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Conservation of Momentum in Electrical Conductivity

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In the theory of the electrical conductivity of metals, the conservation theorem of Peierls establishes the invariance of the sum of the electron wave numbers and a function of the lattice vibrations. This is shown to be accompanied by other related conservation theorems based on the periodicity of the system. One of these refers to the difference between Peierls' integral and the total momentum of the system. The connection between Peierls' integral and the integral of momentum has usually been obscured by the use of electron states that do not represent a definite value of the momentum. When the system is treated as a whole it can be shown that the transfer of momentum from the electrons to the lattice need not be a process involving one electron at a time, but may involve a large number. One can say that the *umklapprozesse* involve a whole group of electrons rather than a single one. As a result one can understand the establishment of a steady state in a conducting metal without considering the *umklapprozesse* of single electrons as introduced by Peierls.

1. Introduction

Since the work of Peierls[1] the theory of the electrical conductivity of metals has been marred by the presence of a conservation theorem whose physical significance has not been clearly understood. It has been interpreted as indicating that the conductivity of a metal should be very much larger, at low temperatures, than is observed, unless the electron energy as a function of wave number is restricted to a closely prescribed form. This seems an undue restriction, especially in the case of the alkalis. It is proposed to show here that this interpretation of the theorem is not justified, and that Peierls' conservation law presents no real obstacle to the understanding of the processes of conduction and resistance.

The theorem in question is that, in the absence of an external electric field, the quantity

$$I_1=(1/G)\{\sum k_j+\sum fm_f\}, \tag{1}$$

is a constant of the motion, or can change by integral amounts only. Eq. (1) is written for the special case of a one-dimensional lattice in which d is the distance between the equilibrium positions of the ions, G is the number of ions in one cycle, f is the integer characterizing a lattice vibration, and m_f is an integer associated with the excitation of the vibration f. A positive value of m_f represents a wave traveling through the lattice in the positive direction. The quantities k_j are integers that characterize the wave functions of the individual electrons. These functions are of the form $u(x_j)\exp(2\pi i/Gd)k_jx_j$, where $u(x_j)$ is periodic with the period d. To satisfy the Pauli exclusion principle the k_j must be different for different electrons. Since the k_j for each electron changes under the influence of an electric field, and since an electric field cannot directly set up vibrations in the lattice, there has seemed to be no way to prevent I_1 from increasing indefinitely during the flow of a current. For the same reason, after a current has been started and the electric field removed, there has seemed to be no way in which the interaction with the lattice could stop the current without establishing a value of Σfm_f far different from the equilibrium value of zero.

To avoid these difficulties, Peierls invoked the processes by which a single k_j changes by the amount G. Such transitions are not prohibited by the conservation theorem, and since they may represent a reversal in the direction of motion, Peierls called them *umklapprozesse*. However, he also showed that the effective intervention of such processes at low temperatures requires a prescribed form of the electron energy as a function of the wave number k. That all metals, and in particular the alkalis, should have this particular form seems quite improbable.

It will be shown here, however, that the restriction imposed by Peierls' theorem applies

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[1] R. Peierls, Ann. d. Physik **12**, 154 (1932).

to all of the electrons together and not to each one by itself. In addition to the *umklapprozesse* of Peierls there can take place processes by which a number of electrons change their total Σk_j by the amount G. Such a process does not require the presence of a lattice vibration, but is due to the interaction with the undisturbed lattice. This provides a tremendous number of processes in which different numbers of electrons take part. Included in them are the processes in which each electron present changes its k_j by a single unit.

Transitions involving two or more electrons might be regarded as very improbable if the interaction between the lattice and the electrons were really a small perturbation. It is, however, a very large interaction, and conclusions characteristic of only the first-order perturbation treatment are not necessarily valid. The fact that the theorem of Peierls imposes a restriction on only the sum of the k_j can be shown by consideration of the properties of the Hamiltonian function.

The treatment of the complete problem shows other integrals of the motion in addition to I_1. One of these is trivial since it expresses merely the cyclic boundary condition. It is that

$$I_2 = k_j \tag{2}$$

can change by integral amounts only. Since k_j is restricted from the beginning to integral values, this is of little importance. The origin of this integral, however, is very similar to that of the others. The third is that

$$I_3 = (1/G)(s - \sum_f f m_f) \tag{3}$$

can change by integral amounts only. In this, s is an integer that represents the momentum of the lattice as a whole. The last is the sum of two others,

$$I_4 = I_1 + I_3 = (1/G)(\sum_j k_j + s), \tag{4}$$

and is associated with the motion of the system as a whole. The presence, in this sum, of both Σk_j and s indicates that all of these integrals are closely connected with the momentum integral.

