Application of Heat Transfer Data to Arc Characteristics

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The chief experimental results of a study of the electric gradient E (v/cm) and current density I (amp./cm^2) in the arc in various gases and pressures can be correlated by means of conduction-convection heat loss data from solid bodies in fluids, and lead to an explanation of the variation of E and I with current i and pressure p which is in good agreement with the measurements. For this purpose we neglect radiation and use an equation of the Nusselt type

$$(W/k\Delta T\pi)=f(D^3\rho^2 g\Delta T\beta_0/\eta^2)=f(x),$$

where W = watts per unit length of arc column; k = conductivity; D = diameter; ρ = density; β_0 = expansion coefficient; and η = viscosity. In the range of interest in arcs $f(x)=x^\alpha$, where $\alpha=0.1$ is a satisfactory approximation. On the assumption that (1) the heat loss from the arc column is given by an equation of the Nusselt type and (2) the variation of arc temperature with i is a small effect, the $E-i$ relation at constant p is found to be E=const. i^{-n} where $n=(2-3\alpha)/(2+3\alpha)$. The exponent n_{exp} for nitrogen $=0.6$, while the $n_{\text{calc}}=0.74$. When p varies, the arc temperature variation becomes important and the effect is included implicitly in the theory, leading to E=const. p^m, and D=const. $p^{-\gamma}$. From heat transfer data and N_2 arc experiments we find that $m_{\text{exp}}=0.32$, $m_{\text{calc}}=0.31$, $\gamma_{\text{exp}}=0.38$, and $\gamma_{\text{calc}}=0.28$. For cases of forced convection we apply correlation data for forced convection cooling in the form $W/\Delta Tk\pi=f(DV\rho/\eta)=(DV\rho/\eta)^\varphi$, where V is the fluid velocity. As above, this leads to the $E-i$ expression for an arc in forced convection E=const. $i^{-n'}$ where $n'=(2-\varphi)/(2+\varphi)$. The exponent $n'_{\text{calc}}=0.54$, while n'_{exp} is not known. For arcs in a variable gravity field g (as in the experiments of Steenbeck), we find E=const. g^q where $q=2\alpha/(2+3\alpha)$. The $q_{\text{calc}}=0.087$, while q_{exp} is not known.

1. INTRODUCTION

Any comprehensive theory of the electric arc must be consistent with the general character of the experimental results on the electrical properties of arcs in various gases.[1] In broad terms, the fundamental observations are (1) the gradient-current relationship* of the form $i=a_1E^{-n}$ where the exponent is always negative and lies in the numerical range $0.25<n<0.70$, (2) the gradient-pressure variation of the form $E=a_2p^m$ where the exponent is always positive and numerically of magnitude $0.15<m<0.32$, and (3) the gradient-gas dependence which shows an increasing gradient through the series Hg, A, N_2, He and H_2.

An arc may be considered as a mechanism of energy transfer. First the electrical energy input is used to dissociate and ionize the gas molecules and atoms and to raise the arc temperature to a high value. The second stage of the energy transfer consists in the dissipation of the thermal energy by radiation, conduction and convection. We consider radiation to be a negligibly small factor in the energy balance. From the low luminous efficiency of these arcs in the visible radiation, and from the total radiation measurements[2] which have been taken on a relatively efficient radiator, the Hg arc, it seems to be a reasonable assumption. Only for very high pressure mercury arcs does the radiation amount to an appreciable fraction of the energy input.[3]

In considering conduction and convection heat loss we are immediately confronted with the question of deciding whether convection can be neglected in the treatment. Elenbaas has shown in an interesting series of papers[4] that the important quantitatives of the high pressure mercury arc can be accounted for satisfactorily on the assumption that the chief energy loss takes place by a process of pure conduction. These mercury arcs are confined to tubes which for stabilizing purposes must necessarily interfere with free convection, so that this result is not evidence that convection heat loss can be neglected as a factor in the treatment of arcs in other media, as for example under conditions of free convection in air. An excellent direct means of investigating the importance of convection effects is to measure the electric gradient E in a

[1] C. G. Suits, Phys. Rev. **55**, 561 (1939).
* A nomenclature table appears at the end of the paper.

