Hyperfine Structure of Manganese I and Nuclear Magnetic Moment

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New measurements of the structure of the green lines of Mn I combined with the data already available make possible a determination of 24 hyperfine interval factors. Interval factor formulas derived by the method of energy sums are in reasonable accord with much of the data. This permits an approximate determination of the nuclear magnetic moment of Mn⁵⁵, which is found to be 3.0 nuclear magnetons.

INTRODUCTION

 ${f M}^{{\it EASUREMENTS}}$ on the hyperfine structure of the visible lines in the arc spectrum of manganese were first reported in 1909 by Janicki¹ who measured twenty-one structures using a Lummer Plate. These lines and others have since been remeasured by Wali-Mohammad,² by White and Ritchl³ and by Wali-Mohammad and Sharma.⁴ White and Ritchl, whose measurements are the most extensive gave an analysis of the hyperfine structure showing the nuclear spin to be $2\frac{1}{2}$ units. They reported values for the total separations of several of the hypermultiplets and pointed out that the prominent splittings are associated with configurations in which an unbalanced s electron occurs. Using the vector model of the atom they developed formulas for the separations expressing the intervals in terms of coupling factors related to the individual electrons of the configuration. Following White and Ritchl's work Wali-Mohammad and Sharma measured many of the lines used by White and Ritchl in their analysis and reported disagreements in some of the measured structures. We have remeasured a number of the lines in the green region concerning whose structure the existing data seemed least conclusive. An analysis of these data along with those of the previous observers yields values for the hyperfine intervals differing in some details from those obtained by White and Ritchl and permits calculations leading to an approximate value for the nuclear magnetic moment.

EXPERIMENTAL DATA

Table I gives the results of our measurements upon thirteen lines in the green region together with the data of the previous observers. The interval in wave numbers separating each measured component from the strongest line in its pattern is always given. The data of the other observers have been converted to the same units. Since many of the structures are only partially resolved, the measured positions may refer only to points of maximum intensity in the pattern. Components and maxima are designated by numbers specifying their relative intensities in the pattern. When a structure terminates in an unresolved "tail" the terminus is designated by an e in the intensity column. White and Ritchl also have indicated the extent of unresolved patterns.

For our observations the manganese was excited in helium in a water-cooled metal hollow cathode discharge tube. Resolution was by means of a ruled Fabry-Perot interferometer⁵ and small Hilger glass prism spectrograph. Interferometer spacers of 2.37, 5.28 and 10.03 mm were employed. Three or more spectrograms were taken with each spacer so that each pattern reported in Table I was verified by its appearance under different resolutions and in different intensities, with the exception of the lines $\lambda\lambda 5399.5$ and 5413.7 which were measurable on only two plates because of their low intensities.

It may be seen (Table I) that in most cases in which the new measurements differ materially from the older the new observations reveal additional components not before resolved. This

¹L. Janicki, Ann. d. Physik 29, 833 (1909)

 ² Wali-Mohammad, Astrophys. J. 39, 185 (1914).
 ³ H. E. White and R. Ritchl, Phys. Rev. 35, 1146 (1930).
 ⁴ Wali-Mohammad and P. N. Sharma, Phil. Mag. 18, 1144 (1934).

⁵ R. C. Machler and R. A. Fisher, J. Opt. Soc. Am. 25, 315 (1935).

is illustrated by the first line of the table, λ 5537.8, in which the pair of components at 0.129 and 0.221 cm⁻¹, respectively, were observed as a single component in an intermediate position by Wali-Mohammad and by Wali-Mohammad and Sharma. The same is true of the pair at 0.320 and 0.377 cm⁻¹. White and Ritchl did not resolve structure in this line and so gave only the initial and terminal points of the pattern. The most serious disagreement in data⁶ appears in con-

 6 The close grouping of lines in the vicinity of 5400A has led to some confusion in their identification in the

nection with $\lambda 5341.4$ for which White and Ritchl reported the same pattern as we, but upon a scale some 10 percent larger while Wali-Mohammad and Sharma found a similar pattern on a

interferograms. It is evident from Janicki's published spectrograms as well as from his measured separations that the three relatively strong lines identified by him as $\lambda\lambda 5394.7$, 5399.5 and 5407.4, respectively, are in reality $\lambda\lambda 5399.5$, 5407.4 and 5420.4, respectively. Wali-Mohammad apparently followed Janicki's example in this identification. We have taken the liberty of shifting these measurements by both Janicki and Wali-Mohammad in the table, thus bringing them into close agreement with those of the later observers.

