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Galactic Rotation and the Intensity of Cosmic Radiation at the Geomagnetic Equator

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The angle of deflection experienced by a primary cosmic particle moving in the plane of the geomagnetic equator is calculated for different energies and the result used to find the diurnal variation in the intensity of cosmic rays arriving vertically at the geomagnetic equator. If it is assumed that the number of primaries varies inversely as the cube of their energy, the calculation shows that there should be a diurnal variation in the vertical intensity of 0.17 percent, if all primaries are positive, with a maximum at 13 hr. 20 min. sidereal time. With a ratio of three positive to one negative primary, and the same distribution law, the amplitude of the diurnal variation should be 0.1 perçent with a maximum at 12 hr. 30 min. sidereal time, while if the primary radiation as a whole is neutral (one positive particle to each negative) the amplitude should be 0.06 percent with maximum at 8 hr. 40 min. sidereal time. If the number of primaries is an exponentially decreasing function of their energy, the amplitude of the diurnal variation should be 0.24 percent with maximum at 18 hr. sidereal time, assuming all primaries are positive; if the primary radiation is neutral the amplitude should be 0.19 percent and the maximum should occur at 20 hr. 40 min. sidereal time. The expected diurnal variations for several values of the lower limiting energy are also discussed.

1.

A FUNDAMENTAL question in the theory of cosmic radiation is whether the radiation comes from outside our own galaxy. A possible way of answering this problem was first suggested in 1935 by Compton and Getting.[1] They pointed out that, as a consequence of the motion of rotation of our galaxy as a whole, there should be a small diurnal variation of the intensity depending on sidereal time. Since then they, and a number of others, have assiduously sought to establish experimentally the existence of this effect, as yet with results which are in part contradictory and largely inconclusive.

Compton and Getting developed the theory of the galactic rotation effect without regard to the deflection of charged primary particles by the earth's magnetic field. They then made a rough estimate of the error introduced by neglecting this deflection, and pointed out that it should result in still further decreasing the small expected diurnal variation in the absence of a magnetic field. In view of the importance of the question at issue, it appeared desirable to develop the exact theory of the galactic rotation effect even if at present the calculations can be carried out rigorously only for the particularly simple case of particles moving in the plane of the geomagnetic equator. A calculation due to van Wijk[2] already showed that, under the assumption that the number of primary particles varies as an exponentially decreasing function of their energy,

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[1] A. H. Compton and I. A. Getting, Phys. Rev. 47, 817 (1935).

[2] L. A. van Wijk, Physica 3, 769 (1936).

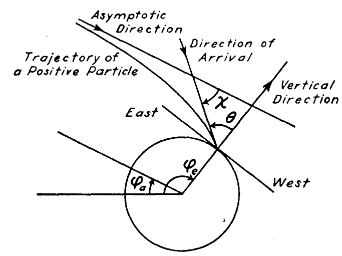

FIG. 1. The figure is drawn for an observer looking down on the equatorial plane from the north pole.

the maximum value of the diurnal variation for particles coming vertically at the geomagnetic equator should be about 0.25 percent, instead of 0.3 percent which would be expected if the earth's magnetic field were absent. He did not attempt to calculate the phase of the diurnal variation.

In the present paper we calculate the deflection of cosmic particles moving in the equatorial plane and then, by taking into account the motion of the earth due to the galactic rotation, we find the diurnal variation in the intensity of cosmic rays arriving vertically at a point in the geomagnetic equator. Methods similar to those used here yield the diurnal variation in any direction in the east-west plane at the geomagnetic equator. The results are similar to those reported here.

2.

