g—Acceleration of gravity (cm sec.$^{-2}$)
η—Viscosity (g sec.$^{-1}$ cm^{-1})
c_p—Specific heat at constant pressure (cal. g^{-1} deg.$^{-1}$)
ΔT—Temperature difference (max.) between hot surface and surrounding fluid (deg.)
α—Exponent of the "Grashoff" term in the approximate expression for convection heat transfer.
M—Molecular weight
p—Pressure (g cm^{-2})
T—Temperature absolute (deg.)
R—Gas constant (cm deg.$^{-1}$)
π—3.1416
W—Heat loss per unit length of arc column (cal. cm^{-1})
$-n$—Exponent of the current in the $E-i$ relation
n_0—Number of electrons per unit volume (cm^{-3})
μ—Mobility of the electron (cm^2 sec.$^{-1}$ volt^{-1})
e—Electronic charge (coulombs)
N—Molecules per unit volume (cm^{-3}) (Loschmidt number)
V_e—Effective ionization potential (volts)
β—Exponent of p in the n_0-p relation
m_0—Mass of the electron (g)
λ—Mean free path (cm)
c—Average velocity (cm sec.$^{-1}$)
$-\gamma$—Exponent of p in the $D-p$ relation
m—Exponent of p in the $E-p$ relation
V—Fluid velocity (cm sec.$^{-1}$)
φ—Exponent of the term $(DV\rho/\eta)$ for forced convection heat transfer
$-n'$—Exponent of i in $E-i$ expression for forced convection cooling
q—Exponent of g in the $E-g$ relation
e_A—Anode drop (volts)
e_c—Cathode drop (volts)

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Electron Scattering and Plasma Oscillations*

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The dependence of electron scattering upon plasma oscillations was studied in a mercury arc discharge tube containing a movable probe. The best conditions for studying these relations were found when the vapor pressures and arc currents were small. Probe volt-ampere characteristics showed the presence of ultimate electrons with a Maxwell-Boltzmann distribution corresponding to about 30,000°K and a superposed stream of fast electrons emitted from the cathode. These fast electrons were scattered without appreciable change in direction in well-defined, approximately plane regions only a few tenths of a millimeter wide. The plasma oscillations were studied with a crystal detector in the probe circuit supplemented by a Lecher wire system. In addition to "turbulent" disturbances with no measurable frequencies found throughout the tube, narrow regions were found in which stable periodic oscillations of considerable magnitude were detected. These regions and the scattering regions were, in general, equal in number and coincided except for a small shift of the former in the direction of the anode. The observed frequencies agreed well with the formula derived by Tonks and Langmuir. The scattering regions and the regions in which periodic oscillations were observed became less marked and moved toward the cathode as either the vapor pressure or arc current were increased. The periodic oscillations were found only in regions traversed by the fast electrons. The results are interpreted as showing that scattering is due to plasma oscillations which receive their energy from the fast electrons. The process by which the oscillations were detected by the probe is discussed to account for the shift of the regions in which oscillations were observed with respect to the scattering regions.

INTRODUCTION

In 1924 Langmuir and Mott-Smith[1] showed that in an electric discharge in a gas or vapor the primary electrons emitted by the cathode suffer scattering, resulting in a redistribution of velocities. Starting with a homogeneous primary beam of electrons, they found scattered electrons with velocities considerably in excess of the original value as well as those with much less velocity, although the mean energy in the beam was not much changed. Dittmer[2] has shown that the region of scattering approaches the cathode as the discharge current is increased. Penning[3] found oscillations of frequencies from 3×10^8 to

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[1] I. Langmuir and H. Mott-Smith, Gen. Elec. Rev. **27**, 449, 538, 616, 762, 810 (1924).

[2] A. F. Dittmer, Phys. Rev. **28**, 507 (1926); I. Langmuir, Phys. Rev. **26**, 585 (1925).

