the determination of the proper form for the lattice-electron interaction energy, their results do not throw much light on the role played by the single electron *umklapprozesse.*

Another consequence of these considerations is the suggestion of a different approach to the problem of computing the resistance. It would seem that the treatment of the electrons as independent, and the inclusion of the Pauli exclusion principle in the integral equation for the electron distribution, might not be the best procedure. It might be possible to treat the motion of the center of mass of all the electrons and to compute the probability of a change of its motion under the influence of the electric field and of the lattice interaction.

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# The Meson Theory of Nuclear Forces

H. A. BETHE
*Cornell University, Ithaca, New York*

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THE meson[1] theory of nuclear forces[2,3] predicts two kinds of interactions between heavy particles (protons and neutrons). The first kind does not depend on the spin of the heavy particles and has the form

$$U=g_1^2e^{-\lambda r}/r, \qquad (1)$$

where $r$ is the distance between the two interacting heavy particles, $g_1$ is a constant, $\lambda$ the reciprocal Compton wave-length of the meson,

$$\lambda=\mu c/\hbar \qquad (2)$$

and $\mu$ the meson mass. Besides (1), there will be a spin-dependent force,

$$V=V_1+V_2, \qquad (3)$$

$$V_1=\tfrac{2}{3}g_2^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,e^{-\lambda r}/r, \qquad (3a)$$

$$V_2=-g_2^2\left(3\frac{\boldsymbol{\sigma}_1\cdot\mathbf{r}\;\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\times\frac{e^{-\lambda r}}{\lambda^2r^3}(1+\lambda r+\tfrac{1}{3}\lambda^2r^2), \qquad (3b)$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators of the two heavy particles, and $\mathbf{r}$ the vector from one to the other. The constant $g_2$ has the same value in (3a) and (3b) but is independent of $g_1$ in (1).

[1] For the name "meson" and the arguments in its favor, see H. J. Bhabha, Nature **143**, 276 (1939).
[2] H. Fröhlich, W. Heitler and N. Kemmer, Proc. Roy. Soc. **166**, 154 (1938); H. J. Bhabha, Proc. Roy. Soc. **166**, 501 (1938).
[3] N. Kemmer, Proc. Camb. Phil. Soc. **34**, 354 (1938).

The spin-dependent interaction has been split into one part, $V_1$, which does not depend upon the relative position of the two particles (direction of $\mathbf{r}$), and another part, $V_2$, whose average over all directions of $\mathbf{r}$ gives zero. It is usually argued[2,3] that the latter part has no influence on spherically symmetrical states such as the ground state of the deuteron so that only the spherically symmetrical interactions $U$ and $V_1$ are considered. In order to obtain agreement with the observed positions of the singlet and triplet states of the deuteron, it is then necessary[3] to choose $g_1\approx 0.6g_2$.

The neglecting of $V_2$ cannot actually be justified. This interaction destroys the spherical symmetry of nuclear states and makes the ground state of the deuteron a mixture of an $S$ and a $D$ state. Quite generally, the orbital momentum $L$ of the nucleus will cease to be quantized when a "tensor" interaction of the type $V_2$ is present. $V_2$ vanishes identically only for singlet states of the two-body problem; for any triplet state, including the ground state of the deuteron, it must be considered.

In fact, $V_2$, as it stands, will give an infinite binding energy for the ground state of the deuteron, for it represents an inverse cube potential which is attractive for a certain linear combination of an $S$ and a $D$ state, and for such a potential the Schrödinger equation does not possess a lowest eigenvalue. The only remedy seems to be to "cut off" the interaction potential at a small distance, $r_0$. This is not very satisfactory from the esthetic point of view but seems justifiable in view of the large size of the interaction constant $g(g_2^2/hc \approx 1/10)$. The cutting-off distance $r_0$ turns out to be about 0.3 to 0.45 of the "range" of the nuclear forces, $1/\lambda$.

If the potential $V_2$ is cut off at a certain distance $r_0$, the lowest eigenvalue of the deuteron will be finite, its exact value depending on $r_0$ and $g_2$. The tensor interaction $V_2$ will tend to lower the triplet state but it will not influence the singlet level. *This interaction can therefore be used to obtain the correct relative position of singlet and triplet state of the deuteron without introducing the spin-independent interaction U at all.* This would mean a great simplification of nuclear theory: There would be only a single force $V$, given by (3), between nuclear particles, instead of the four different forces (Wigner, Majorana, Heisenberg and Bartlett) used previously.

It seems very probable that the meson potential (3) satisfies the saturation requirement since its average over the spins of the interacting particles is zero. It also appears likely that the relative positions of the energy levels of heavier nuclei will remain qualitatively the same as with the old forces. The meson potential is superior to the old forces in *predicting a quadrupole moment for the deuteron* because the ground state of this nucleus will now be a mixture of an $S$ and a $D$ state rather than a pure $S$ state. Such a quadrupole moment has been observed by Kellogg, Rabi, Ramsey and Zacharias.[4] This cannot be explained on the basis of the old "central" nuclear forces (Majorana, etc.) but requires the introduction of a fifth force, of the same tensor character as our $V_2$ (Schwinger[5]).

