent less than that of the proton, due to slight admixture of neutron spin; (e) possible to understand the observed angular momentum (sign of spin-orbit coupling) of N^{15} in either of the models. The discrepancies (a) and (c) are quite small and suggest especially that the intrinsic spin magnetic moments of protons and neutrons are constant to within a few percent, as does (d). The discrepancy (b) is larger but involves more particles. The discrepancies may plausibly be attributed either to influence of the binding forces on the spin magnetic moments, or perhaps to a term in the interaction containing the angles between the spin directions and the displacement vectors between the particles.

THE new magnetic resonance method[1a] yields direct precision values[1] of nuclear magnetic moments which previously could be measured only indirectly by means of their elusive coupling to atomic moments. The few magnetic moments of the simpler nuclei which are now reliably known[1] fall into two groups: those with an "extra proton" (H^1, Li^7, F^{19}), which have large magnetic moments ($\sim 3\mu_N$); and the "odd-odd" nuclei (H^2, Li^6, N^{14}), with comparatively small magnetic moments ($<\mu_N$). In the present unsatisfactory state of nuclear theory, it is gratifying to have at least this rough confirmation of the simple concept of neutrons and protons as nuclear building-stones, endowed with intrinsic spin magnetic moments, and bound by forces whose saturation is linked to the exclusion principle and to spin compensation. To seek further interpretation may be meaningless, but quantitatively understandable nuclear data are so sparse as to encourage the attempt to find some meaning even in the small differences of the related magnetic moments. We shall ask whether these differences are of a sign and order of magnitude which might plausibly be the result of the simplest assumptions that one may make about the forces and intrinsic moments, or whether they are more reasonably attributed to deviations from the simple assumptions, such as might be expected from a field theory of the forces and intrinsic moments.[2] Although there are other indications of the inadequacy of the simple interactions (perhaps the earliest in reference 3b, the most direct in the recently reported $1d$ quadripole moment of the deuteron), it may be expected that knowledge of their various successes and failures will aid in the selection of the necessary modifications.

Of the representations available to facilitate approximations to a theory, the central-field ("Hartree") model[3] is advantageous for the simplicity with which it deals directly with the assumed interactions between the protons and

[1] (a) I. I. Rabi, J. R. Zacharias, S. Millman and P. Kusch, Phys. Rev. **53**, 318 (1938); (b) **53**, 495 (1938); (c) S. Millman, P. Kusch and I. I. Rabi, Phys. Rev. **54**, 968 (1938); (d) J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Abstract 24 as reported at the Washington meeting of the American Physical Society, December 27, 1938; (e) S. Millman and P. Kusch, Abstracts 27 and 28 as reported at the Washington meeting of the American Physical Society, December 27, 1938.

[2] W. E. Lamb and L. I. Schiff, Phys. Rev. **53**, 651 (1938); and others there cited.

[3] (a) M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937); Erratum: The authors agree that their Eq. (35) should contain $V/(E-E_0)$ quadratically, thereby decreasing their estimated theoretical difference between the Li^6 and N^{14} moments; (b) D. R. Inglis, Phys. Rev. **51**, 531 (1937); (c) Phys. Rev. **53**, 882 (1938). The details of the model there used are relatively unimportant for the magnetic moment of Li^6, which depends on the fact that the ground state is an ^3S, as must follow from almost any model, and on the order of magnitude of the spin-orbit coupling and of the spin dependence of the forces.

neutrons themselves, and because of the possibility of carrying out the second order of a perturbation theory to account in part for the interdependence of the motions of the various particles. The alpha-particle model has the advantage that one may introduce the large binding of the alpha-particle, and other parameters concerning alpha-particles, to begin with, thus avoiding the necessity of accounting for them in the same step which calculates further nuclear properties. The alpha-particle model agrees with experiment better than does the central-field model in two cases in Li^7 where comparison is possible. First, it interprets the 440 kv$=0.8mc^2$ separation of the two observed low states as caused by a difference in "orbital" angular momentum, so as to be compatible with the dependence of the reactions on the velocity of the incident particle.[4] In the central-field model this was interpreted as spin-orbit coupling (doublet separation) of a low p state, which happened to agree with the intensity ratio at the incident velocity used in the earliest experiments.[4] It is noteworthy that the argument originally advanced for the applicability of the central-field model, with perturbations, to Li^6 was based primarily on this now doubtful interpretation—otherwise the minimum that one obtains in the energy, while varying a parameter related to the average distance between particles, may be merely a local minimum (the only one within the range of short distances where the method apparently converges), an absolute minimum at larger distances then corresponding vaguely to the alpha-particle picture.[5] Second, the alpha-particle model gives a value of the magnetic moment of Li^7 more nearly in agreement with experiment than does the central model, as will be discussed further below. In applying the convenient central-field model to $\mu(Li^6)$ nevertheless, we may at least repeat that the truth is likely to lie between the two models, and that the central approximation should have more meaning for Li^6 than for Li^7, judging by the ratio of the internal binding of the deuteron or triton to its binding to the alpha-particle.[5]