The angle of deflection of a primary cosmic-ray particle reaching the earth from infinity can be easily calculated in the case we are now considering. We shall confine our attention to positive particles; the deflection of negative particles will be equal in magnitude but in the opposite direction to that of positive particles with the same absolute charge, mass, and energy. From Fig. 1 we see that the angle of deflection χ is given by

$$\chi = \varphi_e - \varphi_a - \theta, \tag{1}$$

where φ_e and φ_a are the polar angles of the trajectory for $r = r_e$, the radius of the earth, and for $r = \infty$, respectively, and θ is the direction of arrival of the particle at the earth. We are using Störmer's normalized coordinates so that r_e is the ratio of the radius of the earth to the radius of the circular periodic orbit of the particle, and it is a measure of the energy of the particle. For the angles we have adopted the following conventions: φ is measured positive westward from the polar axis to the radius vector; θ is measured positive eastward from the vertical to the reversed direction of arrival; χ is measured positive clockwise from the asymptotic direction to the direction of arrival (Fig. 1).

The angles φ_e and φ_a are obtained by putting $r = r_e$ and $r = \infty$, respectively, in the polar equation of the trajectory.[3] For $\gamma_1 < 1$ ($2\gamma_1$ is the moment of momentum of the particle, in the equatorial plane, at infinity and is a constant of the motion) we have

$$\begin{aligned} \varphi_e &= \sin^{-1}(k \sin \psi_e) + (\gamma_1/\sqrt{2})F(\alpha, \psi_e), \\ \varphi_a &= \sin^{-1}(k \sin \psi_a) + (\gamma_1/\sqrt{2})F(\alpha, \psi_a), \end{aligned} \tag{2}$$

where

$$\psi_e = \cos^{-1}(r_e\gamma_1^{-1})/(r_e(1+\gamma_1^2)^{\frac{1}{2}}),$$
$$\psi_a = \cos^{-1}\gamma_1(1+\gamma_1^2)^{-\frac{1}{2}},$$
$$k = \sin\alpha = [\tfrac{1}{2}(1+\gamma_1^2)]^{\frac{1}{2}},$$

and $F(\alpha, \psi)$ is the elliptic integral of the first kind. For $\gamma_1 > 1$ we have

$$\begin{aligned} \varphi_e &= \psi_e + \gamma_1(1+\gamma_1^2)^{-\frac{1}{2}}F(\alpha, \psi_e), \\ \varphi_a &= \pi/4 + \gamma_1(1+\gamma_1^2)^{-\frac{1}{2}}F(\alpha, \pi/4), \end{aligned} \tag{3}$$

where $\psi_e = \sin^{-1}[(2r_e\gamma_1 + r_e^2 - 1)/2r_e^2]^{\frac{1}{2}}$,

and $\sin\alpha = [2/(1+\gamma_1^2)]^{\frac{1}{2}}$.

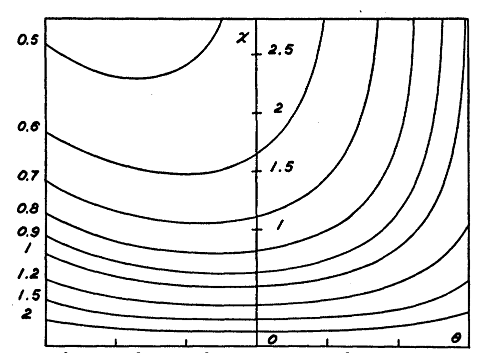

FIG. 2. The angle of deflection as a function of the angle of arrival, for different energies (expressed in störmers). $r_e = 0.5$ for $\theta = 0$ corresponds to an asymptotic orbit for which $\chi \to \infty$.

[3] C. Graef and S. Kusaka, J. Math. Phys. 17, 43 (1938). See also, L. A. van Wijk and H. Zanstra, Physica 3, 75 (1936).

Now θ and γ_1 are related by Störmer's equation[4]

$$\sin \theta = 2\gamma_1/r_e - 1/r_e^2, \tag{4}$$

so that for any given value of r_e, χ is a function of θ alone.

We have calculated χ for all possible values of θ ($-90°$ to $+90°$) and for several values of r_e. The result is plotted in Fig. 2 where χ is given in radians and θ in degrees.