2. The Hamiltonian Function

The most satisfactory treatment of the theorem of Peierls was given by Bloch,[2] and is based on the invariance of the Hamiltonian function to various transformations. His general procedure will be followed here.

The problem can be treated adequately in one dimension as the other dimensions add only nonessential complications. Let the lattice be composed of G ions of mass m_I, arranged in a straight line, and separated by a distance d, in their equilibrium positions. These equilibrium positions will have the coordinates nd, with n an integer. Only displacements that lie in the line of the ions will be considered. Transverse waves need not be treated since they are of little consequence in the resistance. Let u_n be the displacement of the nth ion from its equilibrium position, and assume that the interaction between the ions can be represented by springs connecting them. It is immaterial whether this interaction is restricted to adjacent ions or whether it extends also to those more distant. It is essential, however, that the lattice as a whole be free to move. This interaction, which is represented by springs, is in addition to that produced by the interaction with the conduction electrons, and it can be regarded as caused by the electrons in the closed shells. Such a specification makes, of course, a very much idealized model of a metal crystal, but it contains the elements essential to the present discussion and it is desirable to be very specific as to the model that is under consideration. In particular it is to be emphasized that only harmonic forces between the ions are being considered. It is also to be understood that the problem is treated as cyclic, and that the G ions constitute one cycle.

If p_j and x_j are respectively the momentum and the coordinate of the jth electron, the Hamiltonian function for the system in the absence of any external field, is

$$H = (m_I/2)\sum_{n=1}^{G} \dot{u}_n^2 + \sum_{n,\,n'} b_{nn'} u_n u_{n'} + \sum_{j=1}^{G} \{ (p_j^2/2m) + \sum_{n=1}^{G} V(x_j - nd - u_n) + \sum_{j'=1}^{G} V_e(x_j - x_{j'}) \}. \tag{5}$$

The $b_{nn'}$ are constants that can be expressed in

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[2] F. Bloch, J. de phys. et rad. **4**, 486 (1933).

terms of the force constants of the springs. $V(x_j-nd-u_n)$ is the potential energy of interaction between the jth electron and the nth ion. The use of a general function, instead of the first few terms of a power series expansion, is necessary to give a Hamiltonian function that guarantees the conservation of momentum.

Following Bloch we may introduce new coordinates ξ_f and η_f defined by the equations

$$u_n=\sum_{f=0}^{G/2}\{\xi_f\cos(2\pi fn/G)+\eta_f\sin(2fn/G)\}. \tag{6}$$

The apparent presence of $G+2$ coordinates for the specification of the positions of G ions need not overdetermine them, since η_0 and $\eta_{G/2}$ are always multiplied by zero coefficients in Eqs. (6). They may therefore be set equal to zero without imposing any restriction on the motion of the ions. Expressed in these coordinates the Hamiltonian function is

$$\begin{aligned}H=(M/2)\{\dot{\xi}_0{}^2+(\tfrac{1}{2})\sum_{f=1}^{G/2-1}[\dot{\xi}_f{}^2+\dot{\eta}_f{}^2+\omega_f{}^2(\xi_f{}^2+\eta_f{}^2)]\\ +\dot{\xi}^2{}_{G/2}+\omega^2{}_{G/2}\xi^2{}_{G/2}\}+\sum_{j=1}^{G}\{(p_j{}^2/2m)\\ +\sum_{n=1}^{G}V(x_j-nd-\sum_{f=0}^{G/2}[\xi_j\cos(2\pi fn/G)\\ +\eta_f\sin(2\pi fn/G)])+\sum_{j'}V_e(x_j-x_{j'})\},\end{aligned} \tag{7}$$

with $M=Gm_I$.