[4] L. H. Rumbaugh and L. R. Hafstad, Phys. Rev. **50**, 681 (1936); L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **54**, 657 (1938).

[5] See "discussion of the central approximation" in reference 3b, p. 542.

The Magnetic Moment of Li^6

The accuracy recently attained[1] in the measurement of the magnetic moments of the deuteron and of Li^6 shows that they differ slightly $[\mu(H^2)-\mu(Li^6)\approx 0.03\mu_N]$, although within the previous limits of error they had been equal.[6] The approximate equality of the two moments is surely to be attributed in some way to the stability and nonmagnetic nature of the alpha-particle that must be added to one to produce the other. Insofar as protons and neutrons have constant intrinsic (spin) magnetic moments and insofar as spin-orbit coupling and Coulomb energy are negligible, one would, in fact, expect theoretically that $\mu(H^2)=\mu(Li^6)$ exactly. We propose here to show that the observed difference is too large to be attributed either to the Coulomb energy (judging by the central-field model) or to the usual (implicit) spin-orbit coupling. It is therefore not explained by the usual simple scheme. Whether the difference is chiefly due to an influence of the binding forces on the "intrinsic" magnetic moments of the heavy particles (which would not be surprising[2]) or to an explicit spin-orbit coupling term in the binding interaction, such as a term with a factor $(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})$ suggested by the Yukawa ("mesontron") theory of the forces, is here left undecided.

The ground state of Li^6 in the central-field model[3] is ssp; ssp, $^3S(^2p;\ ^2p)$ (symbols before a semicolon refer to neutrons, after it to protons), and the lowest (i.e., "doubly excited") states which contribute to the magnetic moment in second order are the $^3S(^4P;\ ^2P)$ and $^3S(^2P;\ ^4P)$ arising[3bc] each from each of the excitations $s\rightarrow s'$ and $s\rightarrow d$. The reason that these contributions do not exactly cancel out in pairs is that the state arising from neutron excitation ($ss'p$; ssp, for example) is slightly higher than the corresponding state arising from proton excitation (ssp; $ss'p$), because of their different Coulomb terms.[7] Excited states having different parity from, or the same magnetic moment as, that of the ground state do not contribute.

In an equivalent and even simpler representation, the corresponding states arising from the

[6] J. H. Manley and S. Millman, Phys. Rev. **51**, 19 (1937).

[7] R. F. Bacher, quoted in reference 3a, p. 212.

excitation $s \rightarrow s'$ are $s^+s'^+p^+$; $s^+s^-p^-$ and $s^+s^-p^-$; $s^+s'^+p^+$ (interacting with the ground state $s^+s^-p^+$; $s^+s^-p^+$). The first of these has the spin magnetic moment $\mu_1=\mu_0+2(\mu_\nu-\mu_\pi)$ (where μ_0 is that of the ground state, $\mu_\nu+\mu_\pi$), and the second, $\mu_2=\mu_0-2(\mu_\nu-\mu_\pi)$. The second-order contribution of these two states to the magnetic moment is then*

$$\delta\mu=2(\mu_\nu-\mu_\pi)\{[H_{01}'/(E_1-E_0)]^2 - [H_{01}'/(E_2-E_0)]^2\}.$$

Here the matrix element is $H_{01}' \approx g_0(s'p/J/sp)$, where g_0 (supposedly $\approx 2/9$) is a measure of the spin dependence of the unlike-particle forces. Using $\psi_{s'}=3^{-\frac{1}{2}}(H_{200}+H_{020}+H_{002})e^{-\rho^2/2}$ and (14) of reference 3b, we have

$$H_{01}'=-g_0(f_{2101}+2f_{2000}f_{1010})Bu = 2^{-\frac{1}{2}}g_0(3-5/\tau)Bu/\tau,$$

in the notation and with the assumptions of reference 3bc. Since $E_1-E_0=2\alpha\sigma\approx 53mc^2$, we have

$$\delta\mu=2(\mu_\nu-\mu_\pi)(3-5/\tau)^2(g_0Bu/\tau)^2(E_2-E_1)/(2\alpha\sigma)^3\approx -3\times 10^{-4}((\mathrm{E}_2-E_1)/mc^2)\mu_N.$$