3.

We shall define the intensity of cosmic radiation, I, as the number of particles received per second per unit area. Then if $f(E)$ is the energy distribution function at infinity, the intensity of cosmic rays arriving in the vertical direction at the earth when it is considered to be at rest with respect to the distribution of cosmic rays is, by Liouville's theorem,

$$I = \int_{E_0}^{\infty} f(E) dE, \tag{5}$$

where E_0 is the lowest energy the particle can have to arrive at the given point of the earth in this direction. When the motion of the earth is taken into account the intensity is given by[5]

$$I' = \int_{E_0}^{\infty} \frac{f(E)dE}{(1-\beta \cos \omega')^3} \tag{6}$$

where β is the ratio of the velocity of the earth to the velocity of light, ω' is the angle between the velocity of the earth and the reversed tangent to the trajectory at infinity. From Fig. 3 it is seen that

$$\omega' = \chi + \omega, \tag{7}$$

where ω is the angle between the velocity of the earth and the vertical at the point of observation. From the analysis[6] of Swann, Heitler, Nordheim and others, a likely distribution function of the primaries is

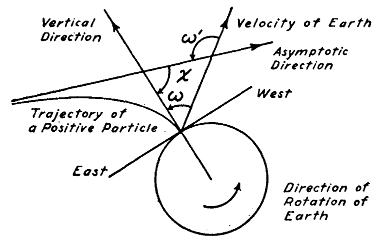

FIG. 3. Diagram of the angles used in the text.

$$f(E) = c/E^3. \tag{8}$$

Now the energy in störmers written in terms of the energy of the particle in electron volts is[7]

$$r_e = R\left(\frac{E}{300Mc\epsilon Z}\right)^{\frac{1}{2}}\left(1+\frac{600m_0c^2}{E}\right)^{\frac{1}{4}}, \tag{9}$$

which for very high energies ($E \gg m_0c^2$) reduces to

$$r_e = KE^{\frac{1}{2}}, \tag{10}$$

where K is a constant. Hence

$$f(E) = K^6/r_e^6 \tag{11}$$

and so

$$I = 2K^4 \int_{r_0}^{\infty} dr/r^5, \tag{12}$$

and, from (6),

$$I' = 2K^4 \int_{r_0}^{\infty} (1+3\beta \cos \omega') dr/r^5, \tag{13}$$

where r_0 is the value of r_e corresponding to E_0. Thus r_0 is the least energy for vertical arrival at the equator, that is, 500 millistörmers.[8] Now

[4] C. Störmer, Zeits. f. Astrophys. **1**, 237 (1930). This paper contains references to his previous work.

[5] A. H. Compton and I. A. Getting, reference 1. The lower limit of the integral (6), according to W. F. G. Swann (Phys. Rev. **51**, 718 (1937)), should be $E_0/(1+\beta \cos\omega_0')$, obtained by applying a Lorentz transformation to the energy E_0 of the particle at infinity. We are unable to agree with this line of reasoning. With respect to the terrestrial observer the limiting energy in the earth's magnetic field is E_0 and the distribution at infinity is anisotropic, as given by the denominator in (6). With respect to the extragalactic observer the limiting energy is not E_0 because the particle moves with respect to him in a combined electric and magnetic field, and its kinetic energy is neither E_0 nor is it conserved, but the distribution of particles with respect to him is of course isotropic. If Swann's suggestion were correct, I', and therefore $\Delta I/I$ (Eq. 15), would be indeterminate by the amount 2β, since the limiting angle ω_0' corresponding to the limiting energy E_0 is infinite because the limiting trajectory described by this particle is asymptotic to the circular (periodic) orbit. We are indebted to Professor G. Lemaitre for illuminating conversations on the correct approach to this problem.

[6] W. F. G. Swann, Phys. Rev. **50**, 1103 (1936); W. Heitler, Proc. Roy. Soc. **A161**, 261 (1937); L. W. Nordheim, Phys. Rev. **51**, 1110 (1937); **53**, 694 (1938).