[3] F. M. Penning, Physica **6**, 241 (1926); Nature **118**, 301 (1926).

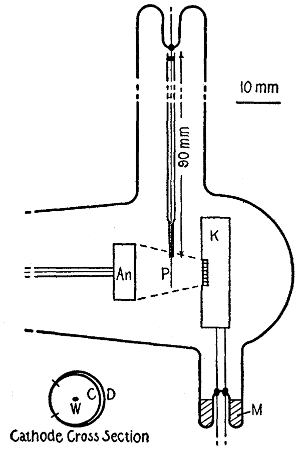

FIG. 1. Experimental tube.

6×10^8 and observed that electron scattering was accompanied by oscillations. Druyvesteyn and Warmoltz[4] observed that the changes in velocity distribution, space potential and electron concentration occur in distinct narrow regions. The phenomenon is especially interesting in its relation to arc discharges with a hot cathode, particularly as it effects the distribution of excitation and ionization.

The most satisfactory explanation, suggested by Dittmer, for the scattering is that it is caused by rapidly varying potential gradients in the discharge due to oscillations of the plasma electrons. Tonks and Langmuir[5] have developed a theory for these oscillations and have shown that, if the reaction of the tube walls and electrodes be negligible, the frequency of oscillation is given by the simple relation $\nu=(n\epsilon^2/\pi m_\epsilon)^{\frac{1}{2}}$ where ϵ and m_ϵ are the electronic charge and mass, respectively, and n is the concentration of the plasma electrons. Such oscillations may produce alternating potential gradients of many volts per cm, and a stream of primary electrons traversing these regions would be scattered accordingly.

In the investigation described in this paper the relation between the plasma oscillations and the scattering of the primary beam was further studied. With a probe which could be moved by very small steps to any part of the discharge and selecting those low pressures and small currents which gave most clearly defined effects, it was possible to study the process in greater detail than has been possible in earlier investigations.

EXPERIMENTAL

The experimental tube is shown in Fig. 1. The indirectly heated oxide-coated cathode K had an active emitting surface, 4×6 mm. A piece of fine nickel mesh of these dimensions was welded to the convex surface of a nickel cylinder C, 6 mm in diameter, inside of which was a tungsten heater W. The mesh was coated with oxide, and care was taken to avoid spreading of the coating on neighboring parts of the cylinder. To further insure a sharp limitation of the emitting surface, a second cylinder D slightly larger in diameter was placed outside the first and in contact with it along the front. The second cylinder had a window, also 4×6 mm, opposite the oxide-covered mesh. Since it was cooler than the inner cylinder, there was no stray emission from it due to sputtered oxide coating. The anode An was a hollow nickel cylinder 12 mm in diameter and 6 mm deep.

A small pool of mercury M in an arm below the cathode supplied the vapor for the arc. This was kept at the temperature corresponding to the desired pressure by immersing it in an oil bath. To prevent condensation, the tube was enclosed in a superheating oven, into which it was rigidly clamped.

The probe P was a tungsten wire, 0.05 mm in diameter and 6 mm long, projecting out of a quartz tube, the lower 1.5 cm of which had a diameter of 0.6 mm. This was supported at the top by what was equivalent to a universal joint, by hanging it on a short section of fine tungsten wire with small clearance between it and the glass wall. To move the probe relatively to the discharge, the oven was tilted either about a horizontal axis parallel to the anode-cathode axis or about one perpendicular thereto. As the probe was easily disturbed by jars and was little damped, the oven was placed on a heavy platform hung from the ceiling. Since changes as small as 0.1 mm in the position of the probe along the anode-cathode axis resulted in marked changes in the observed results, this motion was

[4] M. J. Druyvesteyn and N. Warmoltz, Physica **4**, 51 (1937).