Since (E_2-E_1), the change of the Coulomb energy by proton excitation, is of the order of $-mc^2/10$, this with a few smaller contributions from other pairs of excited states is very much too small to account for the discrepancy—so much too small that the result is probably valid even if the model used is not. The sign of the change arises qualitatively thus: spreading out the protons reduces the Coulomb energy, so excited-proton states lie below the corresponding excited-neutron states in Li^6, proton spins thus contributing more than neutron spins, which would make $\mu(Li^6)>\mu(H^2)$ (since μ_π is positive, μ_ν negative). It is observed,[1] on the contrary, that $\mu(Li^6)<\mu(H^2)$.

* The ground state is an S state, and we consider only that magnetic level with maximum projection $M_S(=1)$. If an excited state a (having $M_{Sa}\neq M_{S0}$ but $M_{La}+M_{Sa}=M_{S0}$) is admixed to the ground state 0 so as to make the wave function $\psi_0+c_a\psi_a$, with $c_a=H_{0a}'/(E_a-E_0)\ll 1$, then the contribution to the magnetic moment is

$$\delta\mu=\int(\psi_0+c_a\psi_a)^*(g_LL_z+g_SS_z)(\psi_0+c_a\psi_a)/\int(\psi_0+c_a\psi_a)^*(\psi_0+c_a\psi_a)-\mu_0=(\mu_0+c_a^2\mu_a)/(1+c_a^2)-\mu_0\approx c_a^2(\mu_a-\mu_0).$$

The property of the excited state is thus substituted for that of the ground state to the extent c_a^2. There is no term linear in c_a (nor bilinear in c_a and c_b) because L_z and S_z are diagonal in this representation.

It has been assumed that, excepting the Coulomb force, the interactions between neutrons are the same as those between protons. This assumed symmetry is based, aside from its aesthetic appeal, both on estimates of the expected neutron excess of heavy elements and on relative binding energies of pairs of light isobars. It seems very unlikely that the interactions could contain a dissymmetry capable of having a much greater effect than has the Coulomb force, *and of opposite sign*, in the present calculation, and yet an effect of the same order of magnitude as has the Coulomb force, and not reversing the sign, in the estimate of the neutron excess of heavy elements and of energy differences of isobaric pairs.

If spin-orbit coupling is neglected, the p neutron and p proton of the central-field model of Li^6 give rise to the states $^{1,3}S, P, D$, of which the ^3S is the ground state. The spin-orbit coupling in its simplest form[8a] $H'=\Sigma a(\mathbf{l}\cdot\mathbf{s})$ has matrix elements between states differing by $\Delta l=\pm 1$, since $\mathbf{l}$ is a vector operator (as is the corresponding operator appearing in its place in Breit's more general theory of spin-orbit coupling[8b]). It therefore mixes the ^1P and ^3P with the ground state. These have the magnetic moments $\frac{1}{2}\mu_N$ and $[\frac{1}{4}\mu_N+\frac{1}{2}\mu(H^2)]$, respectively, or roughly $\mu-\mu_0\approx -\frac{1}{4}\mu_N$. Their elevation in energy above the ground state by a simple space-exchange ("Majorana") interaction is given by Feenberg and Phillips[9] as $E-E_0=2L-K\approx 13K \approx 21mc^2$. (Other values of the constants,[3c] which agree better with the newest scattering data,[10] give $28mc^2$.) This separation can be understood in order of magnitude as follows: In the alpha-particle the forces are saturated and the size of the alpha-particle is essentially the range of the forces. The average value of one interaction in the alpha-particle is thus roughly half of its maximum depth, or about $40mc^2$. The binding energy of each particle in the alpha is about

[8] (a) D. R. Inglis, Phys. Rev. **50**, 783 (1936); (b) G. Breit, Phys. Rev. **51**, 248 (1937); (c) G. Breit and J. R. Stehn, Phys. Rev. **53**, 459 (1938).

[9] (a) E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937); (b) E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

[10] G. Breit, H. M. Thaxton and L. Eisenbud, Abstract 66, as reported at the Washington meeting of the American Physical Society, December 27, 1938, based on data of Tuve, Heydenburg, Hafstad; Herb, Kerst, Parkinson and Plain.