[2] W. Elenbaas, Physica **4**, 413 (1937).
[3] W. Elenbaas and W. deGroot, Physica **2**, 807 (1935).
[4] W. Elenbaas, Physica (a) **1**, 211 (1933–34); (b) **1**, 675 (1933–34); (c) **2**, 169 (1935); (d) **2**, 757 (1935); (e) **2**, 787 (1935); (f) **4**, 279 (1937); (g) **4**, 747 (1937); (h) **4**, 761 (1937),

gravity-free chamber, as in the ingenious experiment of Steenbeck.[5] Steenbeck found that for arcs in air (in the four-ampere range) the total voltage e decreased by a factor of two to three times; while the current density I decreased from two to five times in the gravity-free chamber. This is very important direct evidence that convection plays an important role in the energy balance. Steenbeck measured total arc voltage e; if the measurements had been made only on the positive column, a larger reduction of gradient might have been found. When this experiment is performed[6] with the atmospheric pressure mercury arc, the reduction of arc voltage in the gravity-free chamber is only one percent, which justifies the neglect of the convection factor in calculations involving these lamps.

The available evidence indicates quite conclusively the necessity for considering both conduction and convection in the energy balance. For this purpose we shall apply the methods which have been developed in the solution of a similar problem—the conduction and convection heat loss from solid bodies.

2. Convection Heat Loss

The general character of conduction-convection heat loss from solid bodies has been studied by numerous investigators.[7] It is found that the convection currents, which are zero at the surface of the hot body and at a distance of several centimeters, rise to a maximum at a distance of several millimeters from the surface. The full drop in temperature takes place in this region of a few centimeters, with the steepest temperature gradients being directly at the surface of the hot body.

For small temperature differences, approximate solutions of the equations of viscous fluid flow and of the heat conduction and convection equation have been obtained for the free convection problems of a vertical plate as well as of around a horizontal cylinder, based on the Prandtl boundary layer equations.[7(f), 8] They explain the general form of temperature and velocity distribution curves.

The most comprehensive correlation of heat transfer measurements from hot cylinders has been carried out by means of dimensional analysis. By expressing the heat dissipation in terms of dimensionless units, there results, as shown by Nusselt,[7(g)]

$$\left(\frac{hD}{k}\right)=f\left[\left(\frac{D^3\beta_0\rho^2 g\Delta T}{\eta^2}\right),\left(\frac{c_p\eta}{k}\right)\right],\qquad(1)$$

where h = coefficient of heat transfer (heat dissipation per unit area per unit time, per unit temperature difference), D = diameter, k = thermal conductivity, ρ = density, β_0 = temperature coefficient of volume expansion, g = gravity acceleration, η = viscosity, c_p = specific heat, and ΔT = temperature difference between body and ambient fluid. The three dimensionless constants (the terms in parentheses) occurring in (1) are sometimes referred to as the "Nusselt," the "Grashoff," and the "Prandtl" numbers, respectively. The experimental results for liquids and gases prove further that the "Nusselt" number hD/k depends only on the *product* of the Grashoff and Prandtl numbers:

$$\frac{hD}{k}=f\left[\left(\frac{D^3\beta_0\rho^2 g\Delta T}{\eta^2}\right)\left(\frac{c_p\eta}{k}\right)\right].\qquad(2)$$

A survey of all available data on free heat convection from solids (vertical cylinders and planes, horizontal cylinders) confirms this equation with an average scattering of not more than 10 percent in the heat transferred; the data covers a density range of $1:10^4$, a pressure range of $1:10^3$, and a range of the product of the Grashoff and Prandtl of $1:10^{17}$. The form of the curve (2) can be found in McAdams' textbook.[9] Because of the large range of variables, the

[5] M. Steenbeck, Zeits. f. tech. Physik **18**, 593 (1937). In this experiment the arc chamber is allowed to fall freely over a vertical path of 7 meters. The measurements are made during the free fall or gravity-free period.

[6] By private communication from Dr. Carl Kenty, of the General Electric Vapor Lamp Co.

