Shape of the Domains in Ferromagnetics

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(Received November 28, 1938)

IT is generally agreed that a demagnetized ferromagnetic crystal consists of domains each magnetized uniformly to saturation, the apparent demagnetization being the result of variation in these directions from one domain to another. The domains are estimated to contain 10^{12} to 10^{15} atoms each, but there is no agreement as to their shape. The common assumption that they are roughly cubical can scarcely be justified. It seems worth while to restate the objection to this assumption in a more concrete form; then the condition which must be satisfied by the equilibrium positions of domain boundaries in an ideal cubical crystal will be thrown into a new form.

In a demagnetized iron crystal one-sixth of the domains are commonly assumed to be magnetized to saturation in each of the directions parallel to the three axes. Now if such a domain were more or less round or cubical in form, it would necessarily be surrounded by a local field of the order of I_0, the intensity of magnetization within a domain; this becomes evident upon recalling the formulas for a sphere. This local field might happen to be offset at some points by fields due to other sources, but the cancellation cannot be universal; for over the part of the surface where one domain abuts upon another in which the direction of magnetization differs by a right angle or more, there is a sheet of poles of density roughly equal to I_0, and hence a jump in the normal component of H roughly equal to $4\pi I_0$. In an iron crystal, with $I_0=1800$, the resulting local fields would be of the order of 2000 oersteds. Yet it is observed that an applied field of less than 10 oersteds suffices to magnetize the crystal as a whole to saturation. The conclusion seems inescapable that cubical domains could not be stable; they would tend to alter each other's magnetization so as to become lined up into rows forming long slender domains of larger size. Thus we reach by elementary reasoning the old conclusion that *the domains in iron, and presumably in all ferromagnetic crystals, must have the form either of slender filaments or of thin plates, magnetized longitudinally.*[1] The plate form was suggested for cobalt by Landau and Lifschitz.[2]

The domain boundaries are assumed to be so located as to minimize the total energy due to the Weisz-Heisenberg or quantum field and to magnetic interaction. Considering only cubic crystals, let us assume all fields to be weak enough so that each domain is magnetized sensibly along one axis. Then the quantum energy varies only because there is an excess of it in the domain boundaries, proportional to their area. Presumably there will be a transition layer, through which the direction of the local magnetization varies rapidly from that of one axis to that of another; additional excess energy will be present here due to the occurrence of spins lying oblique to the directions of easiest magnetization in the crystal. These excess energies together are equivalent to a *surface tension* T tending to contract the boundaries.

The effect of the magnetic energy can sometimes be seen directly from the usual expression,

$$U=-\int \mathbf{I}\cdot\mathbf{H}_0 d\tau+\int (\mathbf{H}_\mathrm{I}^2/8\pi)d\tau,$$

where $\mathbf{I}$ is the magnetization and $\mathbf{H}_\mathrm{I}$ the field due to it (both macroscopic, but on the scale of the domains), and $\mathbf{H}_0$ is the field due to all other sources. In the absence of $\mathbf{H}_0$ the magnetic forces will cause the domains to shift so as to obliterate $\mathbf{H}_\mathrm{I}$. But in many situations a criterion involving only conditions at a single point on the boundary is more useful.

To obtain this, suppose that unit area of a boundary, with principal curvatures C' and C'', shifts a distance δs normal to itself. Then by surface-tension theory the boundary energy is

[1] Cf. R. H. de Waard, Phil. Mag. **4**, 641 (1927); F. Bloch, Zeits. f. Physik **74**, 295 (1932).
[2] L. Landau and E. Lifschitz, Physik Zeits. Sowjetunion **8**, 153 (1935).

increased by $(C'+C'')T\delta s$. The increase in magnetic energy is

$$\delta U = -\int \mathbf{H}_0 \cdot \delta \mathbf{I} d\tau - \tfrac{1}{2}\delta \int \mathbf{I} \cdot \mathbf{H}_\mathrm{I} d\tau = -\int \mathbf{H} \cdot \delta \mathbf{I} d\tau,$$

where $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_\mathrm{I}$, and the last expression results from the following transformations, in which we introduce the potential due to $\mathbf{I}$, so that $\mathbf{H}_\mathrm{I} = -\nabla\Omega$, $\nabla^2\Omega = -\mathrm{div.}\ \mathbf{H}_\mathrm{I} = 4\pi\ \mathrm{div.}\ \mathbf{I}$, and then carry out integrations by parts as in the divergence theorem, the integrated terms vanishing always at infinity:

$$\int \mathbf{I} \cdot \delta \mathbf{H}_\mathrm{I} d\tau = -\int \mathbf{I} \cdot \nabla \delta\Omega d\tau = \int \delta\Omega\ \mathrm{div.}\ \mathbf{I} d\tau$$

$$= \int \delta\Omega \nabla^2 \Omega d\tau / 4\pi = \int \Omega\ \nabla^2 \delta\Omega d\tau / 4\pi$$

$$= \int \Omega\ \mathrm{div.}\ \delta \mathbf{I} d\tau = \int \mathbf{H}_\mathrm{I} \cdot \delta \mathbf{I} d\tau.$$