[5] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929); **33**, 990 (1929); I. Langmuir, Proc. Nat. Acad. Sci. **14**, 627 (1928); L. Tonks, Phys. Rev. **38**, 1219 (1931); L. Tonks, Phys. Rev. **37**, 1458 (1931); M. Steenbeck, Zeits. f. Physik **76**, 260 (1932).

controlled by hinging one end of the oven and supporting the other end on a long wire hung from a post on the platform. The thermal expansion of this wire was large, so that by varying its temperature by passing a current through it the probe could be moved in this direction without jarring by steps as small as 0.1 mm. Scales were provided so that the position of the probe in the discharge could be read off directly.

The probe circuit, which was usually completed through the anode, included circuits for detecting and measuring high frequency currents picked up by the probe. Several types of vacuum tubes were tried, but most satisfactory results were obtained with a galena crystal detector, with a circuit similar to that used by Tonks and Langmuir. A Lecher wire was connected in parallel with the detector circuit so that periodic oscillations could be observed and their wavelengths determined. The crystal was calibrated for every set of observations to eliminate errors caused by variations in the sensitivity of the crystal. Tests were made to make certain that these oscillations were independent of the properties of the external circuit.

Before making measurements, the arc was run until it had come to an equilibrium temperature and pressure. When measuring the voltage-current characteristics, it was found that under some conditions the probe disturbed the discharge so that the arc voltage changed by as much as five percent as the probe voltage was varied: in such cases the current controlling rheostats were varied to maintain the arc current constant.

After correcting in the usual manner for the positive ion current to the probe, the resulting electron current-probe voltage curves were analyzed to find the velocity distribution of the electrons. Typical characteristic curves are shown in Fig. 2(A). The curves in this and subsequent figures are numbered in the order in which they were determined in the investigation. In Fig. 2(A), the ordinates are the probe currents plotted to a logarithmic scale and the abscissas the voltages on the probe, with the space potential taken as zero. The arrows marked A and K indicate the potentials of the anode and cathode, respectively. In all of the curves the analysis showed that two groups of electrons were present. For voltages greater than -5 volts the predominate group was the "ultimate" electrons (plasma electrons with random velocity corresponding to a Maxwell-Boltzmann distribution). For voltages less than -5 volts the "primary" and "secondary" electrons became important. The former are the electrons emitted by the cathode which have suffered no scattering, the latter those primary electrons which have been scattered but not sufficiently to have the same distribution as the ultimate electrons. Both primary and secondary electrons will be referred to as "fast" electrons in the remainder of this paper. The ultimate electrons alone give for negative voltages a straight line relation between the log of the probe electron current and the probe voltage, from which the electron temperature is calculated. For positive voltages a straight line relation between current squared and voltage is found,[1] from which the electron concentration and space potential are calculated. The current to a cylindrical electrode in a homogeneous stream of fast electrons of energy V_0 is given by,

$$i = 2rlI(1+V/V_0)^{\frac{1}{2}}; \qquad V > -V_0,$$
$$i = 0; \qquad V < -V_0,$$

where $2rl$ is the projected area of the probe, I the current density, and i the fast electron current to the probe when at the potential V. By the use

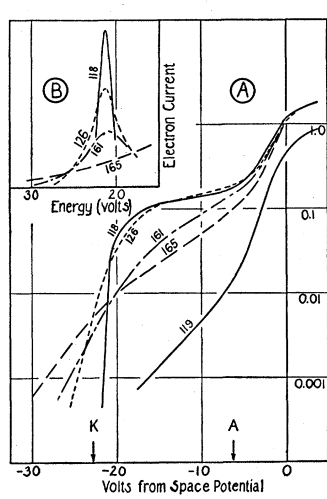

FIG. 2. (A) Typical probe characteristic curves in defined regions of a test mercury discharge: arc vapor pressure, three microns; arc current, 0.042 ampere. (B) Volt-energy distribution of electrons as a function of current in the discharge.