[7] G. Lemaitre and M. S. Vallarta, Phys. Rev. **43**, 87 (1933).

[8] G. Lemaitre and M. S. Vallarta, Phys. Rev. **50**, 530 (1936), Fig. 10.

according to Oort,[9] β is of the order of 0.001 so that we may neglect, as we have done in (13), squares and higher powers of β. From (12) and (13) we get

$$I'-I=2K^4\int_{r_0}^{\infty}3\beta\cos\omega'\,dr/r^5. \tag{14}$$

Hence the fractional variation in intensity is

$$\frac{\Delta I}{I}=\frac{I'-I}{I}=2\beta\left(6r_0^4\int_{r_0}^{\infty}\cos\omega'\,dr/r^5\right). \tag{15}$$

if the primaries were all positive. With an exponential distribution of primaries $f(E)=e^{-KE}$ instead of (11), and

$$\frac{\Delta I}{I}=6\beta e^{r_0^2}\int_{r_0}^{\infty}re^{-r^2}\cos\omega'\,dr \tag{16}$$

under the assumption that all particles are positive. In the actual computation the integrals (15), (16) were found graphically for several values of ω with $r_0=0.5$.

4.

For a mixture of positive and negative particles, we must treat the two components separately. As we have stated earlier, changing the sign of the particle merely changes the sign of χ. Hence, if we assume the same distribution law for negative as for positive particles, our result obtained for positive particle holds for negative particles if the sign of θ is reversed. This follows from Eq. (7) and the fact that ω' enters only as $\cos\omega'$ in Eqs. (15), (16). Now if we use the subscripts + and − to denote quantities referring to positive and negative components of the primary radiation, respectively, then what we want to find is

$$\Delta(I_++I_-)/(I_++I_-)$$

and we know $\Delta I_+/I_+$ and $\Delta I_-/I_-$. Now if there are n times as many positives as negatives in any energy band, then, since the intensity is proportional to the number of particles

$$\frac{1}{n+1}\left[n\frac{\Delta I_+}{I_+}+\frac{\Delta I_-}{I_-}\right]=\frac{\Delta(I_++I_-)}{(I_++I_-)}=\frac{\Delta I}{I}. \tag{17}$$

[9] J. H. Oort, Bull. Astr. Inst. of the Netherlands 6, 155 (1931).

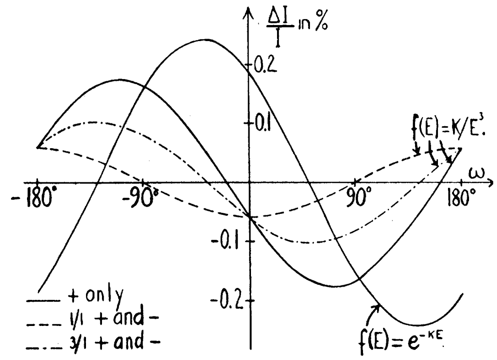

FIG. 4. The galactic diurnal variation of Compton and Getting.

Thus the effect of a mixture of positive and negative primaries is easily found if the ratio of the two components is known.

From Johnson's measurements[10] of the east-west and the north-south asymmetries in Mexico ($\lambda=29°$), the ratio of the positive to the negative primaries in the energy band between 400 and 450 millistörmers has been estimated by Vallarta to be about 3 to 1. It is clear that an established ratio of positives to negatives in any energy interval is not in any way to be interpreted as meaning that the same ratio holds throughout the whole energy spectrum. Assuming, however, that the same ratio holds throughout the primary distribution, we may calculate the variation in the intensity. We simply put $n=3$ in Eq. (17).

5.

