On Thermal Dependence of Elasticity in Solids

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A theoretical study suggests a distinction between the elasticity coefficients for very high frequencies (of the elastic waves representing thermal agitation) and for low frequencies. These last coefficients should show a rapid decrease of rigidity when temperature increases, and the melting point can be said to represent the point at which the macroscopic rigidity becomes zero.

LAST year, while writing a book[1] on elasticity, in explaining the role of thermal agitation in solids, I tried to give a logical and systematic deduction of the theory of specific heat and thermal expansion of solids which obliged me to make some changes and improvements in the generally admitted theoretical scheme.

This brought me once more to the problem: How does temperature modify the elasticity coefficients of a solid body? This problem had troubled me for a long time and I had always felt some link was missing in the reasoning. In the many papers, books, and handbook articles on the subject, everyone, including myself, had started with the study of thermal agitation in an isotropic solid, thus analyzing thermal motion into elastic waves of the two classical longitudinal and transverse types. It was then assumed that the resulting formulas which had been developed for an isotropic solid were valid also for a solid distorted by stresses of any sort. A general thermodynamic theory of solids was built on this basis. This method is wrong because from a study strictly limited to isotropic solids one is not entitled to draw conclusions about what happens when the solid becomes anisotropic as a result of the action of external unsymmetrical forces.

In my book[1] (p. 344, 345) I merely noted this point and have only recently been able to put these general remarks into a more precise form. The necessary, though tiresome, calculations which I have performed entirely justify my point of view. Let us take an isotropic solid body, stretch it along the x direction by applying X forces; its elastic properties along the x direction now differ from those along the y and z direction. The solid has taken the symmetry of a uniaxial crystal. Calculation checks this prediction and shows that elastic waves are propagated in such a stretched solid body in a very peculiar way. There are no longer longitudinal and transverse waves but, for each direction of propagation, there are three different orthogonal polarizations, one of which is nearly longitudinal, another exactly transverse, and the third perpendicular to the first two and nearly transverse. To these three independent polarizations there correspond three different elastic wave velocities.

With this rigorous analysis of thermal agitation in a strained solid body it is possible to develop the whole thermodynamics of solids and to draw conclusions about the influence of temperature on elasticity coefficients. The general results are as follows.

(1) The elasticity coefficients which rule the propagation of hypersonic waves (as Raman calls them) are given by the derivatives of the elastic potential energy and will be affected only in an indirect way by the thermal expansion. They should accordingly show a slow decrease with increasing temperature.

(2) The macroscopic elasticity coefficients as measured in the propagation of acoustical or suprasonic waves are to be derived from the free energy instead of from the purely potential energy. These coefficients are directly influenced by thermal agitation. It is rather difficult to predict the variation of the λ-coefficient but the rigidity coefficient μ should show a very peculiar decrease with increasing temperature. This decrease, while slow at low temperatures, should become faster and faster as the temperature increases. For room temperature and for solids not too near their melting points the decrease is of the order of 50 to 100 times RT/V.

[1] L. Brillouin, *Les tenseurs en mécanique et en élasticité* (Masson, Paris, 1937), Chapter 12.

As the melting point is approached we may guess by extrapolation that the macroscopic rigidity will tend to zero while the microscopic rigidity remains finite. While melting, the body is unable to react to shear (on a macroscopic scale) but will still be able to propagate hypersonic transverse waves. Its specific heat, therefore, will remain about $3R$ just as for a solid body, and this is what really happens.

I wish to recall here an old paper of Sutherland[2] where the curious curve reproduced in

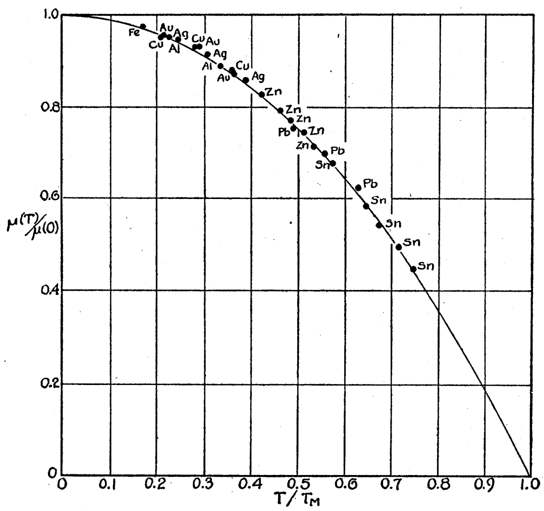

FIG. 1. Sutherland's curve (reference 2) showing the variation of $\mu(T)/\mu(O)$ with T/T_M. The curve is a portion of the parabola $y=1-x^2$.

Fig. 1 is to be found. In this curve the ordinates represent the ratio of $\mu(T)$ to $\mu(O)$ while the abscissas are the ratios of absolute temperature to melting point so that $x=1$ is the melting point. My father repeatedly emphasized that the melting point should be defined as the point at which the rigidity of a solid becomes zero. The preceding deductions seem to bring strong support to this prediction. It is, of course, difficult to imagine that the Sutherland curve should run smoothly up to the melting point since this would mean melting without latent heat of fusion. One must bear in mind, however, that the actual latent heats of fusion are always small compared to latent heats of vaporization. It is generally agreed that the liquid state is often much nearer to the solid state than to the gaseous state. Moreover, it must be remembered that isotropic solids are but a fiction which has been introduced only for simplification because the complete theory of elasticity would be too complicated in the case of crystals. We might guess that for a crystal the different rigidity coefficients would not tend all to zero at the same temperature. When one of these rigidity coefficients becomes zero the whole stability of the crystal is destroyed and this may involve a certain heat of transformation at the melting point. The complete study, a brief account of which is here given, will appear in the *Memorial des sciences mathematiques* (Gauthier-Villars, Paris) and is now in process of publication.

These theoretical remarks suggest experimental research on solids, and first of all, a systematic study of elasticity coefficients as functions of frequency and temperature. The temperature should be varied from very low temperatures up to the melting point. A connection should be sought with the elastic properties of the liquid above the melting point. Such measurements could be carried on by ultrasonic methods as already used by Balamuth, Rose, and Durand[3] and by Goens.[4] In a recent booklet[5] I have discussed the importance of these new methods of measurement which may lead to a better understanding of the properties of solids and the mechanism of fusion.

[2] Sutherland, Phil. Mag. **32**, 42 (1891).

[3] L. Balamuth, Phys. Rev. **45**, 715 (1934); **46**, 933 (1934). F. C. Rose, Phys. Rev. **49**, 50 (1936). M. A. Durand, Phys. Rev. **50**, 449 (1936).

[4] Goens, Physik. Zeits. **37**, 321 (1936).

[5] L. Brillouin, *La structure des corps solides* (A. S. I. No. 549, Hermann, Paris, 1937).