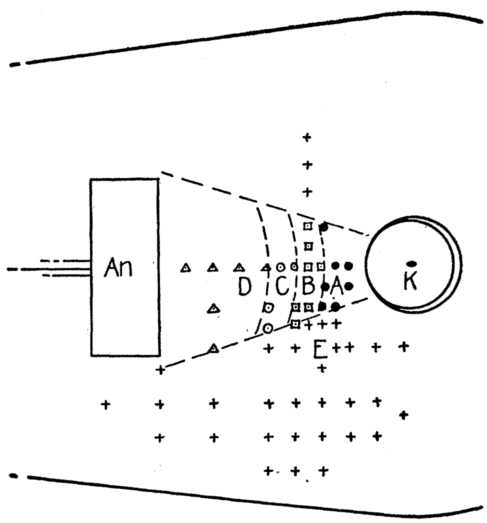

FIG. 3. Probe positions for determining the probe characteristic curves of the test discharge. The discharge is divided into regions, each of which, has a typical probe characteristic current,

of this formula that part of the probe current characteristic caused primarily by fast electrons was analyzed to find their velocity distribution. This was done by dividing them up into a large number of approximately homogeneous groups and building up a synthetic curve, which when added to the curve due to the ultimate electrons, fitted well the experimental results over the whole range of voltages. The precision with which this analysis could be made may be seen from a comparison of curves 423 and 427, in Fig. 5(A), which, although they represent small differences in the electron distribution, were derived from distinctly different probe characteristics. Fig. 2(B) shows the result of such analysis for the curves of Fig. 2(A); the ordinates are the relative number of fast electrons having the energies given in electron-volts by the abscissas. In computing the curves of 2(B), as well as in all similar curves described in this paper, a correction has been made to allow for the increase of the cross section of the arc discharge as the anode is approached. The ordinates in the corrected curves are therefore proportional to the total current of fast electrons rather than their current density.

RESULTS

The temperature and concentration of the ultimate electrons, the velocity distribution of the fast electrons and the space charge oscillations were determined for a large number of points in the axial plane for pressures varying from three to seven microns and for currents between 0.020 and 0.100 amp. Over six hundred complete probe characteristic curves were made in the course of the investigation, and the results were found to be generally reproducible. The results for a number of typical cases are given in Table I. In column (*a*) the pressures of the vapor are given in microns, in (*b*) the arc current in amperes, in (*c*) the arc drop in volts, and in (*d*) the space potential in volts with respect to the anode. Column (*e*) shows the voltage difference between the cathode and space, (*f*) the velocity acquired by electrons falling through this potential difference, and (*g*) lists the observed electron concentrations.

The discharges were all of the well-known type, with cathode drop between 16 to 21 volts and a nearly constant space potential extending almost to the anode, where there was a negative anode

TABLE I. *Data on mercury arc.*

(*a*) Arc Pressure (Microns)	(*b*) Arc Current (amp.)	(*c*) Arc Drop (volts)	(*d*) Space Potential (volts)*	(*e*) Energy Primary Beam (volts)	(*f*) Electron Velocity (cm/sec.)	(*g*) Electron Concentration (No./cc)	(*h*) Oscillation Frequency $8980n^{\frac{1}{2}}$	(*i*) Oscillation Frequency Lecher
3	0.020	11.7	8.1	19.8	2.65×10^8	1.77×10^{10}	1.20×10^9	1.18×10^9
	0.030	12.4	8.4	20.8	2.72	2.56	1.44	1.44
	0.040	13.5	7.3	20.8	2.72	3.33	1.64	1.50
	0.045	14.0	7.0	21.0	2.73	3.72	1.73	—
	0.050	14.7	7.2	21.9	2.78	3.94	1.79	—
7	0.020	9.4	7.5	16.9	2.45×10^8	1.93×10^{10}	1.25×10^9	1.17
	0.030	9.8	7.0	16.8	2.42	3.09	1.58	1.34
	0.040	10.4	6.5	16.9	2.45	3.85	1.76	—
	0.050	11.3	4.8	16.1	2.39	5.65	2.14	—

* Measured from the anode.

drop of from five to eight volts. The part of the discharge in which there were not a negligible number of fast electrons will be referred to as the "arc discharge," and was practically included in a region defined by the active part of the cathode and the periphery of the anode, as shown in Fig. 3. Within this region the paths of nearly all the fast electrons made angles with the cathode-anode axis not greater than 10°.