$14mc^2$. The last two particles in Li^6 are more loosely bound—their average binding is about $4mc^2$. The average value of the interaction between them may be expected to be correspondingly smaller, say $-(4/14)40mc^2=-12mc^2$ in order of magnitude. The space-exchange operation on a P wave function introduces a negative sign because of its antisymmetry, so the P state is elevated and the S state depressed by this amount, separating them by about $24mc^2$.

The strength of the spin-orbit coupling, a, depends on the range of the forces in a very sensitive manner, and therefore also on the validity of the model used, and its theoretical evaluation is quite uncertain. It is thus desirable to determine it empirically. The p particles being less tightly bound in Li^6 than in Li^7, a is surely smaller in Li^6 on the central-field model. If this model applies to Li^7 (which seems very doubtful) it does to Li^6, and the $0.8mc^2$ separation in Li^7 is to be interpreted as equal to $(3/2)a$. This gives an upper limit to a in Li^6 as $0.6mc^2$. We take this to be a measure of the nondiagonal matrix element (since orders of magnitude will suffice). We thus have

$$\delta\mu=(\mu-\mu_0)[H'_{0a}/(E-E_0)]^2\approx-2\times10^{-4}\mu_N,$$

which is much too small. The coupling parameter a in H'_{0a} decreases so rapidly with increasing radius that we may assume that the corresponding effect in the deuteron is even smaller.

In Breit's more general treatment, the spin-orbit coupling caused by a space-exchange interaction is a sum, over the various pairs of particles, of terms of the form

$$H'(1,2)=(a^M-\tfrac{1}{2})(\mathbf{A}_{12}+\mathbf{A}_{21})\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+\tfrac{1}{2}(\mathbf{A}_{21}-\mathbf{A}_{12})\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2).$$

Here $\langle\mathbf{A}_{12}\rangle$ is comparable in its vectorial nature and in order of magnitude (when summed over all other particles) with $a\mathbf{l}$ above, and this treatment reduces in effect to the simpler one when $a^M=1$, except that $\mathbf{A}_{12}$ contains a space-exchange operator. In the interaction of a p particle with the s shell, this space-exchange reduces the average interaction, corresponding to the saturation involved in the effective force which determines the constant a in the simpler treatment. Of the $p-p$ interaction $H'(1,2)$ only the space-antisymmetric term (not containing a^M) enters the $^3S-{}^{1,3}P$ nondiagonal elements. Both a and $\langle\mathbf{A}_{12}\rangle$ depend on the radius so sensitively that there is some arbitrariness in their evaluation. It seems that one must always use questionably small radii in order to get the $0.8mc^2$ splitting in Li^7. Using reasonable radii, one has the alternative of supposing that a^M is rather large. The separation is about $(0.05+0.2a^M)mc^2$, with a typical evaluation of the parameters,[3c] so one[8c] would assume $a^M\approx4$. The term in a^M being the larger, we are effectively determining empirically the product $\langle\mathbf{A}_{12}\rangle a^M$ instead of the constant a in the simpler treatment. Since a^M enters linearly (multiplied by $\mathbf{A}_{12}$) in the nondiagonal elements, the possibility of its being large with $\langle\mathbf{A}_{12}\rangle$ correspondingly small does not introduce a danger of increasing the nondiagonal element exorbitantly. Thus the more general form of the spin-orbit coupling will apparently not explain the difference, in the central-field model.

If, on the other hand, the alpha-particle model applies both to Li^6 and to Li^7, the spin-orbit coupling does not cause the $0.8mc^2$ separation in Li^7, but merely a fine structure of these lines. It is smaller than in the previous interpretation by at least a factor five, judging by the sharpness of the lines[4] (and probably very much smaller—see below). This could be expected to keep the calculated difference small in this case. But there is the further possibility that the alpha-model applies to Li^7 and the central model better to Li^6. In this case the value of Breit's parameter a^M could be very large, since $\langle\mathbf{A}_{12}\rangle$ is extremely small for the alpha-model of Li^7. Values of $a^M\approx30$ would suffice to explain the difference $\mu(H^2)-\mu(Li^6)$, if the central model for Li^6 is used. The theory of spin-orbit coupling does, however, seem very forced with such a value of the arbitrary parameter, so we prefer the opinion that the difference arises either from a slight breakdown of the constancy of the intrinsic heavy-particle moments, or perhaps from a $(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})$ term in the Hamiltonian.