Now let

$$\xi_f=\rho_f\cos\varphi_f,\quad\text{and}\quad \eta_f=\rho_f\sin\varphi_f,$$

with the special provision that $\varphi_0=0$, and $\varphi_{G/2}$ is an integral multiple of π. These provisions preserve the zero values of η_0 and $\eta_{G/2}$. It will also be convenient to introduce the center of mass of the electrons. Hence let

$$X=(1/G)\sum_{j=1}^{G}x_j,\quad\text{and}\quad \xi_j=x_j-X.$$

These coordinates are not all independent since $\Sigma_1{}^G\xi_j=0$. When it is necessary to guard against any ambiguity due to this fact, ξ_1 will be replaced by $-\Sigma_2{}^G\xi_j$. With these coordinates the Hamiltonian function becomes

$$\begin{aligned}H=(M/2)\{\dot{\rho}_0{}^2+(\tfrac{1}{2})\sum_{f=1}^{G/2-1}[\dot{\rho}_f{}^2+\rho_f{}^2\dot{\varphi}_f{}^2+\omega_f{}^2\rho_f{}^2]\\ +\dot{\rho}^2{}_{G/2}+\omega^2{}_{G/2}\rho^2{}_{G/2}+(Gm/2)\dot{X}^2\\ +(m/2)\sum_{j=2}^{G}\dot{\xi}_j{}^2+(m/2)(\sum_{j=2}^{G}\dot{\xi}_j)^2\\ +\sum_{j,j'}V_e(\xi_j-\xi_{j'})+\sum_{j,n}V(X+\xi_j-nd\\ -\sum_{f=0}^{G/2}\rho_f\cos[(2\pi fn/G)-\varphi_f]).\end{aligned} \tag{8}$$

This is not strictly the Hamiltonian function since it contains the velocities instead of the momenta, but the transformation is easily made and does not affect any of the conclusions to be drawn.

In the presence of a uniform electric field there is the extra term

$$H'=GeE(\rho_0-X)=GeEZ, \tag{9}$$

where $Z=\rho_0-X$.

Peierls' conservation theorem is associated with the invariance of H under the transformation

$$x_j\rightarrow x_j+d,\quad \varphi_f\rightarrow\varphi_f+(2\pi f/G), \qquad T_1$$

for all values of j and of f. The increase of all of the x_j by the amount d is equivalent to the increase of X by d with all of the ξ_j unchanged. The terms in the Hamiltonian that are affected by this transformation are those comprising the interaction between the electrons and the lattice. Their invariance under T_1 is due to the cyclic boundary conditions which provide that $V_{n,j}=V_{n+G,j}$. With this provision, the transformation T_1 effects a rearrangement of the terms in the sum.

However the Hamiltonian function is also invariant under the transformations

$$X\rightarrow X+d,\quad \xi_j\rightarrow\xi_j-d,\quad (j\neq i), \qquad T_2$$

$$\rho_0\rightarrow\rho_0+d,\quad \varphi_f\rightarrow\varphi_f-(2\pi f/G), \qquad T_3$$

$$X\rightarrow X+d,\quad \rho_0\rightarrow\rho_0+d. \qquad T_4$$

The transformation T_2 is equivalent to $x_i\rightarrow x_i+Gd$. The invariance of the Hamiltonian function, as well as all wave functions, under this transformation is due to the cyclic boundary conditions and is merely an expression of them. The transformation T_3 represents the displacement of the lattice through the distance d in one direction, and the displacement of all the elastic waves the same distance in the opposite direction. It can be seen from the forms of the transformations that

$$T_4 = T_1 T_3, \tag{10}$$

and that T_4 is associated with the total momentum of the system. To understand the full significance of these transformations it is helpful to examine the behavior of the wave functions under them.

3. Integrals of the Motion

Let the wave function for each electron be of the usual form indicated above. The total electronic wave function can be built up out of products of these of the form

$$U(x_1, \cdots, x_G) = \prod_{j=1}^{G} \exp\left[(2\pi i/Gd)k_j x_j\right] u_j(x_j)$$

$$= \exp\left[(2\pi i/Gd)\sum k_j x_j\right] \prod_{j=1}^{G} u_j(x_j). \tag{11}$$

The wave function of the lattice can be written in terms of the normal coordinates introduced into the Hamiltonian function

$$U(\rho_f, \varphi_f) = \exp\left[(2\pi i/Gd)s\rho_0\right]$$

$$\times \prod_{f=1}^{G/2} R(\rho_f, s_f) \exp(i m_f \varphi_f), \tag{12}$$

where the $R(\rho_f, s_f)$ are the radial functions for a two-dimensional harmonic oscillator, and the s_f and m_f are integers. The Pauli exclusion principle requires that all of the k_j in the electron function be different, but this fact is unimportant for the present considerations. The set of products of these two functions provides a basis for the representation of any state of the combined system.