In Fig. 3 the positions at which probe characteristics were taken are shown for a typical test by the sets of dots, squares, circles, triangles and crosses. The pressure was three microns, the arc current 0.042 amp. and the arc drop 17 volts. Five of the probe current-voltage curves are shown in Fig. 2(A), and are typical of all of the fifty-eight characteristic curves determined for these conditions. The discharge was divided into five regions, marked *A–E* in Fig. 3, in each of which characteristics of one type only were found, 118 in *A* (dots), 126 in *B* (squares), 161 in *C* (cricles), 165 in *D* (triangles), and 119 in *E* (crosses). The first four regions lay entirely in the arc discharge and were separated from one another by surfaces symmetrical to the axis and slightly convex to the cathode. Region *E* included all parts of the tube outside the arc discharge. The boundary of the arc discharge was so sharp that curves such as 118, 126, 161 and 165

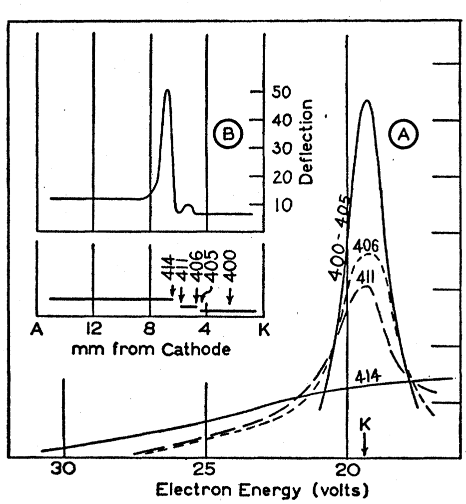

FIG. 4. (A) Volt-energy distribution of electrons to show positions of the regions of scattering: arc vapor pressure, three microns; arc current, 0.020 ampere. (B) The rectified current of the crystal detector circuit as a function of the position of the probe along the axis.

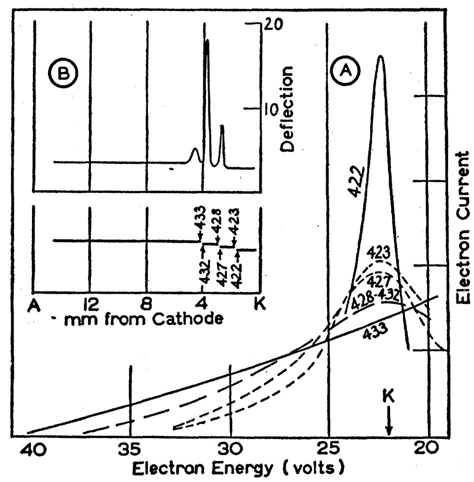

FIG. 5. (A) Volt-energy distribution of electrons to show positions of the regions of scattering: arc vapor pressure, three microns; arc current, 0.045 ampere, (B) The rectified current of the crystal detector circuit as a function of the position of the probe along the axis.

were obtained only 0.5 mm distant from points in *E* for which curves such as 119 were recorded.

Analysis of the ultimate electrons, as discussed above, showed that in every part of the arc discharge and in its immediate environment there was a nearly uniform concentration of the order of 3×10^{10} per cc. The electron temperatures for all points in a given arc discharge and in the neighboring parts of region *E* were practically the same, about 30,000°K.