The Magnetic Moment of N^{14}

In comparing the magnetic moment of N^{14} with that of Li^6, one meets, intensified, the same difficulty as in comparing Li^6 with H^2. The simple theory would have the two moments equal, but their experimental values[1bc] are $\mu(N^{14})=0.4\mu_N$, $\mu(Li^6)=0.82\mu_N$. The calculation of the difference to be expected as a consequence of the Coulomb dissymmetry proceeds exactly as for Li^6 above. The elevation of the excited states considered is again $13K$, with K about the same as before,[9b] (perhaps slightly smaller, because of the larger size of N^{14}, but of the same order of magnitude). The decrease in the Coulomb energy, when one excites a proton, is about three times as great in N^{14} as in Li^6, since one proton interacts with six others in nitrogen and with two in lithium. The estimated $\delta\mu$ is thus roughly three times as great as above, or $\sim 10^{-5}\mu_N$, less than 10^{-4} of the experimental difference (compare erratum to a previous estimate, in footnote 3). Use of the central model for N^{14} is, however, more questionable than for Li^6.

The Magnetic Moment of Li^7

Now that the new value of the proton magnetic moment $\mu=(2.78\pm 0.02)\mu_N$ has been announced,[1d] one may draw the conclusion from a previous paper[3c] that the theoretical value of $\mu(Li^7)$, calculated by the central field model in second order, with any reasonable parameters in the usual four-term symmetrical Hamiltonian, is not greater than $(2.80+0.33)\mu_N=3.13\mu_N$. The experimental difference $\mu(Li^7)-\mu_\pi \geqslant 0.45\mu_N$ is to be compared with a value $<0.33\mu_N$ calculated by the central model. The value from the alpha-model[11] depends slightly on the way the model is handled, but is in any case almost as great[11b] as the value[3c] $(3/7)\mu_N=0.43\mu_N$ given by a "raindrop" model in which all the particles have the same radius (three out of seven being charged). In the calculation by the central model, the second order gave a very small positive contribution, due to excitations of the s shell, indicating a slight tendency toward the alpha-model. Likewise in the alpha-model, one could introduce a correction due to the fact that a "proton hole" is jumping between the alphas. The wave function of the hole has a node between the two alphas, which means that it is relatively unlikely to be near the center of gravity. This increases the probability that a proton is at the center and tends to reduce the average radius of the protons. Even in the extreme in which one proton is put at the center, the magnetic moment would not be reduced below $(1/3)\mu_N$. The fact that the binding due to the hole $(\alpha+H^3-Li'\approx 5mc^2)$ is only about $\frac{1}{8}$ of the magnitude of the binding of the hole to an alpha $(\alpha-H^3-\pi\approx -40mc^2)$ suggests that this extreme should be weighted rather lightly in estimating the influence of this effect on the magnetic moment. The value of μ_2 in the alpha-model might thus be reduced by perhaps $0.01\mu_N$, indicating again that we have a problem of approximation of which the true answer lies between the two models.

There is one further modification of the alpha-model which increases the calculated μ_L slightly. The Coulomb repulsion of the protons tends to distort the triton and alpha-particle in such a way as to make the average radius of the protons larger than that of the neutrons. It is obvious that neither the radial expansion nor the polarization of the compact alpha-particle is as important as the polarization of the triton, so only the latter will be considered in estimating the order of magnitude of the effect. ("Polarization" here means the displacement of the center of charge from the center of mass, multiplied by the charge.) Treating the field of the alpha at the triton as homogeneous, and estimating the polarizability of the triton by the central-field model, as used in reference 3bc, we will show that the effect is negligible. The perturbation caused by the external electric field F is

$$H'=eFx=eF(\alpha\sigma)^{-\frac{1}{2}}H_1[(\alpha\sigma)^{\frac{1}{2}}x],$$

neglecting a factor $\pi^{\frac{1}{4}}/2^{\frac{1}{2}}$, where $H_1[(\alpha\sigma)^{\frac{1}{2}}x]$ multiplied by the wave function of the ground state 0, 0; 0 is the wave function of the "singly excited" state 0, 0; 1.[3b] The matrix element of H' between these states is thus $H_{0a}'=eF/(\alpha\sigma)^{\frac{1}{2}}$. Here we may put

$$\alpha\approx 22mMc^2/\hbar^2=22[1840/(137)^2](mc^2/e^2)^2 \approx 220(mc^2/e^2)^2,$$

and σ may be determined by minimization of the first-order energy $(3/2)\alpha\sigma-(3-4g_0)B \times[\sigma/(\sigma+2)]^{\frac{3}{2}}$, with $B=92mc^2$ which has a minimum of $-5.3mc^2$ at $\sigma=1.3$. The elevation of the excited state is $E_a-E_0=\alpha\sigma$ so the "Stark-

