The Temperature Distribution in the High Pressure Mercury Discharge Tube

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Several arguments are given which support the view that the temperature distribution of the vapor in the high pressure mercury discharge tube is not that given by Adams and Barnes (line A_g of Fig. 1) but has a shape as indicated by the line B_g of Fig. 1.

IN TWO recent articles Barnes and Adams[1] have concluded that the temperature distribution of the vapor in the high pressure mercury arc is as indicated by the line A_g in Fig. 1, in contradiction with the distribution B_g which I proposed some years ago.[2] A_g indicates that the temperature of the vapor is substantially constant over the whole cross section except in the boundary layer which is about 0.8 mm thick (in analogy to the stationary film of Langmuir).[3] The curve B_g was found[2] from the condition that

$$\int_0^R \frac{2\pi r dr}{T(r)}$$

must have a given value (pressure and mean density were known) and from a plausible assumption on the dependence of the coefficient of heat-conduction on temperature (the absolute value was not used in the calculation). As the convection current in the vertical mercury tube is substantially laminar and vertical, it has only little influence on the horizontal temperature distribution (we do not consider the case that the convection is turbulent).[4]

The distribution A_g agrees with the first condition as well. By assuming a constant temperature over the total cross section one finds from the same data which led to the distribution B_g, a temperature of $555/0.223=2500°\text{K}$,[2] which is in agreement with the ditribution A_g of Barnes and Adams. The difference between the two curves thus arises from the second condition according to which I used the equation of heat-conduction (energy streaming through a cylinder one cm high is $2\pi r\lambda dT/dr$) whereas Adams and Barnes use the theory of Langmuir. Furthermore, the distribution of the electron temperature, according to Barnes and Adams, is as substantially indicated by the line A_e, whereas I supposed the electron temperature (curve B_e) to be only slightly higher than the gas temperature, so that calculations can be made on the assumption that a temperature equilibrium between the gas molecules and the electrons exists in the arc.

The difference between the two conceptions is so considerable that we wish to state the following objections against that of Adams and Barnes:

(a) Barnes and Adams find the intensity ratio for a number of spectral lines to be the same in the axis and 8 mm aside (inner tube diameter 36 mm); from this fact they conclude the temperature to be the same in both points. However, this does not agree with my own measurements[5] and with more extensive recent measurements of Fischer and König,[6] who find that the intensity of a line drops more rapidly if its initial level is higher. This indicates that the excitation temperature is a maximum in the axis. As the points through which Barnes and Adams have to draw a straight line, whose slope determines the temperature, scatter much, it is possible that the temperature difference between the axis and 8 mm aside, has escaped their attention.

(b) Even if the temperature in the arc path were uniform, we may not conclude that the temperature over the total cross section (except the boundary layer) is constant (diameter of measured portion 16 mm and PQ about 34 mm).

(c) Langmuir's theory of the film of stationary gas around a heated wire cannot be applied to the

[1] B. T. Barnes and E. Q. Adams, Phys. Rev. **53**, 545 (1938); E. Q. Adams and B. T. Barnes, Phys. Rev. **53**, 556 (1938).
[2] W. Elenbaas, Physica **1**, 211 (1934).
[3] I. Langmuir, Phys. Rev. **34**, 401 (1912).
[4] W. Elenbaas, Physica **3**, 484 (1936). C. Kenty, J. App. Phys. **9**, 53 (1938).
[5] W. Elenbaas, Revue d'optique **15**, 343 (1936).
[6] E. Fischer und H. König, Physik. Zeits. **39**, 313 (1938).

high pressure arc, where the energy is conducted to the surrounding solid part (tube wall) instead of from the solid body (the wire) to the surrounding gas, as in the case studied by Langmuir. This is caused by the fact, that in the case of the wire there can exist a practically uniform temperature field (namely room temperature or temperature of the bulb) beyond the stationary film (by the upward stream of the cold convection gas), but inside the mercury tube there must be a temperature gradient when energy flows to the walls (unless the gas is in a rapid and irregular motion in the part PQ (Fig. 1)); this is, however, not the case as stated under (d). The temperature distribution A_g would therefore only be possible if no energy were transformed into heat in the region between P and Q, but only on the edge PQ. This is very unlikely, as the arc burns in the central part of the tube.