	$\Delta \nu (\mathrm{cm}^{-1})$				$\Delta \nu (\mathrm{cm}^{-1})$										
λ(Α)	INT	New	M&S	W&R	М	J	λ(Α)	Int	New	M&S	W&R	М	J		
5537.8	6 3 3 2	0.000 .129 .221 .320	0.000 .154 .346	0.000	0.000 .150 .340		5413.7	1 6 2 2 e	-0.10 .000 .144 .237 .275		0.000				
5516.8	$ \begin{array}{c c} 1 \\ 6 \\ 1 \\ 1 \\ 3 \\ \end{array} $.376 .000 .061 .150 .220	.000	.000	.000	0.000	5407.4	5 5 3 2 e	.000 .182 .345 .486 .623	0.000 .190 .506	.000 .190 .341 .556	0.000 .185 .357 .505	0.000 .189 .350 .490		
	1 2 5	.264 .372	.247	.240	.230	.240	5399.5	$\begin{array}{c c}1\\4\\1\\3\end{array}$	114 .000 .109 .230			.000 .234	.000 .223		
5505.9	$\begin{vmatrix} 3\\3\\2\\1\\e \end{vmatrix}$.149 .270 .349 .408		.152 .295 .392	.154 .290	.154	5394.7	8 6 4 3 2	$\begin{array}{r} .000 \\114 \\217 \\306 \\383 \end{array}$.000 112 210 289 357				
5481.4	$\begin{array}{c c} 4\\ 3\\ 2\\ 1\\ e \end{array}$.000 .207 .373 .463 .516	.000 .206 .373	.000 .221 .391 .512	.000 .206 .383	.000 .217 .407	5377.6	$ \begin{array}{c} 2 \\ 1 \\ 1 \\ 6 \\ 3 \end{array} $	$410 \\098 \\ .000 \\ .101$		395	.000	.000		
5470.7	4 3 3 e	.000 .191 .350 .516	*****	.000 .182 .327 .495	.000 .184 .347	.000 .187 .351		2 6 5	.130	.000 .161 309	.000 .221 406	.130 .000 .203 .376	.000		
5432.6	6 5 4 3 e	$\begin{array}{r} .000 \\106 \\195 \\266 \\310 \end{array}$		$\begin{array}{r} .000 \\105 \\186 \\254 \\298 \end{array}$			5341.1	5341.1	5341.1	$\begin{vmatrix} 4\\3\\3\\2\end{vmatrix}$.514 .620 .680	.428 .530	.100 .560 .680 .762	.523	.523
5420.4	5 4 3 2 1 <i>e</i>	.000 .190 .341 .440 .510 .59		.000 .197 .356 .475 .627	.000 .185 .356 .503	.000 .195 .359 .492									

TABLE I. Hyperfine structure of green lines of Mn I according to different observers.*

* M&S indicates data of Wali-Mohammad and Sharma; W&R indicates data of White and Ritchl; M indicates data of Wali-Mohammad : J indicates data of Janicki.

λ(Α)	$\Delta \nu (\mathrm{cm}^{-1})$	λ(Α)	Δν(cm ⁻¹)
4018.2	0.0000 .1188 .2147 .2946 .3565	4048.8	0.0000 .0890 .1653 .2251
	.4029e	_	.0000 .0742
4030.7	$\begin{array}{r} .0000 \\0930 \\1724 \\2352 \\ 2826 \end{array}$	4055.5	.1605 .2207 .2633 .2925e
	3138	4063.5	.0000 .0619 .1426
4033.1	0873 1654 2207		.0000
4034.5	$\begin{array}{c}2576 \\ \hline 0.0000 \\ 4034.5 \\1610 \\2083 \\ \hline 0.0000 \\ 0.0829 \\ 4035.7 \\ .1805 \\ .2462 \\ .3113e \end{array}$	4079.2	.0834 .1494 .1986 .2616e
		4079.4	.0000 .1182
4035.7		4082.9	.0000 .0672 .1218 .1638
4041.4	.0000 .1010 .1867 .2577	4083.6	.0000 .0654 .1956e
	.3146 .3513		