In Figs. 4 and 5 the results of two more detailed studies of the scattering process as well as the simultaneous measurements of the plasma oscillations are shown. The probe was moved along the axis of the discharge in steps of 0.2 mm, and particular attention was given to the narrow regions in which scattering was observed. The results shown in Fig. 4 were obtained at three microns pressure, with arc current 0.020 amp., arc drop 11.7 volts and space potential 8.1 volts with respect to the anode. In Fig. 4(A) are given a few of the numerous curves obtained, showing the velocity distribution of the fast electrons at critical points. The position of the probe corresponding to each of these is indicated by arrows on the broken line in the lower part of Fig. 4(B), in which the abscissas are distances from the cathode. In Fig. 4(A), the potential of the cathode relative to space potential is indicated by the arrow *K*, at 19.5 volts on the abscissa axis. It will be seen that up to a point 4.3 mm from the cathode all the fast electrons had energies differing by not more than one volt from that corresponding to the cathode-space difference of potential (cf. curves 400–405). Then, in a region 0.2 mm wide, marked scattering occurred (cf. 405–406); electrons with more than 26 volts energy appeared. From 4.5 to 6.1 mm the energy distribution again remained nearly constant (cf. 406–411). At 6.1 mm a second scattering region was found (cf. 411 and 414) resulting in a distribution in which the maximum at 19.5 volts had disappeared and electrons with energies up to 33 volts were measured. Between this region and the anode the scattering was small.

The boundary between the region by the cathode and the adjoining region in the arc discharge was clearly defined visually, by differences in intensity and color. The boundary between the arc discharge and the region outside was also sharply defined by intensity differences.

In Fig. 4(B) the results of the measurements of oscillations as picked up by the probe circuit are shown. The galvanometer deflections due to the current rectified by the crystal are plotted as a function of the distance of the probe from the cathode. At points between the cathode and a point 5 mm distant there was a constant deflection, corresponding to five millivolts across the crystal. No change in this deflection was observed as the Lecher bridge was moved, so that no frequency can be assigned to these disturbances. Furthermore, as the same deflection was observed for all positions of the probe outside the arc discharge, even near the walls, whether the probe circuit was completed to the anode, the cathode or to an electrode pasted on the outside of the tube, it was concluded that this background disturbance was picked up indirectly and could be neglected in studying the variation in the oscillations with position.

At 5.3 mm from the cathode a small peak in the deflection was observed, and the Lecher system gave measurable response throughout this region with nodes spaced 12.7 cm apart. At 6 mm the deflection was again approximately that of the background and there was no response on the Lecher system. At 6.8 mm a larger peak was observed; periodic oscillations of the same frequency as found in the first peak, 1.18×10^9 cycles, were observed for all currents substantially above the background. From this peak to the anode the deflection had a constant value in excess of the background but without any Lecher response. The excess was definitely caused by turbulence in the arc discharge, since, when the probe was moved over the boundary into the region outside the arc discharge, the deflection fell abruptly to the background value. It will be noted that the peaks in the oscillations occur beyond the corresponding regions of scattering by approximately half the distance between them.

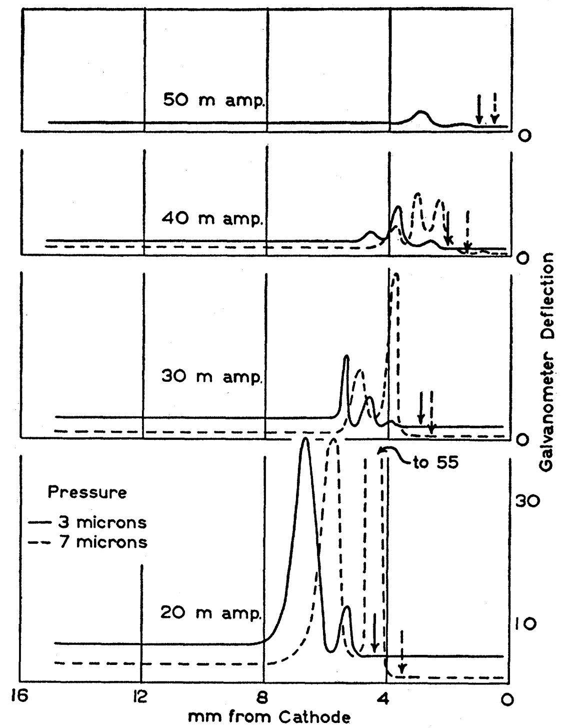

FIG. 6. The rectified current of the crystal detector circuit as a function of the position of the probe along the axis. The arrows show the position of the first region of scattering as found in a family of curves such as shown in Figs. 4 and 5.