(d) Convection is observable on small particles which are moving along with the vapor.[4] They indicate a chiefly laminar streaming which goes upwards in the central part of the discharge and downwards in the surrounding part. The convection cannot cause a uniform temperature across the diameter PQ, as for that purpose only radial components, which practically do not exist, would be useful. If the temperature distribution were as indicated by A_g the convection would be upward in the whole part PQ. Actually the particles are moving downward at a much greater distance from the wall. This phenomenon cannot be caused by electrical forces, as it is the same with alternating and direct current.

(e) For the energy which is conducted to the wall, Barnes and Adams calculate from the Langmuir theory 32 watts per cm of length. The total radiation according to Barnes and Adams is therefore as a maximum $40-32=8$ watt or 20 percent of the input. The total radiation in the above case, however, is greater. For the total radiation S per cm of length of the high pressure mercury discharge in a quartz tube we found in a wide range of diameters and pressures:[7]

$$S=0.7(L-10) \text{ watt}, \tag{1}$$

where L is the input per cm of length. Applying this to the tube of 20 mm diameter, mentioned by Barnes and Adams, on which I did the measurements,[2] we find $S=0.7(40-10)=21$ watt ($L=40$ and not 43.8 as stated in reference 2, where the energy losses at the electrodes had not been taken into account). Therefore, at most only 19 watts could be conducted to the wall. The interpretation given of Eq. (1) leads even to a smaller amount of energy which is conducted to the wall, namely 10 watts per cm of length (we supposed that of the input L, 10 watts are conducted to the wall and $L-10$ watts are radiated, of this radiation only 70 percent passes the quartz wall). This energy of 10 watts is in agreement with measurements on the influence of

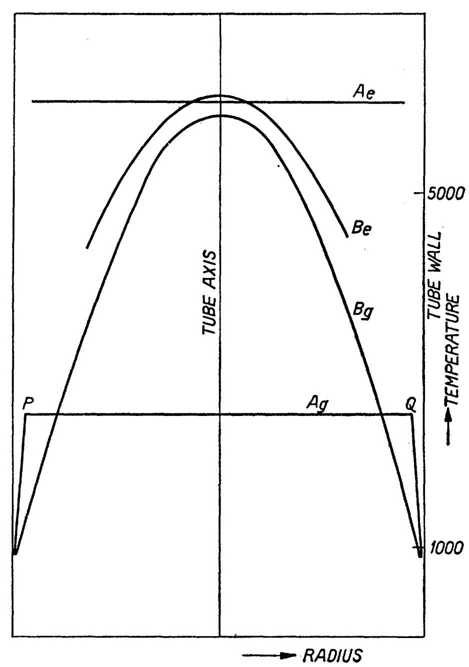

FIG. 1. Gas temperature A_g and electron temperature A_e as functions of the distance to the axis according to Barnes and Adams. B_g and B_e are the corresponding temperatures according to the author.

noble gases on the mercury arc[8] and with the behavior of the gradient as a function of L at a given mean vapor density.[9] If we put this energy of conduction in the formulas of reference 2, we find for the coefficient of heat conduction λ for mercury vapor $2.1\times10^{-7}T^{3/4}$. Extrapolating to 476° (203°C) we find a value of 21×10^{-6} for λ, whereas a value of 18.5×10^{-6} has been measured

[7] W. Elenbaas, Physica **4**, 413 (1937).

[8] W. Elenbaas, Physica **3**, 219 (1936).

[9] W. Elenbaas, Physica **2**, 757 (1935).

by Schleiermacher.[10] Thus the loss of 10 watts by conduction *is in agreement with the slope of curve* B_g. As the convection currents in this tube were laminar, the energy loss by convection may be neglected (only at the first part from the bottom there appears an additional loss caused by the heating up of the colder convection gas).