TABLE II. Hyperfine structure of violet lines of Mn I from measurements of White and Ritchl.*

* White and Ritchl gave intervals between successive components in $\rm cm^{-1}$. These intervals have been added to give positions with respect to the strongest component of the pattern. *e* following a position indicates the end of the pattern rather than a component.

scale 17 percent smaller. The new measurements on this line are, however, in close agreement with those of Janicki and of Wali-Mohammad.

Other data of use in this analysis are the excellent measurements of White and Ritchl upon the group of lines in the vicinity of 4000A listed in Table II. The measurements of other observers in this region are too fragmentary for inclusion in the table. White and Ritchl have also given microphotometer curves depicting the complex unresolved patterns found for $\lambda\lambda4754.1$, 4783.4 and 4823.5 with a wave-length scale.

ANALYSIS OF HYPERFINE STRUCTURE

The term classification of the Mn I lines⁷ here of interest is shown by Table III. The wavelengths of the lines appear within the rectangles and the terms from which they originate along the margins; even terms are above and odd terms to the left.

All of the information now available is in complete accord with White and Ritchl's assigned value for the nuclear spin, $I=2\frac{1}{2}$. Accepting the value of I as established and assuming that the interval rule for hypermultiplets is obeyed within the limits of observation, we may proceed to apply the graphical method of hyperfine structure analysis described by Fisher and Goudsmit.⁸ The scheme employed throughout the analysis of the twenty-nine hyperfine structures has been to construct for each pattern what may be called a "pattern graph" representing the observed positions of components on a varying scale by means of properly spaced diverging pencil lines. This pattern graph which is on semi-transparent paper is then laid over the appropriate analysis graph and shifted about until the position is found at which the corresponding lines of the two graphs coincide. This determines the ratio between the interval factors of the two hypermultiplets. The relation of the two graphs for λ 5537.8 is illustrated in Fig. 1.

TABLE III. Classification of Mn I lines.

	J	$\begin{array}{c c} 3d^{6} \ 4s \ {}^{6}D \\ 4\frac{1}{2} & 3\frac{1}{2} \end{array}$	2 ¹ / ₂	$1\frac{1}{2}$	$\frac{1}{2}$
3d ⁵ 4s(⁵ S)4p ⁶ P ⁰	$\begin{array}{c} 1\frac{1}{2} \\ 2\frac{1}{2} \\ 3\frac{1}{2} \end{array}$	5420.4 5341.1 5407.1	5481.4 5470.7	5516.8 5505.9	5537.8
3d ⁶ 4p ⁶ D ⁰	$\begin{array}{c} 4\frac{1}{2} \\ 3\frac{1}{2} \\ 2\frac{1}{2} \\ 1\frac{1}{2} \end{array}$	4041.4 4079.2 4018.2 4055.5 4035.7	4083.6 4063.5 4048.8	4082.9	4079.4
<u>ar</u>	J	. 3d ⁵ 4s ² ⁶ S ₂		3d ⁵ 4s 5s	8.S31
3d ⁵ 4s 4p ⁸ P ⁰	$\begin{array}{c} 2\frac{1}{2} \\ 3\frac{1}{2} \\ 4\frac{1}{2} \end{array}$	5432.6 5394.7		4754. 4783. 4823.	1 4 5
3d ⁵ 4s(⁷ S)4p ⁶ P ⁰	$\begin{array}{c}1\frac{1}{2}\\2\frac{1}{2}\\3\frac{1}{2}\end{array}$	4034.5 4033.1 4030.7			
	J	3d	5 4s 5s 4S	11	
3d ⁵ 4s 4p ⁴ P ⁰	$2\frac{\frac{1}{2}}{1\frac{\frac{1}{2}}{\frac{1}{2}}}$	5377.6 5399.5 5413.7			

⁸ R. A. Fisher and S. Goudsmit, Phys. Rev. 37, 1057 (1931).