The above functions have been selected to be characteristic of the operators T_1, T_2, T_3 and T_4. The characteristic values are evident from the following equations:

$$T_1 U(k_j, m_f, s_f, s)$$

$$= \exp\left[(2\pi i/Gd)\left(\sum_i k_i + \sum_f f m_f\right)\right] U(k_j, m_f, s_f, s),$$

$$T_2 U(k_j, m_f, s_f, s) = U(k_j, m_f, s_f, s),$$

$$T_3 U(k_j, m_f, s_f, s) = \exp\left[(2\pi i/Gd)\left(s - \sum_f f m_f\right)\right]$$

$$\times U(k_j, m_f, s_f, s),$$

$$T_4 U(k_j, m_f, s_f, s) = \exp\left[(2\pi i/Gd)\left(s + \sum_i k_j\right)\right]$$

$$\times U(k_j, m_f, s_f, s).$$

Since these operators commute with the Hamiltonian function, any stationary state that is a product of the functions (10) and (12) can be specified by a characteristic value of each of these operators. Furthermore, if the system is in a state represented by any one of the above products, it will make transitions to only those other states that have the same characteristic values of the above four operators. In such transitions the integrals I_1, I_2, I_3 and I_4 either do not change at all or change by integral amounts.

4. Discussion of the Conservation Laws

It is important to notice that the integrals I_1 and I_4 contain the sum of the k_j, and not the individual k_j themselves. The Hamiltonian function is not invariant to the operation which increases one of the x_j by the amount d, changes the φ's by the amount indicated in T_1, and leaves the other x_j unchanged. The same thing is true of the operation on the angles φ_f. It is only when they are all changed that the Hamiltonian function is restored to its original condition. This is not due essentially to the presence of a term giving the interaction between the electrons but is due to the form of the interaction between the electrons and the lattice. It is because all of the electrons interact with all of the modes of vibration. It is true that this interaction consists of a sum over the electrons, so that if the interaction is treated as a perturbation and only the first approximation is considered, the electrons act independently. Since the interaction is really a large one, this apparent independence may be quite misleading.

The original derivation of the integral I_1 was based upon this independent treatment of the electrons and so suggested that changes in I_1 by the amount G must be made up of a change in one m_f and one k_j only. Such an interpretation is not at all required by the general treatment given above. Nevertheless it remains to be shown that transitions involving two or more electrons actually do occur. For this purpose it will suffice to treat a special case.

Consider the situation in which all of the s_f and m_f are zero and do not change, and let the electron functions be those for free electrons. The second-order treatment will involve the square of the interaction energy and this will contain terms which are products of the interaction with two electrons. Such matrix elements have the form

$$\begin{aligned}&(k_1', k_2', s' | V^2 | k_1, k_2, s)\\&\sim \sum_{n_1, n_2} \int \exp\left[(2\pi i/Gd)\{(k_1-k_1')x_1\right.\\&\left.+(k_2-k_2')x_2+(s-s')\rho_0\}\right]\prod_{f=1}^{G/2}|R(\rho_f, 0)|^2\\&\times V(x_1-n_1d-\sum_0^{G/2}\rho_f\cos[(2\pi f n_1/G)-\varphi_f])\\&\times V(x_2-n_2d-\sum_0^{G/2}\rho_f\cos[(2\pi f n_2/G)-\varphi_f])\\&\qquad\times \rho_f d\rho_f d\varphi_f dx_1 dx_2. \qquad (14)\end{aligned}$$

For ease in computation take the special case in which the interaction between an ion and an electron is limited to a very small region and let it be approximated by a δ-function. Then integration over x_1 and x_2 gives

$$\begin{aligned}&(k_1', k_2', s' | V^2 | k_1, k_2, s)\\&\sim \sum_{n_1, n_2} \int \exp\left[(2\pi i/Gd)\{(k_1-k_1')\right.\\&\times(n_1d+\rho_0+\sum_1^{G/2}\rho_f\cos[(2\pi f n_1/G)-\varphi_f])\\&+(k_2-k_2')(n_2d+\rho_0+\sum_1^{G/2}\rho_f\cos[(2\pi f n_2/G)-\varphi_f])\\&\quad\left.+(s-s')\rho_0\}\right]\prod_{f=1}^{G/2}|R(\rho_f, 0)|^2\rho_f d\rho_f d\varphi_f d\rho_0. \qquad (15)\end{aligned}$$