[7] (a) L. Lorenz, Wied. Ann. **13**, 582 (1881); (b) W. Nusselt, Gesundh.-Ing. **38**, 477 (1915); (c) I. Langmuir, Phys. Rev. **34**, 401 (1912); (d) C. W. Rice, J. A.I.E.E. **42**, 1288 (1923); (e) E. Griffiths and A. H. Davis, Report 9, Food Invest. Bd., H. M. Stationery Off., London (1922); (f) E. Schmidt, Zeits. f. Gesamte Kalt-Ind. **35**, 213 (1928); Forschung **3**, 181 (1932); (g) W. Nusselt and W. Jürges, Zeits. V.D.I. **72**, 597 (1928); (h) E. Schmidt and W. Beckmann, Tech. Mech. and Thermo. **1**, 341 (1930); **1**, 391 (1930); (i) R. H. Heilman, Trans. Am. Soc. Mech. Eng. **51**, 257 (1929); (j) K. Jodlbauer, Zeits. V.D.I. **76**, 1031 (1932); (k) R. B. Kennard, Nat. Bur. Stand. J. Research **8**, 787 (1932).

[8] (a) K. Jodlbauer, Forschung **4**, 168 (1933); (b) R, Hermann, Physik. Zeits. **34**, 211 (1933).

[9] W. H. McAdams, *Heat Transmission* (McGraw-Hill, 1933), p. 248.

curve (2) is commonly plotted on log-log scales. The resulting slope α is found to vary from 0.04 to 0.25. Hence, over several ranges (2) can be given the form

$$\frac{hD}{k}=\text{const.}\left[\left(\frac{D^3\beta_0\rho^2g\Delta T}{\eta^2}\right)\left(\frac{c_p\eta}{k}\right)\right]^\alpha, \quad (3)$$

where α varies from 0.04 to 0.25; the multiplicative constant also varies.

The product $c_p\eta/k$ is constant for gases; it is 0.73 for diatomic gases and 0.67 for monatomic gases, and can be absorbed into the constant. Also, for perfect gases, $\beta_0=1/T$, $\rho=Mp/RT$. Hence

$$\frac{hD}{k}=\text{const.}\left[\frac{D^3M^2p^2g\Delta T}{T\eta^2R^2T^2}\right]^\alpha. \quad (4)$$

3. Application to Arcs

In applying the above formula to arcs[10] we are faced with several difficulties. The relation (2) is based on free convection around solids; at the boundary of the latter there is no flow; near an arc, however, the convection velocity is nearly a maximum. Thus the relative heat transfer by convection and conduction from an arc must differ from the transfer around a solid cylinder. Direct measurement of the convection velocities around arcs in air[11] have shown that the actual heat convected upward within the luminous core may be as small as seven percent of the total energy loss. No gross error will be involved if we neglect this portion of the convection loss, for which there is no analog in the heat loss from a solid cylinder.

Another difficulty is due to the large temperature variation within the film; this causes both β_0 and ρ to vary considerably, as well as η, k and c_p, with a resulting uncertainty in the dimensionless constants. While no difficulty arises in the product $c_p\eta/k$, which remains constant in spite of the variation of its components, large variations do occur in hD/k and in $D^3\beta_0\rho^2/\eta^2$. In plotting the available data on heat transfer for purposes of deriving (2), it is customary to evaluate the gas constants at the mean film temperature T_f:

$$T_f=T_{\text{ambient}}+\Delta T/2.$$

It must be kept in mind, however, that the extension of (2) to arc temperatures constitutes a distinct extrapolation.

By replacing h by $W/\pi D\Delta T$, where W is the loss in calories per unit length of arc, and converting to watts, we have $h=Ei/4.18\Delta T\pi D$, and (4) takes the form

$$Ei=(\pi k_f\Delta T)\ \text{const.}\left(\frac{D^3M^2p^2g\Delta T}{T\eta^2R^2T^2}\right)^\alpha. \quad (5)$$