⁷ See R. Bacher and S. Goudsmit, *Atomic Energy States* (McGraw-Hill Book Company), for references.

The indicated ratio of interval factors is $A({}^{6}D_{i})/A({}^{6}P_{1i}{}^{0}) = -0.87$. From the measured spacings we then get $A({}^{6}D_{i}) = 0.028$ cm⁻¹ and $A({}^{6}P_{1i}{}^{0}) = -0.032$ cm⁻¹.

The pattern of λ 5537.8 is the most completely resolved of the observed structures, since five of the expected six components are measurable. Using this line as key we have carried through the analysis and obtained values for the interval factors of all states involved. These interval factors appear in Table IV. Since most of the patterns are less completely resolved than λ 5537, an unambiguous determination of the interval factors cannot always be made from a single structure. However, the requirement of consistency in the interpretation of related patterns provides in every case more conditions than are necessary for a unique assignment of values to the interval factors. It is to be recognized that the interferograms contain significant information having to do with the contours of the patterns which cannot conveniently be tabulated, but which nevertheless can be used to advantage in the graphical method of analysis when the

TABLE IV. Hyperfine interval factors, Mn I.

State	J	INTERVAL FACTOR (CM ⁻¹)
3d ⁵ 4s ² ⁶ S	$2\frac{1}{2}$	0.000 B
3d ⁶ 4s ⁶ D	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 4 \\ 1 \\ 2 \end{array}$	$\left.\begin{array}{c} 0.0278\\.0165\\.0145\\.0135\\.0145\end{array}\right\}\pm0.0005,A$
3d ⁶ 4s 4p ⁸ P ⁰	$\begin{array}{c} 2\frac{1}{2} \\ 3\frac{1}{2} \\ 4\frac{1}{2} \end{array}$	$\left. \begin{array}{c} .0222\\ .0205\\ .018 \end{array} \right\} \pm \ .001, \ A$
3d ⁶ 4s 4p ⁴ P ⁰	$1^{\frac{1}{2}}_{2\frac{1}{2}}$	$ \left. \begin{array}{c}056 \\023 \\020 \end{array} \right\} \pm .001, \ A $
3d ⁵ 4s(⁷ S)4p ⁶ P ⁰	$\begin{array}{c} 1 \frac{1}{2} \\ 2 \frac{1}{2} \\ 3 \frac{1}{2} \end{array}$	$\begin{array}{c} .0230\\ .0185\\ .0160 \end{array} \right\} \pm \ .001, \ A$
3d ⁵ 4s(⁵ S)4p ⁶ P ⁰	$\begin{array}{c} 1\frac{1}{2} \\ 2\frac{1}{2} \\ 3\frac{1}{2} \end{array}$	$ \left. \begin{array}{c}0315 \\0215 \\0170 \end{array} \right\} \pm \ .001, \ A $
3d ⁶ 4p ⁶ D ⁰	$\begin{array}{c} 1\frac{1}{2}\\ 2\frac{1}{2}\\ 3\frac{1}{2}\\ 4\frac{1}{2} \end{array}$	$\left.\begin{array}{c}005\\002\\ .002\\ .001\end{array}\right\} \qquad \qquad C$
3d ⁵ 4s 5s ⁸ S	$3\frac{1}{2}$.0265 \pm .0005, A
3d ⁵ 4s 5s ⁴ S	1 ¹ / ₂	$049 \pm .001, B$



FIG. 1. Graphical analysis of $\lambda 5537.8$ of Mn I. Above: Analysis graph showing theoretical line distribution for transition $J = \frac{1}{2} \rightarrow J = 1\frac{1}{2}$ with varying interval factor ratio, $A(\frac{1}{2})/A(1\frac{1}{2})$. Theoretical relative intensity of each component is indicated. Below: Pattern graph showing observed pattern on varying scale. Estimated relative intensity of each component is indicated. Coincidence between the two graphs occurs at $A(\frac{1}{2})/A(1\frac{1}{2}) = -0.87$.

plates are available for study. Aside from the number of significant figures with which they are expressed the interval factor data cannot all be given equal weights because of the varying element of judgment which is represented in the interpretation of the patterns. We have indicated the degree of our confidence in the data of Table IV by grading them A, B and C, A indicating that we regard the figure as comparatively reliable, C that we regard it as somewhat dubious.