Let $x_f=\rho_f\cos\varphi_f$, and $y_f=\rho_f\sin\varphi_f$. In the state of the oscillators for $s_f=0$,

$$R(\rho_f, 0)=\exp[-\alpha_f(x_f^2+y_f^2)/2].$$

The integrals over x_f and y_f can then be carried out. The constancy of I_4 is evident from the coefficient of ρ_0 in the exponent. The integral over ρ_0 vanishes unless

$$(k_1'+k_2'+s')=(k_1+k_2+s).$$

If this condition is satisfied the integral can be evaluated to give

$$\begin{aligned}&(k_1', k_2', s' | V^2 | k_1, k_2, s)\\&\sim\exp[-(\pi/Gd)^2\{(k_1-k_1')^2+(k_2-k_2')\}]\\&\quad\sum_{f=1}^{G/2}(1/4\alpha_f)\sum_{n_1, n_2}\exp\left[(2\pi i/Gd)\right.\\&\times\{(k_1-k_1')n_1+(k_2-k_2')n_2\}\\&-(\pi/Gd)^2\sum_1^{G/2}\{(k_1-k_1')(k_2-k_2')/2\alpha_f\}\\&\qquad\left.\times\cos(2\pi f/G)(n_1-n_2)\right]. \qquad (16)\end{aligned}$$

Since α_f increases with f, the sum in the last exponent decreases rapidly with the absolute value of (n_1-n_2). For $n_1=n_2$ there remains the sum

$$\sum_{n=1}^{G}\exp[(2\pi i/G)(k_1-k_1'+k_2-k_2')n],$$

which vanishes unless $(k_1'+k_2')-(k_1+k_2)=wG$, with w an integer. The same is true for any fixed value of (n_1-n_2). This represents the conservation of I_1 when there is no change in the state of vibration of the lattice. When the conservation laws are satisfied, the above matrix element is not generally zero, and such transitions can be expected to take place.

The change of a single k_j by the amount G has often been compared to Bragg reflection from the undisturbed lattice. This is true for a single particle. The change of the sum of the k_j for a group of electrons may be likened to the Bragg reflection of an atom or molecule from the lattice. In the case at hand, however, the interaction between the electrons need not be considered as due to their Coulomb fields, but merely as taking place through their interaction with the lattice.

Some of the difficulty in understanding the significance of Peierls' conservation theorem has been associated with the fact that the electron functions ordinarily used, $u(x)e^{(2\pi i/Gd)kx}$, are not characteristic of momentum. The conservation law involves the k, which represents momentum for a free particle but not for a particle in a periodic field. However, it must be remembered that even though the function used is not characteristic of momentum, it is a sum of momentum characteristic functions whose momenta differ by just the amount h/d. Such a sum is just the ordinary Fourier expansion,

$$u(x) \exp(2\pi i/Gd)kx = \sum_{n=-\infty}^{\infty} A_n \exp(2\pi i/Gd)(k+nG)x.$$

Since all of the conservation theorems apply to k modulo G, the distinction between the k and the momentum is not really significant. The distribution of the momentum over the various values is not at all restricted by the integrals.

The connection of Σk_j with the momentum is demonstrated by the existence of the integral I_4. Any change in Σk_j (modulo G) is accompanied by a corresponding change in s, and s clearly represents the momentum of the lattice. According to I_3, s and $\Sigma f m_f$ must increase together so that the conservation of momentum can be expressed in the form of I_1. It is the integral I_3 that gives the connection between $\Sigma f m_f$ and the momentum of the lattice. This integral shows that it is impossible for the electrons to excite an elastic traveling wave without a transfer of momentum to the lattice.

When the whole interaction between the electrons and the lattice is treated as a perturbation it is quite clear that the magnitude of the interaction is such that a first-order treatment is inadequate. However, the usual procedure has been to consider the unperturbed system as including the interaction between the electrons and the lattice in its equilibrium configuration. The departures from regularity were then treated as the perturbation. This procedure tends to obscure the momentum relations, since the lattice is usually treated as fixed.

Another difficulty with the usual method is that the contribution, to the perturbation, of the term proportional to the first power of the displacement of the ions is very small. Because of the symmetry of the interaction between an ion and an electron, the average of the first derivative will vanish. If the second derivative is considered, terms in the square of the displacement must be included. This leads at once to processes in which two of the m_f change together. As a matter of fact it can be shown that unless an infinite series is used to express the interaction between one electron and one ion, the Hamiltonian function cannot have the proper periodicity and at the same time provide for the conservation of momentum.