4. $E-i$ Relations

The uncertainties regarding the mean temperature and the proper values for β_0, k, η, ρ disappear if we assume the arc temperature T to be constant and vary only the remaining quantities E, i, R, p (provided it is assumed that the mean values are not affected by these variations). Holding p constant, we have $Ei=\text{const.}\ D^{3\alpha}$. From the Saha equation, n_0, the number of electrons per unit volume, is a function of temperature T only.* The arc current is given by

$$i=(\pi D^2/4)n_0e\mu E, \quad (6)$$

where n_0 is the number of electrons per unit volume, e the electron charge and μ the electron mobility. Since n_0 depends on the temperature only, $i=\text{const.}\ D^2E$; eliminating D, we get

$$i/E=\text{const.}\ (Ei)^{2/3\alpha}, \quad (7)$$

and solving for E, we have

$$E=\text{const.}\ i^{-n},\ \text{where}\ n=(2-3\alpha)/(2+3\alpha). \quad (8)$$

Thus, on the assumptions (1) that the heat loss from the arc column is chiefly by conduction and convection, (2) that the relationship of the physical factors is the same as for heat loss from solid bodies, and (3) that the variation of temperature with i at constant pressure is a negligible factor, we obtain an expression for the gradient E as a function of i.

[10] The first attempts in this direction were made by one of us late in 1934. A summary of results was published in 1937 (Phys. Rev. 52, 136 (1937)). Publication of the complete paper was delayed for more accurate experimental data.

[11] C. G. Suits, Phys. Rev. 55, 198 (1939).

* The arguments which lead us to apply the Saha equation appear in much recent arc literature and will not be reiterated here.

From experimental data on arcs the exponent of i in (8) lies in the range $0.54<n<0.73$ for common gases, being 0.6 for N_2, CO_2 and air. For n to be negative, in agreement with experiment, α must be less than 0.66. Now, the extreme range of α is $0.04<\alpha<0.25$, while for the portion of the heat transfer curve of interest for arcs ($\log hD/k$ near 0), α has the range $0.09<\alpha<0.2$. For the entire range of fluid properties represented by (2) the exponent n of the gradient Eq. (8) would thus be negative, in agreement with experimental data on arcs. For the range of $0.04<\alpha<0.25$, corresponding to the fluid properties and temperatures found in high pressure arcs, the calculated exponent n varies from 0.54 to 0.76, which can be compared to the measured values between 0.54 and 0.73. These values include all of the gases for which free convection arc gradient measurements exist, except Hg vapor. The n for Hg has the value 0.26; in the tubes for which these measurements apply, conditions of free convection are only partially satisfied, as noted previously, which may be to some extent accountable for this result. For air at arc temperatures the heat transfer curve has a slope $\alpha=0.1$; the corresponding value of n is thus -0.74, which can be compared to the experimental value of $n=-0.60$. In summary, it can be said that the fundamental property of a negative $E-i$ arc characteristic at constant pressure can be derived from an application of the conduction-convection heat transfer laws of solid bodies to the arc column if its temperature variation is neglected; and that the exponent of the gradient characteristic is correctly predicted.

5. Variation of D and E with Pressure

In a similar way the $E-p$ and $D-p$ relations can be obtained. The current density I is, of course, given by (6). Pressure p as a variable is included as follows: From the Saha equation,

$$\log_{10}\left(\frac{n_0^2}{N}\right)=-\frac{5040\,V_e}{T}+1.5\log_{10}T+15.38. \quad (9)$$

At constant temperature, $n_0\sim p^{\frac{1}{2}}$; however, the increase in arc temperature with pressure is not a negligible effect in this instance, since n_0 depends upon both p and T. From experimental data on the variation of T with p, we can calculate from (9) the electron density n_0 as a function of p. This has been done for the nitrogen arc $T-p$ data at $i=10$ amperes,[1] as shown in Fig. 1. In general we can put

$$n_0=\text{const. }p^{\beta}, \quad (10)$$

where $\beta=1.44$ from Fig. 1.

The dotted curve of this figure expresses the variation of electron density n_0 with pressure as calculated from the ionization equation on the assumption that the arc temperature is constant, while for the upper (solid) curve this variation of n_0 with pressure is calculated from the same equation, but we use at each pressure the corresponding arc temperature from arc data. The importance of taking the temperature change into account is illustrated by the following calculations. If we neglect the temperature change, the electron density (compared to one atmosphere) will increase 3.16 times at 10 atmospheres. But as a result of the increase of arc temperature with pressure, the actual increase of n_0 with p is 27.5 times at 10 atmospheres. The discrepancy is thus 8.7 times for this pressure interval.