NUCLEAR MAGNETIC MOMENT

In order that the nuclear magnetic moment may be found from hyperfine structure data it is necessary that formulas expressing the observed interval factors in terms of the coupling between the nucleus and the individual electrons be available. Goudsmit⁹ has shown how such for-

⁹ S. Goudsmit, Phys. Rev. 37, 663 (1931).

mulas may be obtained for configurations through application of the method of energy sums. Breit and Wills¹⁰ in extending the theory have shown that the electron "coupling constants" used by Goudsmit are in general not constant for all states of a configuration but depend upon the inner quantum numbers of the states. They derived improved interval factor formulas for a few two- and three-electron configurations and Crawford¹¹ and Crawford and Wills¹² have applied the same method to the derivation of formulas for the d^2s and p^3s configurations. Computations by the method of Breit and Wills have so far not proved feasible for the important configurations of Mn I, namely $d^{6}s$, $d^{5}s \not p$, $d^{5}s \cdot s$ and d^6p . We then have no choice but to rely upon the admittedly approximate method of Goudsmit. There is some justification for confidence in this method as applied to Mn I in view of the fact that the hyperfine structure may largely be attributed to the 4s electron and that Mn I is a celebrated example of Russell-Saunders coupling.

The method of energy sums depends upon the assumption that an externally applied magnetic field may remove some couplings completely and leave others entirely undisturbed. Thus explicit expressions may be written for the various interactions in each stage of decoupling. It is assumed that the interactions between the individual electrons and the nucleus may be expressed by coupling factors associated with individual electrons. These factors are assumed to be constant within a given configuration but may vary from configuration to configuration. Without giving in detail the rather laborious algebraic steps leading to the interval factor formulas we will mention briefly the stages of decoupling which it has been necessary to consider in order to obtain each set of formulas appearing in Table V.

 $3d^{6}(^{5}D)^{4}s$ ^{6}D . Here it is necessary to consider two stages of decoupling. The first is that in which merely the coupling between the nuclear magnetism and the total electron configuration is broken, the configuration itself being coupled

as before. The second stage assumed is that in which the 4s electron is broken away from the configuration leaving the strongly coupled $3d^{6}(^{5}D)$ group acting as an entity having the quantum numbers characteristic of a ⁵D state.

 $3d^{5}(^{6}S)4s 4p ^{8}P^{0}$. Here also two stages must be considered, i.e., (1) the nuclear spin decoupled from the configuration which retains the properties of an ^{8}P , and (2) both the 4s and 4p electrons decoupled from the $3d^{5}(^{6}S)$ group which retains the properties of a ${}^{6}S$.

 $3d^{5}({}^{6}S)4s \ 4p \ {}^{6}P^{0}(\alpha)$ and ${}^{6}P^{0}(\beta)$. Two ${}^{6}P^{0}$ states arise in this configuration. The symbols α and β will be used to designate the ${}^{6}P$ states of lower and of higher absolute energy respectively. One,

TABLE V. Interval factor formulas derived by method of energy sums.

