Excited State of He^3

Recent experiments of Bonner[1] indicate the existence of a bound excited state of He^3 1.9 Mev above the ground state. Since this is probably a P state, it involves interactions that are not determined by the normal state energies of the lightest nuclei or by existing data on scattering. It is therefore of considerable interest to see what information concerning the P interactions between like and unlike particles is implied by the existence of a P state of He^3 about 1.9 Mev above the ground state. It would seem *a priori* that there should be no excited state for a much larger distance above the ground state, since the saturation property of nuclear forces requires the interaction between two particles in an antisymmetric state to contribute much less to the binding than the interaction in a symmetric state.

We shall try here to make this argument somewhat sharper by using the variational method to compute the energies of the normal state and the lowest group of excited states. We assume for the moment that many-body and spin-orbit forces are negligible. The interactions are then:

$$V_{\nu\pi}(r) = [(M+H\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')P^M + W + H\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}']J(r),$$
$$V_{\pi\pi}(r) = (W'+B'\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')J(r) + e^2/r, \qquad J(r) = \exp(-\gamma r^2),$$

where the $\boldsymbol{\sigma}$'s have unit amplitude; with $\gamma^{-\frac{1}{2}} = 2.25\times 10^{-13}$ cm, the conditions $M+W=-32.0$, $H+B=-3.5$, $W'-3B'=-21.4$, all in Mev, insure agreement with data on the deuteron and on scattering.[2] As variational wave-functions we use:

$$\psi_0 = (4\alpha\beta/\pi^2)^{3/4} \exp(-\alpha R^2 - \beta r^2)\cdot {}^2\Sigma_-,$$
$$\psi_1 = 2\beta^{\frac{1}{2}} x (4\alpha\beta/\pi^2)^{3/4} \exp(-\alpha R^2 - \beta r^2)\cdot {}^4\Sigma_+,$$
$$\psi_2 = 2\beta^{\frac{1}{2}} x (4\alpha\beta/\pi^2)^{3/4} \exp(-\alpha R^2 - \beta r^2)\cdot {}^2\Sigma_+,$$
$$\psi_3 = 2\alpha^{\frac{1}{2}} X (4\alpha\beta/\pi^2)^{3/4} \exp(-\alpha R^2 - \beta r^2)\cdot {}^2\Sigma_-,$$

where x is a cartesian component of $\mathbf{r}$, which joins the two protons, X is a cartesian component of $\mathbf{R}$, which joins the center of mass of the two protons with the neutron, and the Σ's are appropriate spin functions. ψ_0 is the even ${}^2S$ ground state, ψ_1 an odd ${}^4P$ excited state, and ψ_2 and ψ_3 odd ${}^2P$ excited states. We treat the Coulomb energy as a perturbation throughout. The minimization of ψ_0 gives -5.7 for the ground state energy, as compared with the experimental value -7.6; this result is insensitive to the values of M, H, and W', provided that $M+W=-32.0$, etc. The calculation of the energy of ψ_1 and the lower root of the second order secular equation, obtained from ψ_2 and ψ_3, and by the use of the usual saturating forces ($W=W'=0$, B and H both of the order of -3.5), shows that these quantities have poorly defined minima and are never negative; for reasonable values of α and β they are both greater than $+5$. Now since the excited state indicated by experiment is well bound (3.5 Mev below the continuum of deuteron plus proton) and since the variational method gives good results for the ground state, we may hope that the same method with similar wave functions will work reasonably well for the excited state. Therefore it seems likely that the usual choice of force constants, which leads to a lowest excited state far up in the continuum, is incompatible with the existence of Bonner's state.[3]

In order to show to what extremes it is necessary to go if one wishes to obtain an excited state at anywhere near the indicated place, the variational calculation was carried through for several sets of force constants, subject to the restrictions $M+W=-32.0$, etc. The constants: $M=+18.0$, $W=-50.0$, $H=-3.5$, $B=0.0$, $W'=-53.5$, $B'=-10.7$, satisfy these restrictions and give two bound excited states: the ${}^4P$ at -3.7 (2.0 Mev above the computed ground state), and the lower ${}^2P$ at -2.0 (3.7 Mev above the computed ground state); the other ${}^2P$ state is far up in the continuum. Any significant decrease of $-W$ or $-W'$ would raise both excited levels considerably. These force constants just avoid binding the ${}^1P$ deuteron, and are not far from binding the ${}^3P$ deuteron and ${}^3P$ He^2. Since they are in flagrant contradiction with the saturation requirement and would imply inacceptably large binding energies for nuclei such as He^5 and Li^6, we cannot regard them as being correct.

We have also investigated the possibility of accounting for the excited state in terms of a spin-orbit force of the form $(\boldsymbol{\sigma}\cdot\mathbf{r})(\boldsymbol{\sigma}'\cdot\mathbf{r})J(r)/r^2$. (An interaction of this general form and of magnitude comparable to that of the $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')$ interaction is suggested by dynaton theories of nuclear forces.[4]) However, it turns out that if saturating forces are used, the spin-orbit part of the interaction must be very much larger than the spin-spin part if the excited state is to be properly placed. This is contrary to the usual assumption that nuclear spin-orbit forces are small. We can thus conclude that an explanation of the position of Bonner's state would require the existence of strong attractive interactions which do not manifest themselves in the normal states of the lightest nuclei or in scattering experiments, and which entail far-reaching modifications of the usual ideas concerning nuclear forces.

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[1] Bonner, Phys. Rev. **53**, 711 (1938).
[2] Margenau and Warren, Phys. Rev. **52**, 790 (1937).
[3] These results do not seem to be sensitive to the breadth of the interaction.
[4] Yukawa, Sakata and Taketani, Proc. Phys.-Math. Soc. Japan **20**, 319 (1938); Frohlich, Heitler and Kemmer, Proc. Roy. Soc. **166**, 154 (1938).