Since arc temperature does not enter explicitly as a variable, the pressure variation of temperature appears as a possible error at other points in the treatment. A detailed investigation shows, however, that it is only in the calculation of n from the ionization equation as described above that this factor need be considered. For example, in the mobility equation

$$\mu=\text{const. }\frac{e}{m_0}\frac{\lambda}{c}, \quad (11)$$

we assume that at constant temperature $\mu\sim 1/p$, and we neglect the small dependence of μ on arc temperature T which enters in the terms λ and c. Similarly, in the heat transfer equation, (5), we neglect the dependence of the right-hand member on temperature which appears chiefly in the terms $k_f(\sim T^{3/4})$ and $\Delta T(\sim T)$ in comparison to the much larger effect of the temperature variation in (9).

Including these factors, we find that

$$i=\text{const. }D^2Ep^{\beta-1}. \quad (12)$$

From $W=Ei$ we obtain

$$\frac{dW}{W}=\frac{dE}{E}+\frac{di}{i}. \quad (13)$$

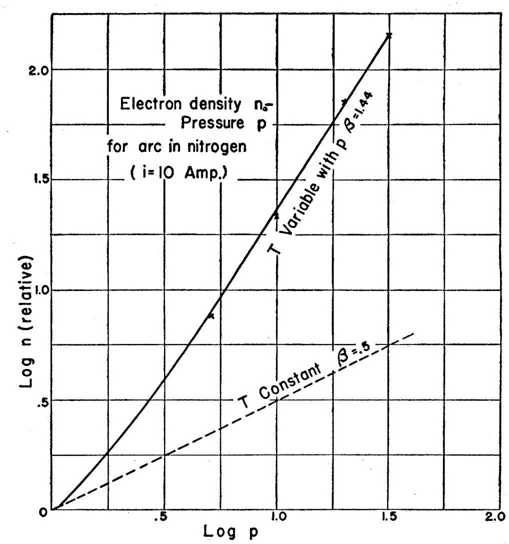

FIG. 1. Electron density as a function of pressure.

Taking logarithmic differentials of (12), we have

$$\frac{di}{i}=\frac{2dD}{D}+\frac{dE}{E}+(\beta-1)\frac{dp}{p}. \quad (14)$$

Similarly, for the heat transfer equation, (5),

$$\frac{dW}{W}=2\alpha\frac{dp}{p}+3\alpha\frac{dD}{D}. \quad (15)$$

Case I

If p is constant, as in the simpler derivation,

$$E=\text{const. } i^{-(2-3\alpha)/(2+3\alpha)} \quad (16)$$

as above.

Case II. *i* constant, *D* as *f*(*p*)

We have, from (13), (14) and (15)

$$2\frac{dD}{D}+\frac{dE}{E}+(\beta-1)\frac{dp}{p}=0, \quad (17)$$

$$3\alpha\frac{dD}{D}-\frac{dE}{E}+2\alpha\frac{dp}{p}=0. \quad (18)$$

Eliminating dE/E and combining terms we obtain

$$\frac{dD}{D}+\frac{dp}{p}\left(\frac{\beta+2\alpha-1}{2-3\alpha}\right)=0. \quad (19)$$

Finally,

$$D=\text{const. } p^{-(\beta+2\alpha-1)/(2-3\alpha)}=\text{const. } p^{-\gamma}, \quad (20)$$

which expresses the dependence of the arc diameter D on pressure p in terms of the exponent α of the heat transfer equation and the experimental constant β for the variation of electron density with pressure and temperature.

Case III. *i* constant, *E* as *f*(*p*)

Eliminating dD/D in (17) and (18)

$$\frac{dE}{E}\left(\frac{1}{2}-\frac{1}{3\alpha}\right)+\frac{dp}{p}\left[\left(\frac{\beta-1}{2}\right)+\frac{2}{3}\right]=0. \quad (21)$$

Finally, we have

$$E=\text{const. } p^{-\alpha[3(\beta-1)+4]/(3\alpha-2)}=\text{const. } p^{m}. \quad (22)$$

Eq. (22) expresses the pressure variation of the arc gradient in terms of the exponents β and α.

