Coulomb Energies and Nuclear Models

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(Received April 22, 1939)

Electrostatic energy differences of isobaric light nuclei, estimated by use of the alpha-model, do not agree with experiment as well as those given by Bethe for the central model. The comparison with experiment of the binding energies of alphas in the light $4n$-particle nuclei, given by Hafstad and Teller for the alpha-model, is improved by inclusion of the Coulomb energy.

IN THE face of the intricacy of nuclear structure, attempts have been made to correlate certain properties by means of simplifying models. In particular, two models have been useful: the central model in which the particles (protons and neutrons) are initially considered as moving independently in a central field;[1] and the alpha-model in which the particles are pictured as clustered into alpha-particles as far as possible.[2] In the $(4n+1)$-particle nuclei, the alpha-model has one particle moving in the field of the alphas. In the $(4n-1)$-particle nuclei, one of the alphas lacks a particle; the deficit is exchanged rapidly between the alphas. Geometrically the two models are opposite extremes, although they exhibit striking similarity where angular momenta are concerned.[2(g)]

The Coulomb energy of nuclei should depend principally on the geometrical properties of the nuclei and not directly on the nature of the binding forces, of which we must still profess ignorance. It is therefore of interest to determine whether calculation of this energy can furnish a criterion of the relative value of the two extreme models. The electrostatic energy differences of isobaric pairs of light nuclei according to the central model have been compared with experimental values by Bethe.[3] He considered two approximations to the central model, (I) one in which the particle density is constant throughout a sphere, and (II) one in which the most loosely bound proton has a preferentially large radius. Using only one value (marked with an asterisk) to determine parameters, he was able to reproduce the other experimental values rather well, as indicated in columns I and II of Table I. In the alpha-model the geometrical environment of the "most loosely bound" particle is quite different in the $(4n-1)$- and $(4n+1)$-particle nuclei, so that comparison of nuclei in different groups involves unjustifiably detailed assumptions about the model. Comparison of the few nuclei within one group, however, involves only the more immediate features of the model.

For the purpose of estimating the Coulomb energy (neglecting exchange, as done in reference 3), a $(4n-1)$-particle nucleus is pictured as $n-1=0$, 1, 2, or 3 alpha-particles and a triton (or He^3); the alpha-alpha distances all have the same value $r_{\alpha\alpha}$ and the triton-alpha distances all

TABLE I. *Coulomb energies involved in isobaric transitions (in mMU).*

n	Isobaric Pair	Central Model[3] I	Central Model[3] II	Alpha-model	Observed[3]
$(4n-1)$ 1	H^3-He^3	0.87	0.84	0.74*	0.74
2	Li^7-Be^7	2.0	1.99	1.92	<1.94
3	$B^{11}-C^{11}$	2.7	3.0	3.1*	3.1
4	$N^{15}-O^{15}$	3.5	3.8	4.28	3.7
$(4n+1)$ 3	$C^{13}-N^{13}$	3.21*	3.21*	(3.54)	3.21
4	$O^{17}-F^{17}$	3.9	3.9	(4.26)	3.9

[1] (a) W. Heisenberg, Zeits. f. Physik **96**, 473 (1935); (b) E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937); (c) M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937); (d) D. R. Inglis, Phys. Rev. **55**, 329 (1939), (erratum: 13 and 14 lines from the end of p. 335, and in the third line of p. 336, the words "positive" and "negative" should be exchanged); (e) H. Margenau and K. G. Carrol, Phys. Rev. **54**, 705 (1938); (f) W. J. Kroeger, Phys. Rev. **54**, 1048 (1938).

[2] (a) J. H. Bartlett, Jr., Nature **130**, 165 (1932); (b) E. Teller and J. A. Wheeler, Phys. Rev. **53**, 778 (1938); (c) L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938); (d) W. Wefelmeier, Zeits. f. Physik **107**, 332 (1937); (e) H. A. Bethe, Phys. Rev. **53**, 843 (1938); (f) B. O. Grönblom and R. E. Marshak, Phys. Rev. **55**, 229 (1939); (g) R. G. Sachs, Phys. Rev. **55**, 825 (1939).

[3] H. A. Bethe, Phys. Rev. **54**, 436 (1938). *Note added in proof* (suggested by Professor Teller):—The central-model calculation of Coulomb energies, taking into account the concentration in angle of the p-protons, was carried out earlier by Feenberg and Wigner, Phys. Rev. **51**, 95 (1937). Their values, in the order of listing in Table I, are 0.74, (1.4), 2.2, 3.0, 2.5, 3.1. These are consistently small, parameters having been determined by other data. They would be increased (the first less than the others) if the better inverse-square-range parameter $\alpha=22$ were used. Simple multiplication of each value but the first by 1.28 gives about as good agreement as does column II.

have the value $r_{t\alpha}$. In this model, the transition from one isobar to the other involves the transition $H^3 \rightarrow He^3$, the energy of which is taken from experiment to be 0.74 mMU. In the field of $n-1$ alphas at a distance $r_{t\alpha}$, the Coulomb energy due to each of the alphas is $D=2e^2/r_{t\alpha}$. The Coulomb energy C involved in an isobaric transition is then

$$C=(n-1)D+0.74 \text{ mMU}.$$

From the experimental difference $B^{11} \rightarrow C^{11} (n=3)$, we determine $D=1.18$ mMU. The consequent values for $n=2$ and 4 are listed in Table I.