$d^{\mathfrak{g}(5D)}$ s $^{\mathfrak{g}D}$	
$\begin{array}{l} A\left(\frac{1}{2}\right) = (7/15)a(s) - (4/3)a(d^{6}) - (4/3)b(d^{6}) \\ A\left(1\frac{1}{2}\right) = (13/75)a(s) + (2/15)a(d^{6}) + (74/105)b(d^{6}) \\ A\left(2\frac{1}{2}\right) = (23/175)a(s) + (12/35)a(d^{6}) + (164/245)b(d^{6}) \\ A\left(3\frac{1}{2}\right) = (37/315)a(s) + (26/63)a(d^{6}) + (122/441)b(d^{6}) \\ A\left(4\frac{1}{2}\right) = (1/9)a(s) + (4/9)a(d^{6}) - (20/63)b(d^{6}) \\ \Sigma A = a(s) \end{array}$	
d5(6S)s\$ 8P0	
$\begin{array}{l} A(2\frac{1}{2}) = (9/49)a(s) - (88/245)a(p) \\ A(3\frac{1}{2}) = (59/441)a(s) + (80/441)a(p) \\ A(4\frac{1}{2}) = (1/9)a(s) + (8/45)a(p) \\ \Sigma A = (3/7)a(s) \end{array}$	
d5(6S)sp 4P0	
$\begin{array}{l} A(\frac{1}{2}) = -\frac{1}{3}a(s) - (8/15)a(p) \\ A(1\frac{1}{2}) = -(11/75)a(s) + (32/375)a(p) \\ A(2\frac{1}{2}) = -(3/25)a(s) + (56/125)a(p) \\ \Sigma A = -\frac{3}{5}a(s) \end{array}$	
$d^{\mathfrak{s}}({}^{\mathfrak{s}}S)sp \ \alpha \ {}^{\mathfrak{s}}P^{\mathfrak{0}}$ and $\beta \ {}^{\mathfrak{s}}P^{\mathfrak{0}}$	
$\begin{array}{l} \alpha A \left(1\frac{1}{2}\right) + \beta A \left(1\frac{1}{2}\right) = (2/25)a(s) - (104/125)a(p) \\ \alpha A \left(2\frac{1}{2}\right) + \beta A \left(2\frac{1}{2}\right) = (62/1225)a(s) + (1696/6125)a(p) \\ \alpha A \left(3\frac{1}{2}\right) + \beta A \left(3\frac{1}{2}\right) = (2/49)a(s) + (136/245)a(p) \\ \Sigma (\alpha A + \beta A) = (6/35)a(s) \end{array}$	
α ⁶ P ⁰	
$\begin{array}{l} A(1\frac{1}{2}) = (4/15)a(s) - (8/25)a(p) \\ A(2\frac{1}{2}) = (124/735)a(s) - (8/1225)a(p) \\ A(3\frac{1}{2}) = (20/147)a(s) + (16/49)a(p) \\ \Sigma A = (4/7)a(s) \end{array}$	
β 6 P 0	
$\begin{array}{l} A(1\frac{1}{2}) = -(14/75)a(s) - (44/75)a(p) \\ A(2\frac{1}{2}) = -(62/525)a(s) + (248/875)a(p) \\ A(3\frac{1}{2}) = -(2/21)a(s) + (8/35)a(p) \\ \Sigma A = -\frac{2}{3}a(s) \end{array}$	
$d^{\mathfrak{g}(5D)}p^{\mathfrak{g}}D^{\mathfrak{g}}$	
$\begin{array}{l} A\left(\frac{1}{2}\right) = -\left(8/225\right)a(p) - (10/9)a(d^{6}) - (8/15)b(d^{6}) \\ A\left(1\frac{1}{2}\right) = -\left(86/1125\right)a(p) + (1/9)a(d^{6}) + (148/525)b(d^{6}) \\ A\left(2\frac{1}{2}\right) = -\left(32/875\right)a(p) + (2/7)a(d^{6}) + (327/1225)b(d^{6}) \\ A\left(3\frac{1}{2}\right) = (142/4725)a(p) + (65/189)a(d^{6}) + (244/2205)b(d^{6}) \\ A\left(4\frac{1}{2}\right) = (16/135)a(p) + (10/27)a(d^{6}) - (8/63)b(d^{6}) \\ \Sigma A = 0 \end{array}$	(6)