[11] (a) L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938); (b) H. A. Bethe, Phys. Rev. **53**, 842 (1938).

effect" energy of the triton is $\epsilon=(H_{0a}')^2/(E_a=E_0)=(eF/\alpha\sigma)^2M/\hbar^2$. The electric polarization of the triton is $\partial\epsilon/\partial F$, and, for the sake of orders of magnitude, we may describe it as caused by the displacement δr of a charge e away from the alpha-particle. We thus have

$$e\delta r=\partial\epsilon/\partial F=2(e/\alpha\sigma)^2FM/\hbar^2.$$

F is not larger than $2e/r^2$ with $r=e^2/mc^2$, so we have, as an upper limit,

$$e\delta r=2Me^3(\hbar\alpha\sigma r)^{-2}.$$

The magnetic moment of the displaced charge is increased from $er^2\omega/2c$ to $e(r+\delta r)^2\omega/2c$ by the distortion (taking the center of gravity about at the edge of the triton) so the increase in magnetic moment is

$$\begin{aligned}\delta\mu &\approx 2r\delta re\omega/2c\approx 8Me^3(\hbar\alpha\sigma)^{-2}\omega/2cr\\ &\approx(e^2/220\sigma\hbar c)^2(8M\omega r^2)e/2mc\\ &\approx\{1840/(137\times 220\sigma)^2\}\mu_N\approx 10^{-6}\mu_N,\end{aligned}$$

which is quite negligible. The small polarizability of the triton in this model is associated with the penetration of the fictitious potential barrier by the excited wave function and the consequent high excitation energy. This model does not give a discrete excited state of the triton, although there is some experimental evidence for its existence. That the negligibility of the estimated Coulomb repulsion might be partly a consequence of artificialities of the model is suggested by the following almost plausible estimate in which the excitation energy is due to kinetic energy only. Let the triton be limited by a deep well of radius $r=e^2/mc^2$. The excited state has momentum $p\sim\hbar/r$ and kinetic energy $E_a-E_0\sim\hbar^2/(2Mr^2)$. Its electric matrix element with the ground state is of the order of magnitude $H_{0a}'\sim Fer\sim e^2/r$ and the energy $\epsilon=(H_{0a}')^2/(E_a-E_0)\approx 2Me^2r^4F^2/\hbar^2$. Thus

$$e\delta r=\partial\epsilon/\partial F=4Me^2r^4F/\hbar^2=4(1840/137^2)er=0.4er$$

and the increase in magnetic moment is $\delta\mu\sim er^2\omega/2c=\mu_N$, estimated by this method which ignores the necessity of explaining the small binding of the deuteron. Although this correction due to the repulsion is negligible according to the better estimate, the discrepancy is small in the alpha-model. It is, in fact, of the same order of magnitude as the Li^6-H^2 discrepancy discussed above, so the two may be associated with the same break-down of the simple assumptions (such as the assumption of constant intrinsic moments).

There is no difficulty in interpreting the observed 30-cm group of protons[4, 11] in the reaction $Li^6(d,p)Li^7$ as being caused by an unresolved ^2P of Li^7: the spin-orbit coupling calculated on the alpha-model, using for definiteness the Thomas precession, is much smaller than the observed width of the 30-cm group, as was remarked above. The splitting factor is $\hbar\omega_T$ with $\boldsymbol{\omega}_T=\ddot{\mathbf{r}}\times\dot{\mathbf{r}}/2c^2$. For a circular orbit, or for a proton riding around the center of gravity of Li^7 on a triton, $\ddot{\mathbf{r}}\perp\dot{\mathbf{r}}$, $\dot{r}=\omega r$, $\ddot{r}=\omega^2 r$, so $\omega_T=\omega^3r^2/2c^2$. In the Hartree model one may put roughly $r\approx e^2/mc^2$, and, for a p state, $M\omega r^2=\hbar$, so

$$\begin{aligned}\hbar\omega_T=\hbar^4/(2c^2M^3r^4)&\approx(\hbar c/e^2)^4(m/M)^3mc^2/2\\ &=\tfrac{1}{2}(137^4/1840^3)mc^2=0.03mc^2.\end{aligned}$$