For nitrogen and air $\beta=1.44$ and $\alpha=0.1$. The calculated values of γ and m are compared in Table I. The experimental values for γ and m show a dependence on current which is not predicted by the theory. This effect arises from the small arc current-arc temperature variation which could be taken into account by evaluating β for different values of current.

The calculated values of m and γ lie in the correct range and, in view of the assumptions, show a satisfactory agreement with the measured values. The lack of comprehensive pressure-temperature data for other gases prevents a calculation of m and γ for these cases at present.

6. Arcs in Forced Convection

It is of some interest to examine the case of arcs in fluids under conditions of forced convection, since in this case the arc temperature T does not appear explicitly in the right-hand member of the heat transfer expression, which has the form

$$\frac{hD}{k}=\frac{W}{\Delta Tk\pi}=f\left(\frac{DV\rho}{\eta}\right)=\text{const.}\left(\frac{DV\rho}{\eta}\right)^{\varphi}, \quad (23)$$

where all the quantities are defined above, except V, which is the fluid velocity. Considering first the case of constant temperature with variable current and pressure, we have from (31)

$$\frac{dW}{W}=\varphi\left(\frac{dD}{D}+\frac{dp}{p}\right). \quad (24)$$

Combining (23) with the previously determined relations above, we obtain, at constant pressure

$$E=\text{const. } i^{-(2-\varphi)/(2+\varphi)}=\text{const. } i^{-n'}. \quad (25)$$

Thus under conditions of forced convection the exponent n' of the current in the $E-i$ relation has the form $(2-\varphi/2+\varphi)$, which can be compared to the same quantity for free convection where $n=(2-3\alpha/2+3\alpha)$. The $E-i$ characteristic will be negative for forced convection cooling of the arc when $\varphi<2$. A probable value of φ estimated from known correlation data is $\varphi=0.6$, for which $-n'=-0.54$. When careful measurements of arc gradient for forced convection conditions are available, it will become of interest to examine them from the viewpoint of the above calculation.

7. Arcs in a Variable Gravity Field

In view of the experiments of Steenbeck[5] and some unpublished experiments of our own in which an arc is subjected to a centrifugal field, it is of interest to calculate the variation of E with g on the basis of the relationships used above. To include g, we write

$$Ei=\text{const. } D^{3\alpha}g^{\alpha}, \quad (26)$$

which may be combined with $i=\text{const. } D^2E$ to obtain

$$E=\text{const. } g^{2\alpha/(2+3\alpha)}=\text{const. } g^{q}. \quad (27)$$

With $\alpha=0.1$ as used previously, this expression becomes

$$E=\text{const. } g^{0.087}. \quad (28)$$

This relationship (27) expresses the dependence of arc gradient E upon the gravity field g. Applying (27) to Steenbeck's experiments shows that E and g both go to zero together. If g is sufficiently small, the arc voltage observed in the gravity-free chamber should be the sum of the anode drop e_A and cathode drop e_c, since the positive column drop will become zero. This could be checked by actually measuring the positive column gradient in the gravity-free chamber by the vibrating electrode method. An alternative check would be a separate measurement of e_A+e_c as, for example, by shorting electrodes. This latter method is somewhat indirect and involves additional postulates.

We have some incomplete data on the variation of E with g where g is increased by centrifuging the arc. The dependence of E on g is so small, however, that very large values of g must be used to obtain accuracy. In the measurements we have available the accuracy is not satisfactory, but it can be said for certain that the coefficient q of the gravity effect is much smaller than the coefficient m of the pressure effect.