The forces (1), (3) are based on the assumption that heavy particles interact only with neutral mesons (neutral theory). It is also possible to assume interaction with charged as well as neutral mesons. A theory of this type, symmetrical in positive, negative and neutral mesons, was proposed by Kemmer[3] (symmetrical theory) and leads to a force between heavy particles differing from (1), (3) by a factor $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ where $\boldsymbol{\tau}_1$ is the operator of isotopic spin for the first heavy particle. For states "symmetrical in charge" such as the $^1S$ state of the deuteron, $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = 1$; for states antisymmetrical in charge, such as the $^3S$ state, $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = -3$.

Both in the neutral and the symmetrical theory, the force between two neutrons ($n$-$n$) is equal to that between two protons ($p$-$p$), and in the $^1S$ state the force between neutron and proton ($p$-$n$) is also equal to the $n$-$n$ and $p$-$p$ force. The equality ($n$-$n$) = ($p$-$p$) is required by general nuclear theory while ($n$-$p$) = ($p$-$p$) follows from the comparison of the scattering of slow neutrons and of protons by protons. With respect to the ground state of the deuteron, the two theories differ: In the neutral theory, the interaction (3a) is repulsive, in the symmetrical theory, it is attractive and equal to the $^1S$ interaction. The influence of (3b) must therefore be greater in the neutral theory.

In the neutral theory, $V_2$ is attractive when the spins $\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$ are in line with the axis $\mathbf{r}$ of the deuteron. Therefore the theory predicts a "cigar shape" for the deuteron, i.e., greater probability of finding the axis $\mathbf{r}$ parallel to the total spin $\boldsymbol{\sigma}$ than perpendicular to it. In the symmetrical theory, the factor $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = -3$ reverses the sign of $V_2$ so that a "pill-box" shape results. The experiments of Kellogg, Rabi, Ramsey and Zacharias[4] give a cigar shape.[6]

We have solved the Schrödinger equation for the ground state of the deuteron using the potential $V$, both for the symmetrical and the neutral theory. The solution was obtained by numerical integration without neglecting any terms. The interaction constant $g_2$ and the cutting-off radius $r_0$ were determined so as to give the correct position for both the singlet and the triplet level of the deuteron. The latter was assumed to be $-2.17$ Mev, the former was deduced from a slow neutron cross section of $18.3 \cdot 10^{-24}$ cm$^2$. The meson mass was taken equal to 177 electron masses, in approximate[7] agree-

[4] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. **55**, 318 (1939).

[5] J. Schwinger, in course of publication.

[6] In order to obtain sign and magnitude of the quadrupole moment from the experiments, it is necessary to know the inhomogeneity of the electric field in the hydrogen molecule near one nucleus. This inhomogeneity has been calculated by Nordsieck using Wang wave functions. It is not known how much error is introduced by the use of such approximate wave functions. Even a reversal of sign can not be completely excluded at present.

[7] The value 177 is convenient for calculation because it makes $h/\mu c$ equal to one-half of the "radius" of the deuteron.

TABLE I. *Summary of results obtained on the basis of the two theories and the two ways of cutting off.*

| Cut-off | Neutral Theory zero | Neutral Theory straight | Symmetrical Theory zero | Symmetrical Theory straight |
|---|---|---|---|---|
| $g_2^2/\hbar c$ | $0.081_2$ | $0.080_0$ | 0.250 | 0.154 |
| $r_0/(\hbar/\mu c)$ | 0.320 | 0.436 | 1.679 | 1.733 |
| Quadrupole moment | 2.73 | 2.67 | $-24.7$ | $-17.8 \quad \times 10^{-27}$ cm$^2$ |
| Percent "$D$" function | 6.88 | 6.68 | 23.46 | 18.52 |

ment with the latest determination of Nishina, Takeuchi and Ichimya[8] (170±9). The cutting off was made in two different ways, *viz.* (a) the potential was taken to be zero inside of $r_0$ (zero cut-off), (b) the potential for $r<r_0$ was taken equal to $V(r_0)$ (straight cut-off). In case (a), the radius $r_0$ must, of course, be taken smaller than in (b) in order to obtain the correct binding energy for the deuteron, but the final value for the quadrupole moment of the deuteron is changed very little.

In Table I, the results[9] are summarized for the two theories and the two ways of cutting off. The values of the cutting-off radius obtained in the neutral theory (0.320 and 0.436 $h/\mu c$) are very reasonable, while those for the symmetrical theory ($\sim 1.7\hbar/\mu c$) are unbelievably high. Correspondingly, the symmetrical theory gives improbably high values for the quadrupole moment (in addition to the wrong sign, i.e., pill-box shape). The neutral theory, on the other hand, gives a value for the quadrupole moment ($\sim 2.7\cdot 10^{-27}$ cm$^2$) in close agreement with that obtained from the experiments of Kellogg, Ramsey, Rabi and Zacharias with the use of Wang wave functions[6, 10] ($2.5\cdot 10^{-27}$ cm$^2$). Although the evaluation of the experiments with Wang functions may not be very accurate, present evidence is definitely in favor of the "neutral theory." In this theory, it is of course difficult to explain the $\beta$-decay and the magnetic moments of proton and neutron.

The last line of Table I gives the percentage of $^3D$ wave function contained in the eigenfunction of the ground state of the deuteron.

[8] Y. Nishina, M. Takeuchi and T. Ichimiya, Phys. Rev. **55**, 585 (1939).

[9] The values given previously (Phys. Rev. **55**, 1130 (1939)) were erroneous.

[10] I am indebted to Dr. Nordsieck and Professor Rabi for communicating this value to me.