In Fig. 5(A), (B) are shown similar results for an arc current of 0.045 amp. The arc pressure was three microns, arc drop 14 volts and the space potential seven volts positive to the anode. With this higher current the regions of scattering were closer to the cathode, the first being only 1.6 mm distant. Three distinct regions of scattering were observed in place of two, and corresponding to these were three oscillation peaks, throughout which the Lecher system showed a definite frequency. The background again gave about 5 mv across the crystal, with no measurable frequency. Between the third peak and the anode the deflection was slightly in excess of the background without any indication of periodicity.

Column (*h*) of Table I gives the frequencies calculated by the Tonks and Langmuir formula for a number of typical cases and (*i*) the corresponding frequencies observed with the Lecher wires. These usually agreed to better than ten percent; the difficulties involved in measuring the concentrations were responsible for much of the lack of precision. For currents greater than 0.040 amp. no frequency is recorded as the response to the tuning on the Lecher wires was so unstable that the frequency could not be determined precisely.

In general, increase of either current or pressure moves the singular regions toward the cathode. In Fig. 6 the observed oscillations are plotted for two pressures, three and seven microns, at four currents, 0.020, 0.030, 0.040 and 0.050 amp. It will be seen that for a given current the curve taken at seven microns pressure has its peaks nearer the cathode than that for the lower pressure and that the effect of increasing the current is also to move the peaks toward the cathode as well as to make them less pronounced. For 0.020 amp. the peaks are exceptionally large; the one nearest the cathode extends to the ordinate value 55, while for 0.050 amp. the peak is very small in the three-micron curve and was unobservable at the seven-micron pressure. The positions of the scattering region nearest the cathode are indicated by solid arrows on the three-micron curve and by broken arrows on the seven-micron curves. It will be noted that these behave in the same way as the oscillation peaks, and that they again occur nearer the cathode.

No measurements of either scattering or oscillation were made at lower arc currents and pressures since the discharges became too unsteady for analysis.

Discussion

The close correlation between the scattering phenomena and the oscillations strongly confirms the conclusion of earlier investigators that the scattering is due to plasma oscillations of the type described by Tonks and Langmuir.[5] The agreement between the frequencies observed and those calculated from their formula (1), $\nu=(n\epsilon^2/\pi m_\epsilon)^{\frac{1}{2}}$, is of particular interest. The fact that oscillations of definite frequencies are observed only in the arc discharge where fast electrons are undergoing scattering leads to the further conclusion that these electrons furnish the energy for the oscillations. There are, however, several important points to be considered in connection with these conclusions.

The scattering regions were observed to be quite narrow, of the order of a few tenths of a millimeter. On the other hand, a single scattering process results in increases in energy of the order of five electron volts. This requires a large electric intensity in the scattering region. Let us assume that a single oscillation region is confined to the space between two parallel infinite planes at $x=o$ and $x=b$, respectively, and that the oscillations are due to motions of the plasma electrons parallel to the X axis. Let the electron concentration at any point be $n=n_0[1+\varphi(t)f(x)]$, where n_0 is the mean electron concentration, $\varphi(t)$ a periodic function of the time, and $f(x)$ some function of x which is zero except in the oscillation region and is such that its instantaneous mean value from o to b is zero. The number of electrons in excess of the mean in the region o to x is