8. Discussion

The method of treatment yields expressions for the chief electrical measurables of the high pressure* arc in terms of heat transfer quantities and one experimental constant depending upon the pressure variation of arc temperature. The important assumption is that the heat loss from the arc column is given by a relationship of the form known to apply to convection loss from solid bodies. The general agreement between the measured and calculated quantities is evidence that for the experimental conditions considered no prominent variables have been omitted. Since all of the quantities involved in convection heat loss as expressed by (5) are not varied explicitly in the experiment, we cannot conclude in general that the heat loss from the

Table I. *Values of γ and m for nitrogen.*

$\gamma_{CALC}=0.28$ $(i=10)$	$m_{CALC}=0.31$ $(i=10)$
$\gamma_{exp}=0.30$ for $i=1$	$m_{exp}=0.29$ for $i=1$
$=0.30$ for $i=2$	$=0.31$ for $i=5$
$=0.35$ for $i=5$	$=0.32$ for $i=10$
$=0.38$ for $i=10$	

* "High pressure," in this sense, refers to the range around and above atmospheric pressure. More fundamentally, the thermal equilibrium considerations apply to discharges in which the energy acquired by the electron per mean free path is small compared to its thermal energy. This criterion is best satisfied at high pressure and high current. From the fact that there is direct evidence from spectroscopic studies that thermal equilibrium is established in the current range of our experiments at one atmosphere pressure in air, it can be inferred that the same is true at higher pressures and currents.

arc column involves all of these quantities in this particular relationship. That p and D enter in the form D^3p^2 is, however, a proper deduction. It is evident that some of the other quantities in (5) could be varied experimentally and some evidence of the result is already known. In the experiments of Steenbeck, for example, g has been reduced to practically zero with the result that could be predicted from (27). More experiments with a gravity-free chamber or an arc centrifuge would be interesting, particularly if E were measured instead of e. The other quantity which could be varied in (5) is the ambient temperature, and so far as is known no data exist on the effect on arc gradient E.

These conclusions apply, of course, to arcs under conditions of free convection (except Section 6). One would expect the effect of pressure on arc gradient to be different if free convection is not possible in the experiment. However, since the convection force increases linearly with pressure for the same temperature difference, an arc space which would be totally inadequate for convection at one atmosphere may permit prominent convection effects at higher pressure. Similarly, in the case of transient arcs, a time interval which would at one atmosphere be short in terms of the thermal processes in the arc may be relatively very long at higher pressure. In other words, the "thermal time constant," which involves the rate of convection heat flow, must be much shorter at higher pressures.

It would be better in general if the theory included the arc temperature T as an explicit variable, but some new relationship not known at present will be required for that purpose. In the case of constant pressure, the inclusion of T is not of great importance since temperature measurements show a small dependence on current. When pressure varies, however, the temperature change is important because a rather large variation of pressure is required to measure gradient changes with accuracy, and for this case a means of including T explicitly would improve the treatment.

In the analysis given, a relationship between D and p of the form (5) results in a satisfactory agreement with experiment. It is important to note that this is not equivalent to saying that (5) can be used to calculate the heat loss from the arc column; that is a relevant but different question which involves the constant of proportionality of (5). An investigation of this subject will be reported in a separate paper. The results show that (5) can be applied to arcs with a suitable interpretation of the constants. The method of interpretation is determined from a study of the data on heat loss from wires at high temperatures, which show systematic deviations from the Nusselt correlative curve at the highest temperatures.

It is clear that the conduction-convection heat loss in a series of gases increases in the same order as the gradient,[1] although the detailed analysis of this question is not given here. Some uncertainty exists in the relative order of He and N_2 in regard to heat transfer. Experimentally E is slightly greater in He than in N_2 for steady-state arcs. For some transient arcs He and N_2 appear to be identical within the errors of measurement. Although the low temperature heat transfer in N_2 is much lower than He, it appears that at high temperature the contribution of nitrogen dissociation to heat transfer balances the more favorable kinetic factors of He.

We may presume that the correlation of heat transfer data and the electrical properties of arcs is a matter of considerable generality and is not limited to the case of free convection. It would be of interest to examine cases of forced convection with that in mind. There seems little doubt that the arc gradient in a forced convection air stream of 1 meter sec.$^{-1}$ velocity would be the same as in an arc in air under free convection velocities[11] which are known to be of this same magnitude.