 ¹⁰ G. Breit and L. A. Wills, Phys. Rev. 44, 470 (1933).
 ¹¹ M. F. Crawford, Phys. Rev. 47, 768 (1935).
 ¹² M. F. Crawford and L. A. Wills, Phys. Rev. 48, 69 (1935)

the lower in energy, may be described as $d^{5}({}^{6}S)s$ $(^{7}S)p \ ^{6}P^{0}$, the other as $d^{5}(^{6}S)s \ (^{5}S)p \ ^{6}P^{0}$. While these designations are useful as a means of distinguishing the states they cannot be regarded as an accurate statement of the manner in which the states are built up since each ${}^{6}P^{0}$ must be thought of as built in part upon the ^{7}S and in part upon the ⁵S of the ion by addition of the pelectron. If we are to obtain interval factor formulas for these states and avoid this ambiguity we may consider only the sums of the interval factors of states having the same J value in the two multiplets. These formulas for the sums of interval factors may be obtained by considering the same stages of decoupling as in treating the $^{8}P^{0}$.

In Mn I the two states ${}^{6}P^{0}(\alpha)$ and ${}^{6}P^{0}(\beta)$ are found to be some 11,000 cm⁻¹ apart. This suggests that the designation mentioned above may be fairly accurate. Proceeding on this assumption we may obtain interval factor formulas for the two multiplets separately. For this purpose it is necessary to consider an intermediate stage in which the 4p electron is decoupled from the $3d^{5} 4s$ (⁷S) or (⁵S) group as the case may be. While this is admittedly a questionable procedure it leads to relations which seem to have some significance when compared with the experimental data. A variation calculation too involved for inclusion here indicates that values for the coupling factors obtained by introduction of the experimental interval factors into formulas for $\alpha^6 P^0$ and $\beta^6 P^0$ will represent lower and upper limits, respectively, for the "correct" value.

 $4d^{5}({}^{6}S)4s 4p {}^{4}P^{0}$. The same considerations as applied to the ${}^{8}P^{0}$ of this configuration lead to the formulas.

 $3d^{6}({}^{5}D)4p {}^{6}D^{0}$. Since several states arise from this configuration the formulas can be obtained only by making a further assumption not used in deriving the other formulas, i.e., that the sums of energies of all states characterized by the same values of M_{L} and M_{S} are independent of the degree of decoupling.

In the formulas of Table V the interval factors of the hypermultiplets are designated by Afollowed by the inner quantum number of the particular state in parentheses. Each electron coupling factor is designated by a followed by parentheses indicating the particular electron or electron group to which it refers. The interaction of the d^6 group is expressed by two factors, one relating to the spin and the other to the orbital interaction. The orbital factor is represented by $a(d^6)$ and the spin factor by $b(d^6)$ in the formulas.

Introducing the experimental interval factors into these formulas we obtain numerical values for the coupling factors. In most cases more relations exist than there are constants to be determined so that consistency provides a test of the accuracy of the formulas. A convenient means of determining the coupling constant for the *s* electron is provided by the common property of all formulas of Table V, that the sum of the interval factors of all states within a given multiplet depends only upon the contributions of unpaired s electrons. The values given below are those found to give best consistency within a given set of formulas. These results are graded A, B and C in accord with their relative reliability. The coupling constants determined for the different states are as follows:

$$\begin{array}{c} 3d^{6} 4s \ {}^{6}D \\ a(4s) = 0.0863 \ {\rm cm}^{-1} \\ a(3d^{6}) = 0.0094 \ {}^{\prime\prime} \\ b(3d^{6}) < 0.0001 \ {}^{\prime\prime} \end{array} \right\} A \\ 3d^{5} 4s \ 4p \ {}^{8}P^{0} \\ a(4s) = 0.143 \ {\rm cm}^{-1}, A \\ a(4p) = 0.009 \ {}^{\prime\prime} , B \\ 3d^{5} 4s \ 4p \ {}^{4}P^{0} \\ a(4s) = 0.162 \ {\rm cm}^{-1}, A \\ a(4p) = 0.006 \ {}^{\prime\prime} , C \\ 3d^{5} 4s \ 4p \ {}^{6}P^{0} \\ a(4s) = 0.095 \ {\rm cm}^{-1}, B \\ a(4p), \ {\rm small}, \ {\rm values \ inconsistent}. \\ \beta \ {}^{6}P^{0} \\ a(4s) = 0.175 \ {\rm cm}^{-1}, B \\ a(4p), \ {\rm small}, \ {\rm values \ inconsistent}. \end{array}$$