The lack of agreement for $N^{15} \rightarrow O^{15}$ is rather striking. It might be associated with the low stability of Be^8 thus: the three alphas in N^{15} may be closer together than the two alphas in B^{11}, and the more crowded alphas might hold the triton further off by the exclusion principle, and thus reduce C for the case $n=4$. However, this is hardly compatable with a rapid exchange of alpha and triton, and such an artificial modification of the model practically destroys its utility.

The case $(4n+1)$ is less definitely determined by the alpha-model, since the extra particle is not pictured as having a definite position in the framework, but merely as having nodes at the alphas.[2(c)] Furthermore, the data on this type are rather sparse. The particle taken to be situated just where a triton would be in the previous case (and taking $r_{\alpha\alpha}=r_{t\alpha}$) gives the values listed in parentheses in Table I, but this has relatively little meaning. It is qualitatively understandable that the values so obtained are too high; the superfluous triton binding has drawn the particle inward unduly.

Such comparison of the ground states of isobaric pairs is not possible for the $4n$ nuclei. The binding energies of the $4n$ nuclei have been correlated by Hafstad and Teller,[2(c)] with the alpha-model. In their interesting considerations, the Coulomb energy was neglected. Although the Coulomb energy is very small compared with the total binding energy in light nuclei, it is actually not so very small compared with the binding between alphas which one treats by the alpha-model. It has therefore some interest to modify their correlation by taking into account the Coulomb energy. In so doing we take $r_{\alpha\alpha}=r_{t\alpha}$ in the $4n-1$ case (as is reasonable because of the exchange), and take for D the average value $D=1.1$ mMU (or $r_{\alpha\alpha}=2.8\times10^{-13}$ cm) determined by both $n=3$ and $n=4$ of the $4n-1$ nuclei. This is probably an underestimate of the Coulomb energy of the $4n$ nuclei; alphas are somewhat smaller and more closely packed than tritons. The geometrical configurations are taken from

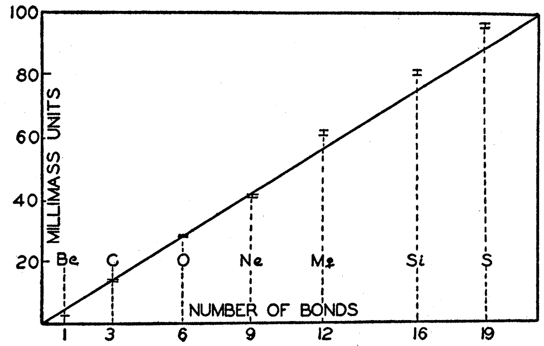

FIG. 1. Bond energy of the $4n$ nuclei (mass defect relative to alphas, plus Coulomb energy).

reference 2(c). The results are plotted in Fig. 1, which is to be compared[4] with Fig. 2 of reference 2(c). In Fig. 1 the experimental values represent the magnitude of the energy due to the binding forces between alphas (mass defect relative to alphas, plus estimated Coulomb energy), which should be proportional to the number of bonds. (The pairs of limiting values indicate the experimental uncertainty only.) The Coulomb energy alone is also proportional to the number of bonds up to O^{16}, after which it increases more rapidly than the number of bonds and raises the experimental lines. The discrepancy for Be^8 is thus not affected in magnitude by these considerations, that for Ne^{20} is removed, and instead one has discrepancies for the heavier elements, for which the use of the model is more questionable. The Be^8 discrepancy is less striking in Fig. 1 than in Fig. 2 of reference 2(c), mostly because of inclusion of the Coulomb energy and consequent change of scale, but also to a slight extent because of more recent experimental data.[5]

The application of the alpha-model to the light $4n$ nuclei is thus rather satisfactory, due probably to the great stability of the alphas. The less satisfactory result for the $(4n-1)$ nuclei may be ascribed to the fact that the internal binding of the triton is small compared to several triton-alpha bonds, insufficient to retain the identity of the triton in N^{15}, for example. It is not small compared to one bond, so this view is compatible with the relative success of the alpha-model in regard to the Li^7 magnetic moment.[2(e), 1(d)]

[4] Note change of scale by a factor 2, due to Coulomb energy.

[5] Allison, Graves, Skaggs and Smith, Phys. Rev. **55**, 107 (1939).