(f) According to Adams and Barnes the energy of 32 watts, which is conducted to the wall per cm of length, can be dissipated just at a temperature of 900°K (outer diameter of the tube 22 mm). With a temperature of 900° they calculate a loss of 5.3 watts by conduction and convection and a loss of 25.6 watts by radiation. They obviously treat the glass wall as a black body. As the maximum of the black body radiation of 900°K lies at 3.2μ and the transmission of most glasses is good till about this wave-length, the emissivity of glass must be considerably smaller than one at this temperature. If we assume that of the 21 watts which are radiated by a quartz tube (formula (1)), 18 watts pass the glass wall (the tube wall was of thin, ultraviolet-transmitting glass), the tube wall has to dissipate 22 watts per cm of length. Subtracting 5.3 watts for conduction and convection, 17 watts remain for radiation. At 875° (actually measured temperature) an emissivity of 75 percent is needed hereto.

(g) A maximum value for the difference between the electron temperature T_e and the gas temperature T_g may be calculated according to Druyvesteyn[11] in the following way: The energy lost by the electrons per cm^3 and per sec. by elastic collisions with the Hg atoms, is according to Cravath[12] (with a few immaterial neglections)

$$Q=8n_e n_a \sigma^2 kT_e\left(\frac{2\pi kT_e}{m}\right)^{\frac{1}{2}}\frac{m}{M}\left(1-\frac{T_g}{T_e}\right). \quad (2)$$

n_e and n_a are the number of electrons and Hg atoms per cm^3, σ the radius of the Hg atoms and m and M the mass of the electron and the Hg atom. We can find n_e from the mobility equation:

$$i=\frac{3n_e e^2 G}{8\sigma^2 n_a(2\pi mkT_e)^{\frac{1}{2}}}, \quad (3)$$

where i is the current density and G the gradient.

Substituting (3) in (2) and putting $n_a = n_0(273/T_g)$, if n_0 is the number of atoms per cm^3 at a pressure of one atmos. and at 0°C (the pressure in the above-mentioned tube was one atmos.), we find as $Q<iG$

$$\left(\frac{T_e}{T_g}\right)^2-\left(\frac{T_e}{T_g}\right)<\frac{3e^2G^2M}{128\pi(273)^2\sigma^4k^2n_0^2m}. \quad (4)$$

If we take for $\pi\sigma^2$ the value 5×10^{-15} and for $G=8$ volt/cm (measured value), we find 0.17 for the right side of Eq. (4). This gives $(T_e/T_g)<1.15$. As only a fraction of the input will be transferred to the atoms by elastic collisions, the real difference between electron temperature and gas temperature will be smaller than 15 percent. The only uncertain quantity in (4) is σ. The value 5×10^{-15} for $\pi\sigma^2$ may be wrong to a factor 3.[13] Taking $\pi\sigma^2$ three times smaller we find $(T_e/T_g)<1.83$, whereas Barnes and Adams need at least a value of 2.4 (6000/2500). If, however, the electron temperature is 6000° and the vapor temperature 2500°, the total excitation is practically caused by the electrons, so that the energy transferred to the atoms by elastic collisions, will be smaller than $\frac{1}{2}iG$ (the measured total radiation is 50 percent of the input, Eq. (1)). A ratio of 2.4 for T_e/T_g, therefore corresponds to a value of $\pi\sigma^2<0.8\times10^{-15}$, which seems to be a very improbable value.

For all these reasons we believe that it is much more likely that the temperature distribution is as indicated by the line B_g than as indicated by the line A_g.

[10] A. Schleiermacher, Wied. Ann. **36**, 346 (1889).
[11] Unpublished. See also R. Mannkopff, Zeits. f. Physik **86**, 161 (1933).
[12] A. M. Cravath, Phys. Rev. **36**, 248 (1930).
[13] W. Elenbaas, Physica **5**, 573 (1938).