As an example of the relative consistency within a set of formulas we may consider the $3d^{6} 4s {}^{6}D$ terms, for which the experimental interval factors are rather accurately known. Substituting the values, $a(s) = 0.0863 \text{ cm}^{-1}$, $a(d^{6}) = 0.0094 \text{ cm}^{-1}$, $b(d^{6}) = 0$, into the five formulas, we obtain for the interval factors $A(\frac{1}{2})$ to $A(4\frac{1}{2})$, respectively, 0.0278, 0.0162, 0.0145, 0.0140 and 0.0138 cm⁻¹. These are to be compared with the experimental values of Table IV.

The only coupling factor that adapts itself to a calculation of the nuclear magnetic moment is that of the 4s electron. In addition to the fact that our formulas are probably less reliable in reference to a p electron, the smallness of the indicated values for a(4p) makes them extremely sensitive to errors in measurement. The value of a(4s) obtained from the $3d^6$ 4s 6D state is probably the most reliable. The values for the same factor determined from the $3d^5$ 4s 4p $^{8}P^{0}$ and ${}^{4}P^{0}$ are considerably larger and in only moderate agreement with one another. That they should be larger is expected, since the screening of a 4p electron must be rather less than that of a 3d electron. The interval factor formulas for the two ${}^{6}P^{0}$ states are the least satisfactory when applied to the observed interval factors. However, if the values of a(4s) obtained from these two multiplets are regarded merely as an upper and a lower limit for the correct value as mentioned above, then these values are in accord with those obtained from the ${}^{8}P^{0}$ and ${}^{4}P^{0}$ of the same configuration. The observed interval factors from the $3d^5 4s 5s \ ^8S$ and 4S terms are not useful because formulas derived by the method of energy sums do not distinguish between the contributions of the two s electrons.

Goudsmit's relation¹³ expressing the nuclear g factor in terms of the coupling between an s electron and the nucleus may be written in the form,

$$g(I) = \frac{3}{8}a(s) \frac{R^{\frac{1}{2}}Z_0 1838}{\alpha^2 Z_i W^{\frac{3}{2}} K(\frac{1}{2}, Z_i)}$$

Here W represents the energy in cm⁻¹ necessary ¹³ S. Goudsmit, Phys. Rev. 43, 636 (1933). to remove the *s* electron from the atom in that stage of ionization in which it is the only outer electron. The other symbols have their conventional meanings. The computation must be made for each configuration since a(s), Z_0 and W are different for different configurations.

Considering the state $3d^6$ 4s 6D we have, a(4s) = 0.0863 cm⁻¹, W = 53,500 cm⁻¹, $Z_0 = 1$, $Z_i = 25$, K = 1.06. W is the approximate energy necessary to remove the 4s electron from an atom in the above state leaving the ion in the state $3d^6$ 5D , the centers of gravity of both states being taken into account.⁷ Putting these numerical values into the formula we obtain

$$g(I) = 1.13.$$

Considering the configuration $3d^5 4s 4p$ we have two indicated values of a(4s), i.e., 0.143 and 0.162 cm⁻¹. Since the larger one is to be favored in such cases¹³ we suppose that the value of 0.160 cm⁻¹ is satisfactory for use in computation. Here W is the energy needed to remove the 4s electron from the configuration $3d^5 4s$ of Mn II leaving the configuration $3d^5$ of Mn III. If centers of gravity are taken into account, W=123,000 cm⁻¹, approximately. Also $Z_0=2$. We now obtain from the formula,

$$g(I) = 1.20.$$

In view of the comparative agreement between these two independent calculations we believe that the value 1.2 may be assigned as the nuclear g factor of Mn⁵⁵ until more precise means for its determination are available. Multiplying by the value of I, we obtain a nuclear magnetic moment of 3.0 nuclear magnetons.