This is only about one-twentieth of the value $(2/3)0.8mc^2$ required by the first interpretation of the separation, so one needs a radius smaller by a factor $20^{-\frac{1}{4}}=\frac{1}{2}$ for this interpretation (or even a factor $60^{-\frac{1}{4}}=0.36$ if one includes the symmetry of the wave function). In the alpha-model the angular momentum is due to all the particles so the angular velocity is much smaller. If in order of magnitude we keep $r=e^2/mc^2$, ω is reduced by a factor $1/7$, for Li^7, making $\hbar\omega_T$ only $10^{-4}mc^2$, about 7^{-3} of the value calculated by the single-particle model. (This estimate might easily be in error by a factor 10, as it is rather sensitive to assumptions about r.) This splitting is, of course, still enormously larger than magnetic energies of nuclear particles in external fields, so excludes any nuclear Paschen-Back effect. The smaller magnetic term in the splitting is correspondingly reduced, and is of the same sign as the term in ω_T for a proton, so the sign of the coupling is unaltered and the ground state has $I=\frac{3}{2}$.

The Magnetic Moment of F^{19}

The experimental value[1b] of the magnetic moment F^{19} is $2.64\mu_N$, differing from that of the proton by $\delta\mu=-0.14\mu_N$. This small difference practically demands that the ground state of F^{19} be a ^2S. The sign of the difference can be understood as due to the fact that the sum of the proton spins is not a perfect constant of the motion, so that the total spin contains a small contribution from the neutron spins (with their negative magnetic moments). This may be described by a perturbation calculation, in which the ground state is a neutron singlet and a proton doublet, the admixed excited state is a neutron triplet and a proton doublet, the total spin being $\frac{1}{2}$ for each state. The magnetic moment is $\mu_0=\mu_\pi$ for the ground state and $\mu_a=(4/3)\mu_\nu-(1/6)\mu_\pi$ for the excited state. The elevation of the excited state is probably caused principally by the fact that the space wave function must be changed from being symmetric to antisymmetric in the two neutrons when one puts the neutron spins parallel. This changes the sign of the leading (space-exchange) term of the interaction between the two neutrons. The nondiagonal matrix element H_{0a}' between the excited state and the ground state arises from the spin-dependent term of this interaction, which is supposedly smaller than the leading term by about a factor $g_0\approx 2/9$. The ratio $H_{0a}'/(E_a-E_0)$ may thus be expected to have the order of magnitude g_0. We may thus estimate

$$\delta\mu=(\mu_a-\mu_0)[H_{0a}'/(E_a-E_0)]^2 \approx[(4/3)\mu_\nu-(7/6)\mu_\pi]g_0^2\approx -0.3\mu_N,$$

agreeing in order of magnitude with experiment.

The fact that the ground state is a ^2S is also understandable in an approximate way. One has less reason to trust the alpha-model here than in the lighter nuclei,[11a] and perhaps more reason to trust the central model beyond the completion of a closed shell. The central model with oscillator potential gives us degenerate s and d states for the last two neutrons and one proton of F^{19}. The configurations giving rise to a singlet-doublet ^2S are then ss, s and dd, d and sd, d, while only the latter would form a triplet-doublet ^2S. The combined singlet-doublet ^2S would be expected to be the ground state for the same reasons of symmetry that must be responsible for the vanishing of the magnetic moments of even-even nuclei. It is of interest to note, however, that a potential which puts s below d, as postulated to simplify the discussion[12] of K^{39} and K^{41}, also leads very simply to a ground state ^2S, and to a smaller estimated $\delta\mu$.

[12] D. R. Inglis, Phys. Rev. 53, 174 (1938).