Nomenclature
(in order of appearance)

E—Electric gradient along the axis of the positive column of the arc (volts cm^{-1})
e—Total voltage drop between the electrodes (volts)
i—Current (amp.)
I—Current density (amp. cm^{-2})
h—Surface coefficient of heat transfer (cal. sec.$^{-1}$ cm^{-2} deg.$^{-1}$)
D—Diameter of cylinder
k—Thermal conductivity (cal. sec.$^{-1}$ cm^{-1} deg.$^{-1}$)
β_0—Temperature coefficient of volume expansion (deg.$^{-1}$)
ρ—Density (g cm^{-3})

g—Acceleration of gravity (cm sec.$^{-2}$)
η—Viscosity (g sec.$^{-1}$ cm^{-1})
c_p—Specific heat at constant pressure (cal. g^{-1} deg.$^{-1}$)
ΔT—Temperature difference (max.) between hot surface and surrounding fluid (deg.)
α—Exponent of the "Grashoff" term in the approximate expression for convection heat transfer.
M—Molecular weight
p—Pressure (g cm^{-2})
T—Temperature absolute (deg.)
R—Gas constant (cm deg.$^{-1}$)
π—3.1416
W—Heat loss per unit length of arc column (cal. cm^{-1})
$-n$—Exponent of the current in the $E-i$ relation
n_0—Number of electrons per unit volume (cm^{-3})
μ—Mobility of the electron (cm^2 sec.$^{-1}$ volt^{-1})
e—Electronic charge (coulombs)
N—Molecules per unit volume (cm^{-3}) (Loschmidt number)
V_e—Effective ionization potential (volts)
β—Exponent of p in the n_0-p relation
m_0—Mass of the electron (g)
λ—Mean free path (cm)
c—Average velocity (cm sec.$^{-1}$)
$-\gamma$—Exponent of p in the $D-p$ relation
m—Exponent of p in the $E-p$ relation
V—Fluid velocity (cm sec.$^{-1}$)
φ—Exponent of the term $(DV\rho/\eta)$ for forced convection heat transfer
$-n'$—Exponent of i in $E-i$ expression for forced convection cooling
q—Exponent of g in the $E-g$ relation
e_A—Anode drop (volts)
e_c—Cathode drop (volts)

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Electron Scattering and Plasma Oscillations*

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The dependence of electron scattering upon plasma oscillations was studied in a mercury arc discharge tube containing a movable probe. The best conditions for studying these relations were found when the vapor pressures and arc currents were small. Probe volt-ampere characteristics showed the presence of ultimate electrons with a Maxwell-Boltzmann distribution corresponding to about 30,000°K and a superposed stream of fast electrons emitted from the cathode. These fast electrons were scattered without appreciable change in direction in well-defined, approximately plane regions only a few tenths of a millimeter wide. The plasma oscillations were studied with a crystal detector in the probe circuit supplemented by a Lecher wire system. In addition to "turbulent" disturbances with no measurable frequencies found throughout the tube, narrow regions were found in which stable periodic oscillations of considerable magnitude were detected. These regions and the scattering regions were, in general, equal in number and coincided except for a small shift of the former in the direction of the anode. The observed frequencies agreed well with the formula derived by Tonks and Langmuir. The scattering regions and the regions in which periodic oscillations were observed became less marked and moved toward the cathode as either the vapor pressure or arc current were increased. The periodic oscillations were found only in regions traversed by the fast electrons. The results are interpreted as showing that scattering is due to plasma oscillations which receive their energy from the fast electrons. The process by which the oscillations were detected by the probe is discussed to account for the shift of the regions in which oscillations were observed with respect to the scattering regions.

INTRODUCTION

IN 1924 Langmuir and Mott-Smith[1] showed that in an electric discharge in a gas or vapor the primary electrons emitted by the cathode suffer scattering, resulting in a redistribution of velocities. Starting with a homogeneous primary beam of electrons, they found scattered electrons with velocities considerably in excess of the original value as well as those with much less velocity, although the mean energy in the beam was not much changed. Dittmer[2] has shown that the region of scattering approaches the cathode as the discharge current is increased. Penning[3] found oscillations of frequencies from 3×10^8 to

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