The principal consequence of the above considerations is that it is not necessary to give a special treatment of the *umklapprozesse* of a single electron in computing the resistance of a metal. Since, as Peierls showed, such processes require very special circumstances for their effective occurrence at low temperatures, they will be very rare, and will be replaced by processes in which a number of electrons make transitions at the same time. Most of the treatments of resistance have not included the *umklapprozesse* of a single electron in a very satisfactory way at the low temperatures where they would be of importance. In his original work Bloch[3] neglected them entirely. Peierls[4] criticized this neglect and undertook to estimate the effect of their inclusion. This was not very precise, but he showed that their inclusion, with the special circumstances under which they might be expected to occur, would not seriously change the temperature dependence of the resistance from that otherwise estimated. In a recent treatment by Peterson and Nordheim[4] they were neglected on the ground that they would be unimportant at high temperatures, although still later they were considered by Bardeen[5] at the same high temperatures. Since the last two papers were principally devoted to

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[3] F. Bloch, Zeits. f. Physik **52**, 555 (1938); **59**, 208 (1930).

[4] E. L. Peterson and L. W. Nordheim, Phys. Rev. **51**, 355 (1937).

[5] J. Bardeen, Phys. Rev. **52**, 688 (1937).

the determination of the proper form for the lattice-electron interaction energy, their results do not throw much light on the role played by the single electron *umklapprozesse.*

Another consequence of these considerations is the suggestion of a different approach to the problem of computing the resistance. It would seem that the treatment of the electrons as independent, and the inclusion of the Pauli exclusion principle in the integral equation for the electron distribution, might not be the best procedure. It might be possible to treat the motion of the center of mass of all the electrons and to compute the probability of a change of its motion under the influence of the electric field and of the lattice interaction.

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The Meson Theory of Nuclear Forces

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THE meson[1] theory of nuclear forces[2, 3] predicts two kinds of interactions between heavy particles (protons and neutrons). The first kind does not depend on the spin of the heavy particles and has the form

$$U=g_1^2e^{-\lambda r}/r, \tag{1}$$

where r is the distance between the two interacting heavy particles, g_1 is a constant, λ the reciprocal Compton wave-length of the meson,

$$\lambda=\mu c/\hbar \tag{2}$$

and μ the meson mass. Besides (1), there will be a spin-dependent force,

$$V=V_1+V_2, \tag{3}$$

$$V_1=\tfrac{2}{3}g_2^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,e^{-\lambda r}/r, \tag{3a}$$

$$V_2=-g_2^2\left(3\frac{\boldsymbol{\sigma}_1\cdot\mathbf{r}\,\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\times\frac{e^{-\lambda r}}{\lambda^2r^3}(1+\lambda r+\tfrac{1}{3}\lambda^2r^2), \tag{3b}$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators of the two heavy particles, and $\mathbf{r}$ the vector from one to the other. The constant g_2 has the same value in (3a) and (3b) but is independent of g_1 in (1).

The spin-dependent interaction has been split into one part, V_1, which does not depend upon the relative position of the two particles (direction of $\mathbf{r}$), and another part, V_2, whose average over all directions of $\mathbf{r}$ gives zero. It is usually argued[2, 3] that the latter part has no influence on spherically symmetrical states such as the ground state of the deuteron so that only the spherically symmetrical interactions U and V_1 are considered. In order to obtain agreement with the observed positions of the singlet and triplet states of the deuteron, it is then necessary[3] to choose $g_1\approx 0.6g_2$.

The neglecting of V_2 cannot actually be justified. This interaction destroys the spherical symmetry of nuclear states and makes the ground state of the deuteron a mixture of an S and a D state. Quite generally, the orbital momentum L of the nucleus will cease to be quantized when a "tensor" interaction of the type V_2 is present. V_2 vanishes identically only for singlet states of the two-body problem; for any triplet state, including the ground state of the deuteron, it must be considered.

In fact, V_2, as it stands, will give an infinite binding energy for the ground state of the deuteron, for it represents an inverse cube potential which is attractive for a certain linear combination of an S and a D state, and for such a potential the Schrödinger equation does not possess a lowest eigenvalue. The only remedy seems to be to "cut off" the interaction potential at a small

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[1] For the name "meson" and the arguments in its favor, see H. J. Bhabha, Nature **143**, 276 (1939).

[2] H. Fröhlich, W. Heitler and N. Kemmer, Proc. Roy. Soc. **166**, 154 (1938); H. J. Bhabha, Proc. Roy. Soc. **166**, 501 (1938).

[3] N. Kemmer, Proc. Camb. Phil. Soc. **34**, 354 (1938).