Figure 4 gives the result of this calculation. The variation in the vertical intensity is plotted as a function of ω. This is just the sidereal time expressed in angular measure plus a constant which is determined from the fact that the velocity of the earth according to Oort[9] is in the direction 20 hr. 55 min. right ascension and 47° N declination. It is found that the zero of ω corresponds to 20 hr. 40 min. The full curves show the percentage variation for positive particles alone; the amplitude is 0.17 percent and the maximum occurs at 13 hr. 20 min. sidereal time, assuming an inverse cube distribution, and 0.24 percent, in agreement with van Wijk's result,[2] with maximum at 18 hr. sidereal time, assuming

[10] T. H. Johnson, Phys. Rev. 47, 91 (1935); 48, 287 (1935).

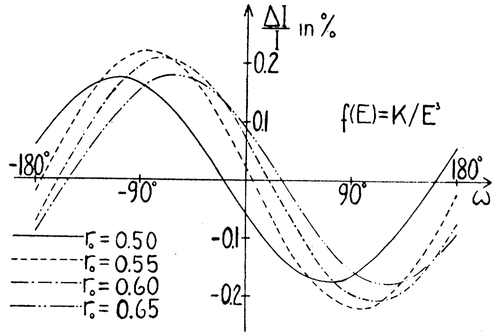

FIG. 5. The galactic diurnal variation for different values of the low energy limit. Inverse cube distribution of positive primaries.

an exponentially decreasing distribution. The other two curves show the expected diurnal variation if there are three positive primaries to each negative, and if there is an equal number of positives and negatives (primary radiation as a whole neutral), assuming in each case an inverse cube distribution. It may be pointed out that if the primary radiation as a whole is neutral, the amplitude of the expected diurnal variation is less than 0.1 percent, for an inverse cube distribution, and consequently its detection, or alternately the proof of its nonexistence, requires a very careful experimental investigation.[11] It is also plain that a knowledge of the experimental diurnal variation as a function of sidereal time would lead to valuable conclusions as to the ratio of positive to negative primaries and as to their distribution law.

It should also be emphasized that, as already indicated by Compton and Getting, the magnitude of the diurnal variation is smaller than would be expected if the deflection of the particles in the earth's magnetic field were neglected, and the phase, except for the case of a neutral primary radiation, in the sense of equal number of positives and negatives, is shifted by a rather large amount. For undeflected particles, the amplitude of the diurnal variation should be 3β or 0.3 percent and the maximum should always occur at 20 hr. 40 min. sidereal time.

The effect of filtering off low energy particles above the magnetic cut-off is exhibited in Fig. 5. It is seen that the effect depends rather considerably as regards both magnitude and phase on the lower limit of the energy if the distribution law goes according to the inverse cube. Such is not the case for an exponentially decreasing distribution, which is rather insensitive to the lower cut-off. The possibility of enhancing the expected effect by filtering off low energy particles, however, should not be overlooked, nor that of obliterating it (for $r_0=0.54$) if the number of positive and negative primaries is the same.

The calculation of the galactic rotation effect for trajectories not lying wholly in the equatorial plane and for latitudes other than the equator is rendered considerably more difficult by the fact that the equations of motion are then no longer integrable. This investigation is being undertaken and the results will be presented in a future paper.

[11] Whether a diurnal variation depending on sidereal time exists or not is a question which does not seem to be decided at the time of writing. Forbush's careful statistical analysis of the intensity measurements made with a Compton-Bennett automatic recording meter at Huancayo, Peru, reported at the University of Chicago's symposium on cosmic rays (June 30, 1938), which are comparable to a good approximation with the theoretical results reported here, would not seem to rule out the existence of an effect such as would be expected for a primary radiation consisting of equal number of positive and negative particles having an inverse cube distribution. The requirement of a neutral radiation, necessary on other grounds, is thus seen to be consistent with present experimental evidence. Forbush's result, however, does seem to rule out an exponentially decreasing distribution of primaries, provided they satisfy the condition of being as a whole neutral. Cf. S. E. Forbush, Phys. Rev. **52**, 1254 (1937).