Now at any point in the transition layer $\delta I_\perp = -I_\perp' \delta s$, $\delta \mathbf{I}_{\parallel} = -\mathbf{I}_{\parallel}' \delta s$; subscripts denote, respectively, the perpendicular component in the direction of δs and the vector parallel component, and primes indicate the space derivative. Hence

$$\delta U = \delta s \int (\mathbf{H}_{\parallel} \cdot \mathbf{I}_{\parallel}' + H_\perp I_\perp') dz$$

integrated through the transition layer. But $\mathbf{H}_{\parallel}' = 0$, and $H_\perp' = -4\pi I_\perp'$ by Gauss' law; thus, if subscripts 2 and 1 indicate values on the two sides of the boundary,

$$\int \mathbf{H}_{\parallel} \cdot \mathbf{I}_{\parallel}' dz = \mathbf{H}_{\parallel} \cdot \int \mathbf{I}_{\parallel}' dz = \mathbf{H}_{\parallel} \cdot (\mathbf{I}_{\parallel 2} - \mathbf{I}_{\parallel 1});$$

$$\int H_\perp I_\perp' dz = -\int H_\perp H_\perp' dz / 4\pi$$

$$= -(H_{\perp 2}^2 - H_{\perp 1}^2)/8\pi = \tfrac{1}{2}(H_{\perp 2} + H_{\perp 1})(I_{\perp 2} - I_{\perp 1}).$$

Hence we can write, since $\mathbf{H}_{\parallel} = (\mathbf{H}_{\parallel 2} + \mathbf{H}_{\parallel 1})/2$,

$$\delta U = \tfrac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot (\mathbf{I}_2 - \mathbf{I}_1) \delta s.$$

The condition for equilibrium is now that the total change in energy must vanish, which requires that

$$\tfrac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot \mathbf{I}_1 = \tfrac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot \mathbf{I}_2 + (C' + C'')T.$$

If the quantity on the left in this equation exceeds that on the right, the boundary shifts away from medium No. 1, and vice versa. (C' and C'' are positive when they mean convexity toward domain No. 2.) A *plane* boundary, therefore, shifts away from the side on which $\frac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot \mathbf{I}$ is the greater, and is in equilibrium when

$$\tfrac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot \mathbf{I}_1 = \tfrac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) \cdot \mathbf{I}_2.$$

Here $\frac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2)$ represents the mean field, due to all other sources than free poles on the part of the boundary under consideration; for equilibrium of a *plane* boundary this mean field must be equally inclined to $\mathbf{I}$ in the two adjacent domains.

The magnitude of the surface tension is not definitely known. According to Bloch[3] the transition layer between domains represents a compromise between the tendency of the quantum forces to decrease the gradient of the local magnetization and the tendency of the crystal forces to turn all spins into the direction of a crystal axis, and T should be of the order of the work required to turn them halfway from one axis to

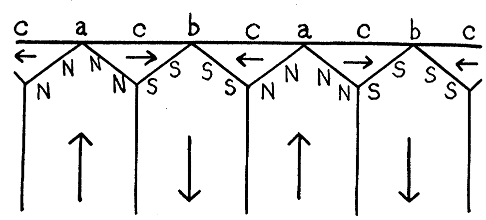

FIG. 1.

another in a layer about 30 crystal spacings thick. In iron this would make $T = 2 \times 10^5 \times 30 \times 3 \times 10^{-8} = 0.18$ erg/cm^2, roughly. If the surface tension is really of this order of magnitude, the shape of the domain boundaries must be determined largely by the local magnetic fields.

The rules obtained above are useful in making qualitative studies. It is readily verified that cubical domains would tend, because of magnetic

[3] F. Bloch, *Handbuch der Radiologie*, Vol. 6, Part 2 (1934), p. 481.

interaction, to change as suggested above. Equilibrium in an ideal crystal seems, moreover, to require that the domains extend to the surface; for any domain ending in the interior tends to shrivel up, both because of the surface tension and because of magnetic action. The field due to the domain itself tends to collapse its sides, and the field due to the poles at one end tends to pull in the other end. In an actual crystal, however, there appears to exist a certain resistance to the motion of domain boundaries, so that it might be sufficient for a domain to minimize its own magnetic field by tapering very gradually to a point, or to a sharp edge.[4]

At the surface, again, the magnetization will adjust itself so as to keep the local fields within bounds. A possible arrangement below a surface perpendicular to one crystal axis seems to be as suggested in Fig. 1. If the surface tension were absent, the magnetization, shown by arrows, would be carried round through triangular prisms lying parallel to the surface, with their oblique faces at 45° to the axis (as suggested for cobalt by Landau and Lifschitz[2]), and there would be no free poles at all. In reality, however, the boundaries must be pulled up a little by their surface tension, with the production of free poles as shown. The local fields due to such poles might be the cause of the powder patterns that are often observed; a magnetic colloid would collect in the regions of strongest field at $ccc\cdots$. If an external field were then applied in the upward direction, the strongest field might be shifted to $aa\cdots$, and the powder ridges would occur there; whereas in a reverse field they would be at $bb\cdots$. Thus the ridges in the presence of a field would be in locations halfway between the no-field positions and twice as far apart; this is just what is observed.[5]

The application of the field would also change the domains somewhat, of course. In the figure an upward field would broaden those under $aa\cdots$ and should probably tend to make their tops dome-shaped, with an increase in the pole strength; whereas the tops of the domains under $bb\cdots$ would become more peaked, with a decrease in the pole strength there, or even a reversal of its sign. These changes would intensify the phenomena just described, at least so long as the field did not become too strong.

Even if the effect of surface tension is negligible, an imperfection in the triangular prisms might result from the resistance which is observed to impede the motion of domain boundaries. It is not so easy, however, to understand how the plate-like domains can turn square corners as freely as they must do to fit the powder patterns. Furthermore, isotropic demagnetization is hard to imagine, for spindles and plates lying in all three directions can scarcely fit together without gaps. In a demagnetized crystal, do the shortest dimensions of all domains perhaps lie in the same plane? If so, the observed magnetization curve should differ according as the crystal is demagnetized by use of a parallel or a perpendicular field. This point might be worth testing experimentally.

[4] K. J. Sixtus and L. Tonks, Phys. Rev. **37**, 930 (1931).

[5] W. C. Elmore, Phys. Rev. **51**, 982 (1937); **53**, 757 (1938).