The Angular Momentum of N^{15}

The magnetic moment of N^{15} has not yet been announced, but the angular momentum has recently been found[13] to be $I=\frac{1}{2}$. This is satisfactory from the point of view either of the central-field model or of the alpha-model. In the central model, which has a p shell closed at O^{16}, the result is due to the simple inversion for an almost-closed shell familiar in atomic spectra: the "hole" in N^{15} has $^2P_{\frac{1}{2}}$ lowest if the proton in Li^7 has $^2P_{3/2}$ lowest. The alpha-model of N^{15} is four alphas with a proton hole jumping between them, in such a way that the nucleus is in a p state. In the extreme case in which the average "orbital" angular momentum is caused almost entirely by nuclear rotation, the hole staying on one alpha, we have simply a triton with spin $\frac{1}{2}$ rotating and accelerated (in a positive electric field) and the sign of the spin-orbit coupling is positive, as in Li^7 and disagreeing with the experimental result. If, on the other hand, the angular momentum is due primarily to the motion of protons between the alphas, the "hole" concept is essential to a description in terms of a one-body problem, and the coupling is negative, agreeing with experiment. In the former extreme case, the angular velocity is smaller than in the latter by about a factor 15 (because the angular momentum is shared by so many particles), so the positive coupling in the former extreme is only about 15^{-3} times as large as the negative coupling in the latter. This suggests that the actual case would have to be extremely close to the former extreme for the actual coupling to be negative, and thus that the calculated coupling would be positive, as observed, unless the alpha-particles retain their identity to a surprising degree in nuclei. Although correlation of binding energies by means of the exchange of the hole is not very exact, it does suggest that the exchange of the hole is associated with only about 1/20 as much energy as the binding of the hole to an alpha ($Q\ll B$ in Fig. 4 of reference 11a, much of the binding being due to the static term R because there is no repulsion between alphas like that between ions in the molecular analog). This may be taken to indicate that the actual case is rather close to the former extreme (which

[13] G. H. Dieke and R. W. Wood, J. Chem. Phys. 6, 908 (1938).

would correspond to no exchange), and casts some doubt on the surmise that the coupling should be positive, in the alpha-model. Since both models agree that the ground state of N_{15} is a ^2P, which is, experimentally, a $^2P_{\frac{1}{2}}$, its magnetic moment is $(2\mu_L - \mu_S)/3$. This is $-0.3\mu_N$ in the central model[3a] with $g_L = 1$, and about $-0.6\mu_N$ in the alpha-model (near the "former extreme") with $g_L \approx 1/2$, and $\mu_S = \mu_\pi$. Since, furthermore, the data on the odd-proton nuclei indicate a tendency[18] for μ_S to be rather near μ_π, especially for the lighter nuclei, and for g_L to be almost unity, one should expect at least to find the N^{15} magnetic moment rather small and negative.

The theory of spin-orbit coupling and the consequent nuclear magnetic moments in the alpha-model will be the subject of a future paper by Sachs, Goeppert-Mayer and Teller.

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The Emission of Secondary Electrons from Metals Bombarded with Protons

JAMES S. ALLEN
Department of Physics, University of Minnesota, Minneapolis, Minnesota
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A study has been made of the yield of secondary electrons from various metals bombarded with protons. For metals that have not been outgassed the secondary electron-proton ratio was about three for protons having energies between 48 and 212 kev. The ratio from outgassed targets of C, Cu, Ni and Pt was about two for protons of the same energies as above. Be gave a yield of about 7.5 electrons per proton. The secondary electron yield was found to be proportional to the cosecant of the angle between the proton beam and the target.

Introduction

IN RECENT years little work has been done on the emission of secondary electrons from metals bombarded with protons. Healea and Chaffee[1] have investigated the secondary emission from thick Ni targets bombarded by protons having energies up to 1600 electron volts. For a hot target the secondary electron-proton ratio increased with the energy of the protons and reached a value of about 22 percent at 1600 volts. Moreover, the ratio increased to 90 percent for targets that had not been outgassed by heating. Schneider[2] has investigated the energy distribution of the secondary electrons ejected by the passage of protons through very thin films of Au and Al. The secondaries produced by 23 and 53 kev protons had a continuous energy distribution with a broad peak at about 20 to 40 electron volts. The total number of electrons ejected from thick targets of Au, Cu and Al placed at an angle of 90 degrees to the proton beam was also determined. The secondary electron-proton ratio for the three metals had a value of approximately four. This ratio did not change as the energy of the protons was varied from 23 to 53 kev. These targets were not outgassed by external heating, but merely by the local heat produced by the proton beam hitting the metal surface.

Because of the lack of information concerning the electron yields from outgassed metals bombarded by protons of greater energies a study was made of the emission from various metals that could be outgassed by heating.

Apparatus

A transformer-kenotron set supplied voltages up to 250 kev to a five-section accelerating tube described by Williams, Wells, Tate and Hill.[3] A resistance-type voltmeter was used to measure the accelerating voltages. After magnetic analysis the proton beam passed through a number of slits into the long Faraday cage shown in Fig. 1. The end of this cage was covered with a glass

[1] Monica Healea and E. L. Chaffee, Phys. Rev. **49**, 925 (1936).
[2] G. Schneider, Ann. d. Physik **11**, 357 (1931).
[3] J. H. Williams, W. H. Wells, J. T. Tate and E. L. Hill, Phys. Rev. **51**, 434 (1937).