$$S=\int_0^x n_0\varphi(t)f(x)dx$$

and the electric field strength at x resulting from this electron distribution is $4\pi\epsilon S$. Now if ξ is the displacement from its mean position of an electron when it is at x, and if we assume for simplicity that all electrons in a plane perpendicular to the X axis have the same velocity, we have $S=-n_0\xi$. The potential difference across the region is then

$$4\pi\epsilon\int_0^b(-n_0\xi)dx.$$

If this is put equal to five volts $=0.017$ e.s.u. and if typical values, $b=0.03$ cm and $n_0=3\times10^{10}$ are used, the mean value of ξ is approximately 0.003 cm. We see then that n in these experiments probably varied throughout the scattering region by as much as ten percent. The precision of measuring the concentration of the plasma electrons was, however, not sufficient to necessitate taking this variation in n into account, and in calculating the values in Table I the mean concentration was used.

It has been pointed out that the peaks in the oscillation curves occurred from one-half to one millimeter beyond the corresponding scattering regions, and that when the probe was in these latter regions no definite frequency could be detected. The actual plasma oscillations have, however, been assumed to be in the scattering region. To explain the shift it is necessary to consider how the alternating potential is impressed on the probe. Only a negligible part of the periodic current picked up by the probe was caused by the ultimate electrons. That this was principally caused by the fast electrons was shown by a study of the oscillation curves taken with different potential impressed on the probe. Only small changes were noted as this potential was decreased from a value slightly positive to the space potential to a value negative to the space potential by an amount equal to the cathode-space difference of potential. For more negative potentials the peaks decreased in height and finally disappeared, leaving only turbulent disturbances such as were always observed outside these regions. From this we conclude that the periodic fluctuation in the current density of the fast electrons accounted for the alternating potential picked up by the probe. Since a scattering region was narrow and the current density before scattering was practically constant, these periodic fluctuations became appreciable only at some distance beyond the region, where the fast electrons had to some extent overtaken those of mean velocity and the slower ones had dropped back. It is interesting to note the result of a calculation of the magnitude of the fluctuations in the current of fast electrons at the probe, for a simple case in which all of the electrons have the velocity v_0 before entering a narrow scattering region at $x=0$, while those emerging have the velocity, $v=v_0-A\cos 2\pi\nu t$. We find that the amplitude of the current density at any point x is equal to $(a_1^2+b_1^2)^{\frac{1}{2}}$ where,

$$a_1=2(2\nu C_0)\int_0^{\frac{1}{2}\nu}\cos(2\pi\nu x/v)\cos 2\pi\nu t\,dt$$

and

$$b_1=2(2\nu C_0)\int_0^{\frac{1}{2}\nu}\sin(2\pi\nu x/v)\cos 2\pi\nu t\,dt;$$

C_0 is the initial current density. Now since A is, in general, small with respect to v_0, it can be shown that $(a_1^2+b_1^2)^{\frac{1}{2}}=2C_0J_1(2\pi\nu xA/v_0^2)$. This expression has a maximum for the argument equal to 1.84. For $A=0.1v_0$, the value of x for maximum amplitude is approximately equal to 6 mm for typical values of ν and v_0. This is much larger than the observed distances between the oscillation peaks and the corresponding scattering regions. This is probably because we have assumed too simple conditions.

One of the assumptions which is most probably in error is that at any point at a given time all the fast electrons had the same velocity. A study of the velocity distribution of these electrons after scattering shows that there were fewer electrons in the faster and slower groups in comparison with those which retained their original velocity than would be expected on this assumption. It is probable, therefore, that there were always present mixtures of fast electrons of different velocities.

While results seem to point to the fast electrons as the source of the energy necessary to maintain the oscillations, attempts to calculate the energy transfer have not been successful.

For larger currents and pressures than those reported here, the electrons emerge from the immediate vicinity of the cathode already scattered, but no measurements could be made to study the details of the process. The above results suggest that a very violent oscillatory disturbance must exist in a very narrow layer close to the cathode.